Principles of Complex Systems, CSYS/MATH 300—Assignment 3
University of Vermont, Fall 2010

Dispersed: Monday, October 4, 2010.
Due: By start of lecture, 1:00 pm, Thursday, October 14, 2010.
Some useful reminders:
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Course website: `http://www.uvm.edu/~pdodds/teaching/courses/2010-08UVM-300`

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

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1. Consider a network with a degree distribution that obeys a power law and is otherwise random.

   Assume that the network is drawn from an ensemble of networks which have N nodes whose degrees are drawn from the probability distribution $P_k = ck^{-\gamma}$ where $k \geq 1$ and $2 < \gamma < 3$.

   (a) Estimate $\min k_{\max}$, the approximate minimum of the largest degree in the network, finding how it depends on N. (Hint: we expect on the order of 1 of the N nodes to have a degree of $\min k_{\max}$ or greater.)

   (b) Determine the average degree of nodes with degree $k \geq \min k_{\max}$ to find how the expected value of $k_{\max}$ scales with N.

2. Determine the clustering coefficient for toy model small-world networks [2] as a function of the rewiring probability p. Find C_1, the average local clustering coefficient:

$$C_1(p) = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2} \right\rangle_i = \frac{1}{N} \sum_{i=1}^{N} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2}$$

   where N is the number of nodes, $a_{ij} = 1$ if nodes i and j are connected, and $\mathcal{N}_i$ indicates the neighborhood of i.

   As per the original model, assume a ring network with each node connected to a fixed, even number m local neighbors ($m/2$ on each side). Take the number of nodes to be $N \gg m$.

Start by finding $C_1(0)$ and argue for a $(1-p)^3$ correction factor to find an approximation of $C_1(p)$.

Hint 1: you can think of finding C_1 as averaging over the possibilities for a single node.

Hint 2: assume that the degree of individual nodes does not change with rewiring but rather stays fixed at m. In other words, take the average degree of individuals as the degree of a randomly selected individual.

For what value of p is $C_1 \simeq 1/2$?

3. Use a scaling argument to show that maximal rowing speed V increases as the number of oarspeople n as $V \propto N^{1/9}$.

Assume the following:

(a) Rowing shells are geometrically similar (isometric). The table below taken from McMahon and Bonner [1] shows that shell width is roughly proportional to shell length ℓ.

Shell dimensions and performances.

No. of oarsmen	Modifying description	Length, l (m)	Beam, b (m)	l/b	Boat mass per oarsman (kg)	Time for 2000 m (min) I	II	III	IV
8	Heavyweight	18.28	0.610	30.0	14.7	5.87	5.92	5.82	5.73
8	Lightweight	18.28	0.598	30.6	14.7				
4	With coxswain	12.80	0.574	22.3	18.1				
4	Without coxswain	11.75	0.574	21.0	18.1	6.33	6.42	6.48	6.13
2	Double scull	9.76	0.381	25.6	13.6				
2	Pair-oared shell	9.76	0.356	27.4	13.6	6.87	6.92	6.95	6.77
1	Single scull	7.93	0.293	27.0	16.3	7.16	7.25	7.28	7.17

(b) The resistance encountered by a shell is due largely to drag on its wetted surface.

(c) Drag is proportional to the product of the square of the shell's speed (V^2) and the area of the wetted surface ($\propto \ell^2$ due to the shell isometry).

(d) Power $\propto$ drag force $\times$ speed (in symbols: $P \propto D_f \times V$).

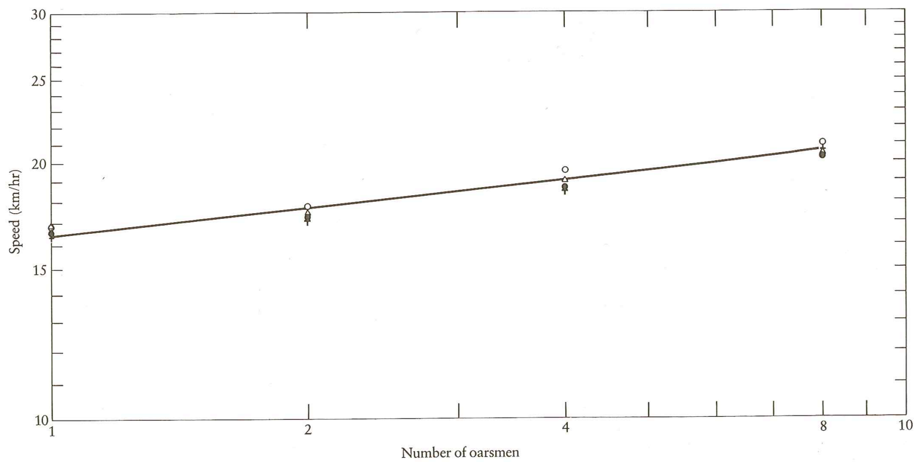
(e) Volume displacement of water by a shell is proportional to the number of oarspeople N (i.e., the team's combined weight).

(f) Assume the depth of water displacement by the shell grows isometrically with boat length ℓ.

(g) Power is proportional to the number of oarspeople N.

2 pt Bonus: find the modern day world record times for 2000 metre races and see if this scaling still holds up. Of course, our relationship is approximate as we have

neglected numerous factors, the range is extremely small (1–8 oarspeople), and the scaling is very weak (1/9). But see what you can find. The figure below shows data from McMahon and Bonner.



References

[1] T. A. McMahon and J. T. Bonner. *On Size and Life*. Scientific American Library, New York, 1983.

[2] D. J. Watts. *Small Worlds : The Dynamics of Networks Between Order and Randomness*. Princeton Studies in Complexity. Princeton University Press, Princeton, 1999.