# Applications of Random Networks

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# Outline

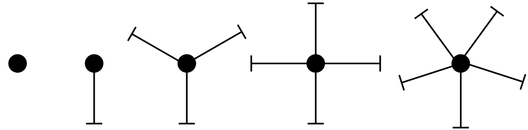
# More on building random networks

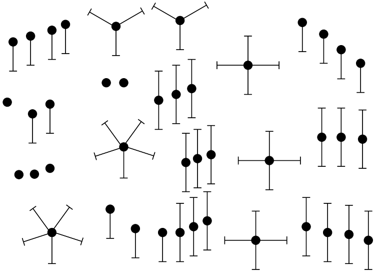
- Problem: How much of a real network's structure is non-random?
- Key elephant in the room: the degree distribution $P_k$.
- First observe departure of $P_k$ from a Poisson distribution.
- Next: measure the departure of a real network with a degree frequency $N_k$ from a random network with the same degree frequency.
- Degree frequency $N_k$ = observed frequency of degrees for a real network.
- What we now need to do: Create an ensemble of random networks with degree frequency $N_k$ and then compare.

# Building random networks: Stubs

## Phase 1:

- Idea: start with a soup of unconnected nodes with stubs (half-edges):



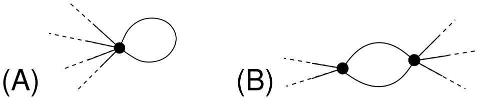


- Randomly select stubs (not nodes!) and connect them.
- Must have an even number of stubs.
- Initially allow self- and repeat connections.

# Building random networks: First rewiring

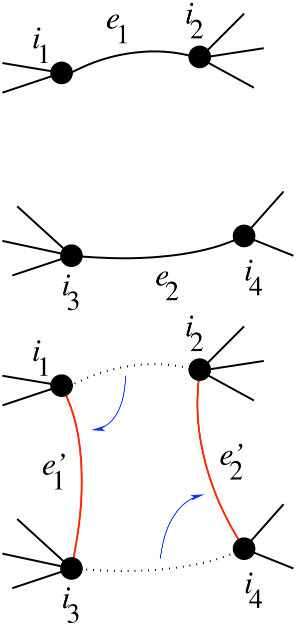
Phase 2:

- Now find any (A) self-loops and (B) repeat edges and randomly rewire them.




- Being careful: we can't change the degree of any node, so we can't simply move links around.
- Simplest solution: randomly rewire two edges at a time.

# General random rewiring algorithm




- Randomly choose two edges. (Or choose problem edge and a random edge)
- Check to make sure edges are disjoint.
- Rewire one end of each edge.
- Node degrees do not change.
- Works if $e_1$ is a self-loop or repeated edge.
- Same as finding on/off/on/off 4-cycles. and rotating them.

# Sampling random networks



## Phase 2:

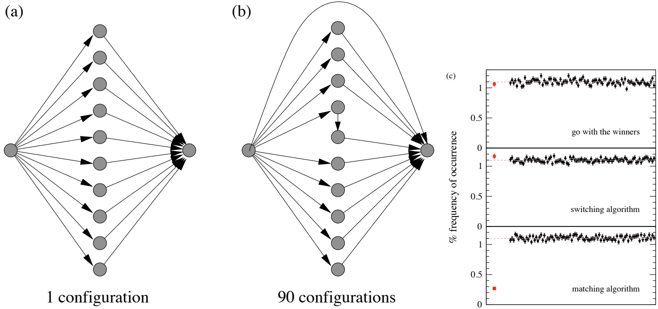
- Use rewiring algorithm to remove all self and repeat loops.

## Phase 3:

- Randomize network wiring by applying rewiring algorithm liberally.
- Rule of thumb: # Rewirings $\simeq 10 \times$ # edges [1].

# Random sampling

- Problem with only joining up stubs is failure to randomly sample from all possible networks.
- Example from Milo et al. (2003) [1]:



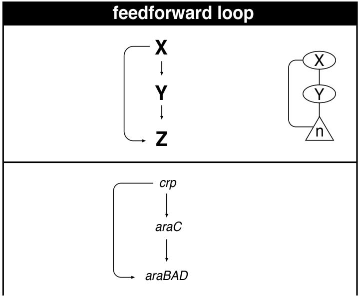
# Sampling random networks

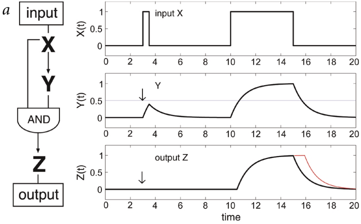
- What if we have $P_k$ instead of $N_k$?
- Must now create nodes before start of the construction algorithm.
- Generate $N$ nodes by sampling from degree distribution $P_k$.
- Easy to do exactly numerically since $k$ is discrete.
- Note: not all $P_k$ will always give nodes that can be wired together.

# Network motifs

- Idea of motifs [2] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.
- Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).
- Used network randomization to produce ensemble of alternate networks with same degree frequency $N_k$.
- Looked for certain subnetworks (motifs) that appeared more or less often than expected

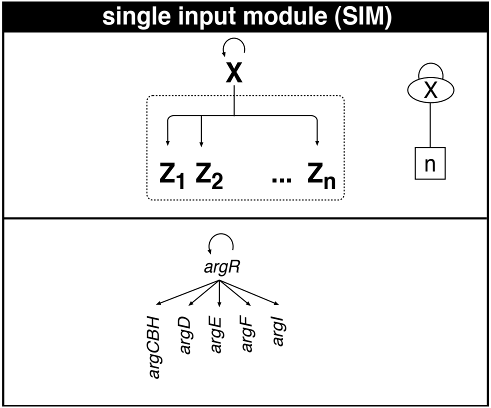
# Network motifs



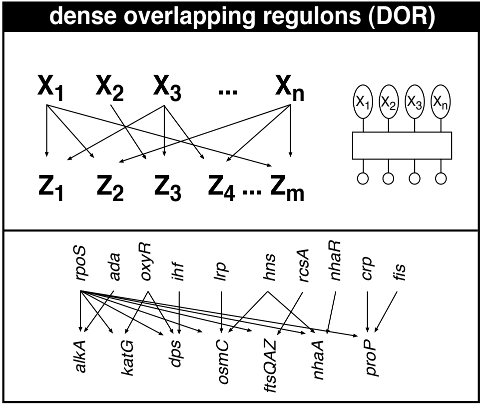

- $Z$ only turns on in response to sustained activity in $X$.
- Turning off $X$ rapidly turns off $Z$.
- Analogy to elevator doors.

# Network motifs




- Master switch.

# Network motifs



# Network motifs

- Note: selection of motifs to test is reasonable but nevertheless ad-hoc.
- For more, see work carried out by Wiggins et al. at Columbia.

# References I

[1] R. Milo, N. Kashtan, S. Itzkovitz, M. E. J. Newman, and U. Alon.
On the uniform generation of random graphs with prescribed degree sequences, 2003. pdf (⊞)

[2] S. S. Shen-Orr, R. Milo, S. Mangan, and U. Alon.
Network motifs in the transcriptional regulation network of *Escherichia coli*.
*Nature Genetics*, pages 64–68, 2002. pdf (⊞)