Overview of Complex Networks

Complex Networks, CSYS/MATH 303, Spring, 2010

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Department of Mathematics & Statistics
Center for Complex Systems
Vermont Advanced Computing Center
University of Vermont

Outline

Class Admin

- Office hours:
  - Tuesday 1:00 pm–2:30 pm (Farrell Hall)
  - Appointments by email.
- Course outline
- Projects

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- Three versions (all in pdf):
  1. Presentation,
  2. Flat Presentation,
  3. Handout (2x2).
- Presentation versions are navigable and hyperlinks are clickable.
- Web links look like this (⊞).
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Basic definitions

Complex System—Some ingredients:

- Distributed system of many interrelated parts
- No centralized control
- Nonlinear relationships
- Existence of feedback loops
- Complex systems are open (out of equilibrium)
- Presence of Memory
- Modular (nested)/multiscale structure
- Opaque boundaries
- Emergence—'More is Different' [1]
- Many phenomena can be complex: social, technical, informational, geophysical, meteorological, fluidic, ...

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Basic definitions

Complex: (Latin = with + fold/weave (com + plex))

Adjective

- Made up of multiple parts; intricate or detailed.
- Not simple or straightforward.

net•work |ˈnetˌwərk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

- a complex system of roads, railroads, or other transportation routes *: a network of railroads.*

2 a group or system of interconnected people or things *: a trade network.*

- a group of people who exchange information, contacts, and experience for professional or social purposes *: a support network.*
- a group of broadcasting stations that connect for the simultaneous broadcast of a program *: the introduction of a second TV network* | [as adj.] *network television.*
- a number of interconnected computers, machines, or operations *: specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
- a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network *: the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career *: the skills of networking, bargaining, and negotiation.*

Thesaurus deliciousness:

network

noun

1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.

2 *a network of lanes* MAZE, labyrinth, warren, tangle.

3 *a network of friends* SYSTEM, complex, nexus, web, webwork.

Ancestry:

From Keith Briggs's excellent etymological investigation: (⊞)

- Opus reticulatum:
- A Latin origin?

[http://serialconsign.com/2007/11/we-put-net-network]

Ancestry:

First known use: Geneva Bible, 1560

‘And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).’

From the OED via Briggs:

- 1658–: reticulate structures in animals
- 1839–: rivers and canals
- 1869–: railways
- 1883–: distribution network of electrical cables
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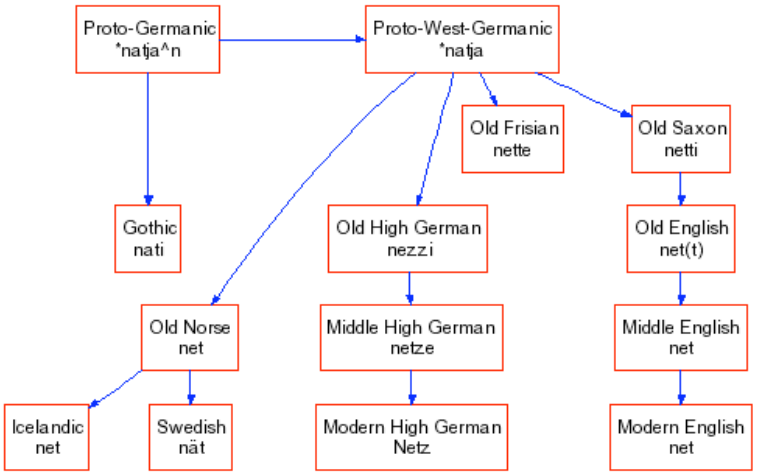
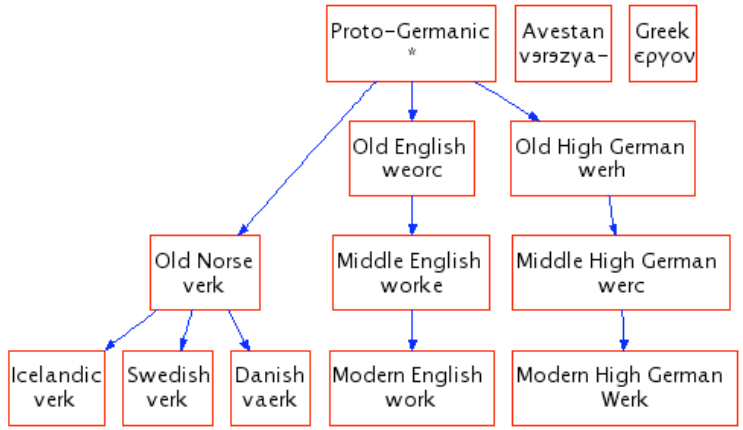
Ancestry:

Net and Work are venerable old words:

- ‘Net’ first used to mean spider web (King Ælfréd, 888).
- ‘Work’ appears to have long meant purposeful action.



The network of Germanic `net' words



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- ‘Network’ = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

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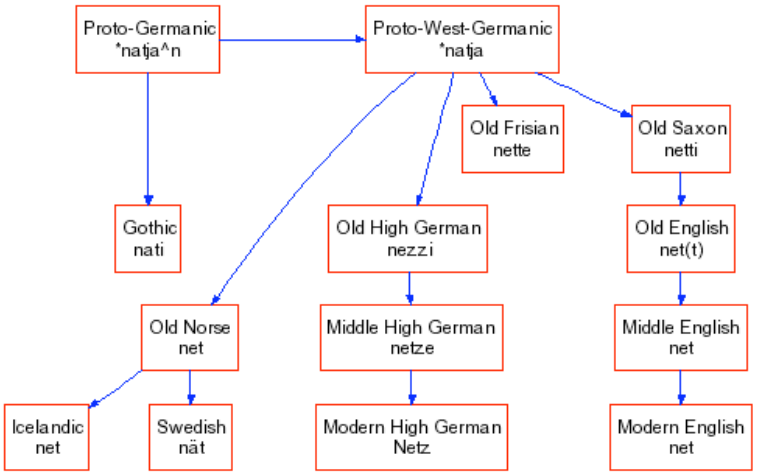

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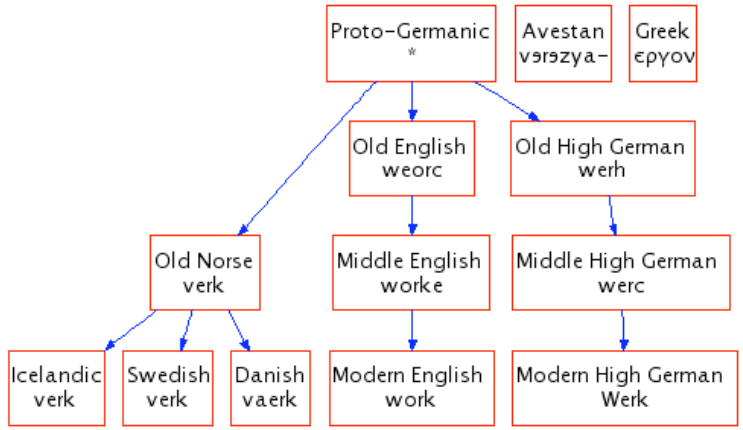

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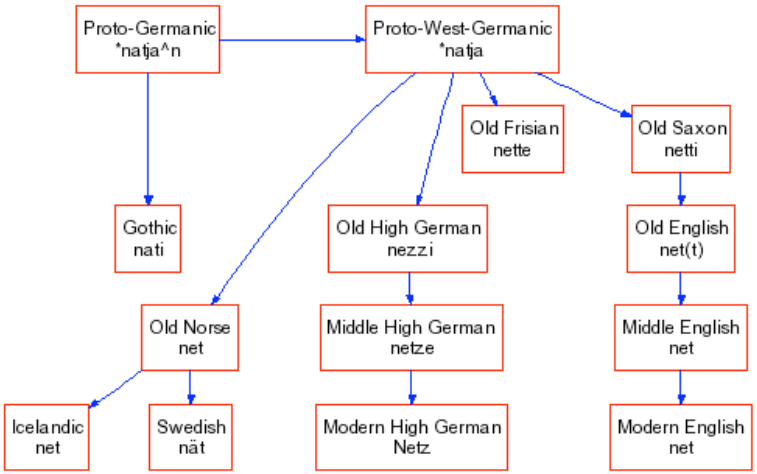

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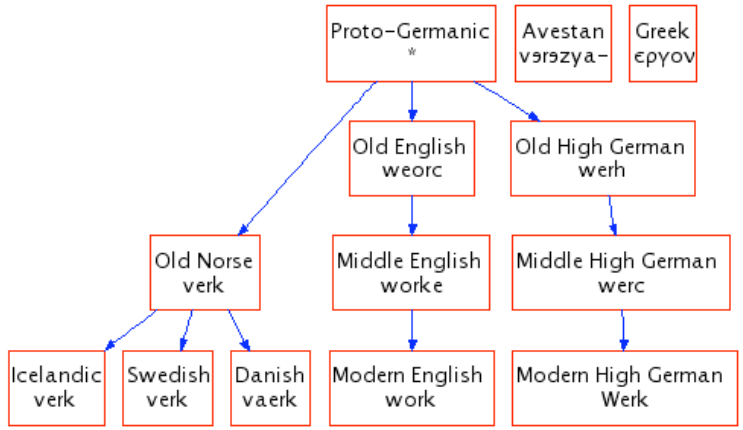

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- Many complex systems
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  of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.
- Mindboggling amount of work published on complex networks since 1998...
- ... largely due to your typical theoretical physicist:

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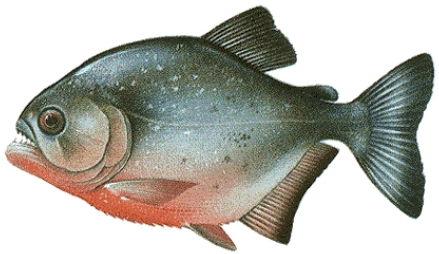

- *Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas
  (see chaos, cellular automata, ...)

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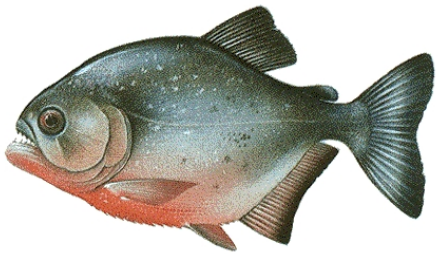

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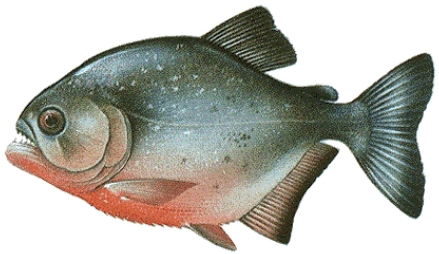

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Popularity (according to ISI)

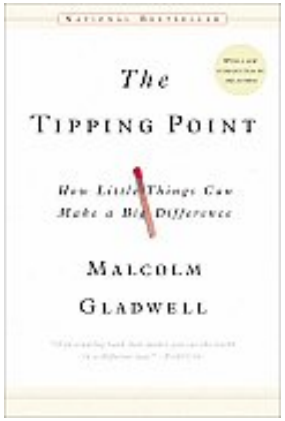
“Collective dynamics of ‘small-world’ networks” [21]

- Watts and Strogatz
  Nature, 1998
- $\approx$ 4100 citations (as of January 18, 2010)
- Over 1100 citations in 2008 alone.

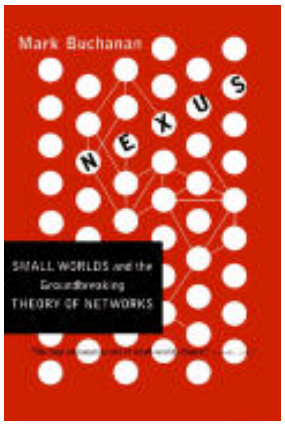
“Emergence of scaling in random networks” [2]

- Barabási and Albert
  Science, 1999
- $\approx$ 4400 citations (as of January 18, 2010)
- Over 1100 citations in 2008 alone.

Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [9]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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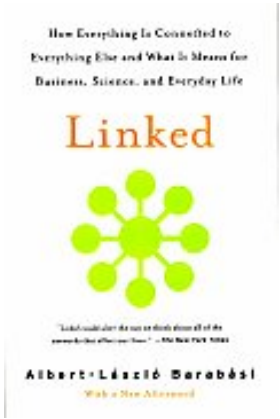
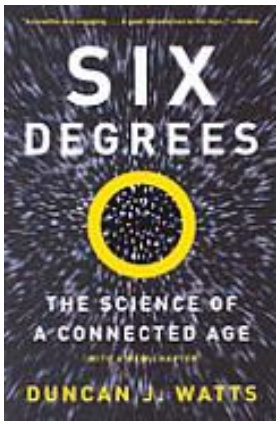


Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [19]

Numerous others:

- Complex Social Networks—F. Vega-Redondo [18]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [15]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [5]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [8]

More observations

- But surely networks aren't new...
- Graph theory is well established...
- Study of social networks started in the 1930's...
- So why all this 'new' research on networks?
- Answer: Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- Real networks occupy a tiny, low entropy part of all network space and require specific attention.
- A worthy goal: establish mechanistic explanations.
- What kinds of dynamics lead to these real networks?

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* *If this is upsetting, maybe string theory is for you...*

More observations

- Web-scale data sets can be overly exciting.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) (⊞)
- "The Unreasonable Effectiveness of Data," Halevy et al. [10]
- c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [22]

But:

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Super basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

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  - real and dynamic (airline routes),
  - abstract with physical impact (hyperlinks),
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- Links may be directed or undirected.
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Node degree = Number of links per node

- Notation: Node i's degree = k_i.
- $k_i = 0,1,2,\ldots$.
- Notation: the average degree of a network = $\langle k \rangle$

- For undirected networks, connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}$$

- For directed networks,

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Adjacency matrix:

- We represent a graph or network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).
- e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

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- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids

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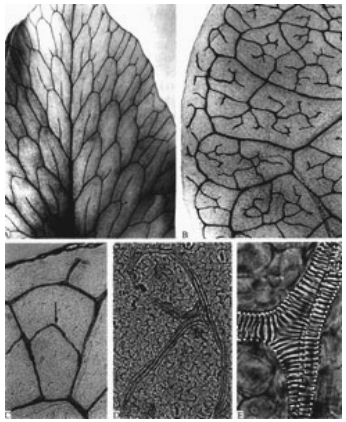

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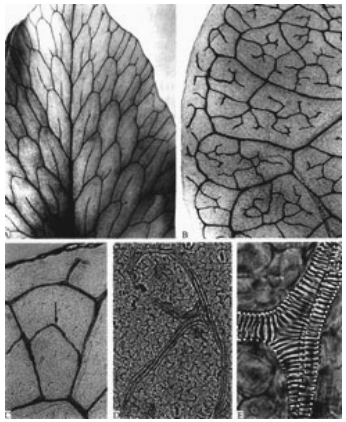

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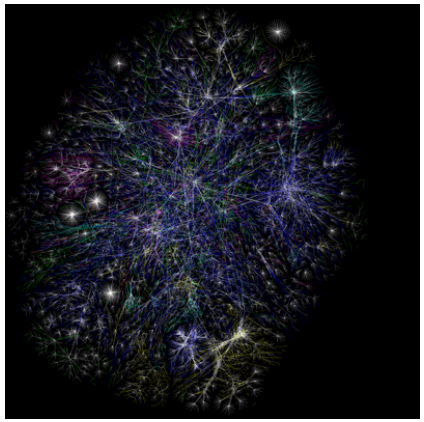

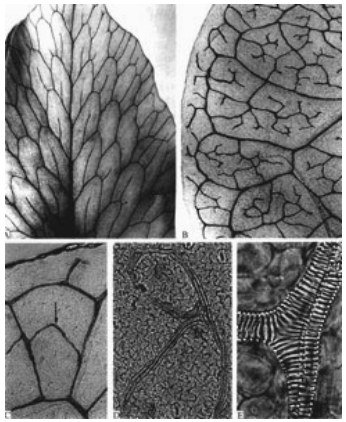

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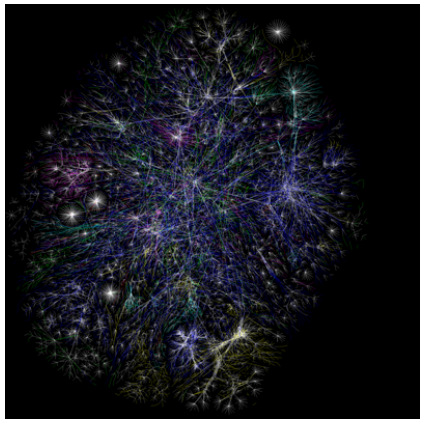

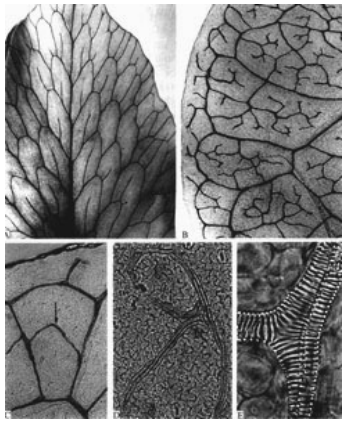

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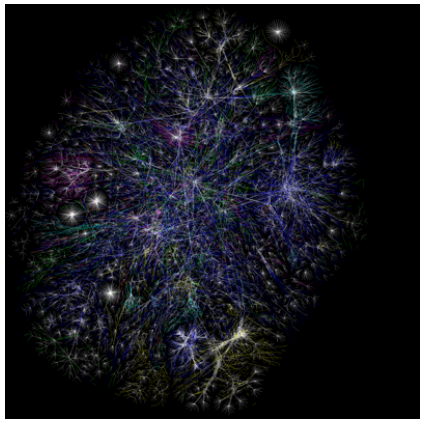

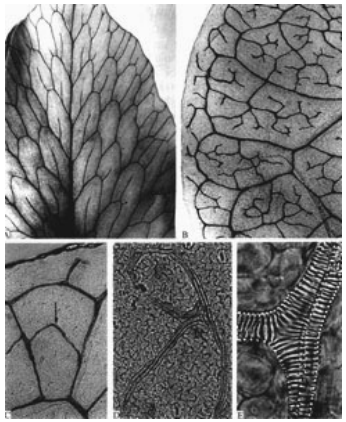

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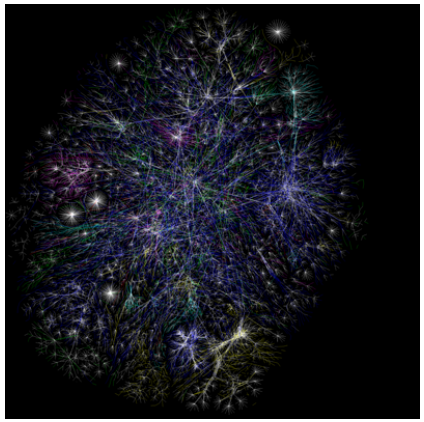

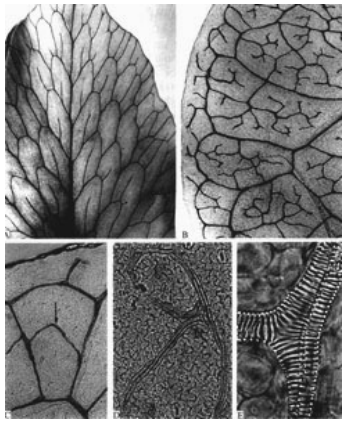

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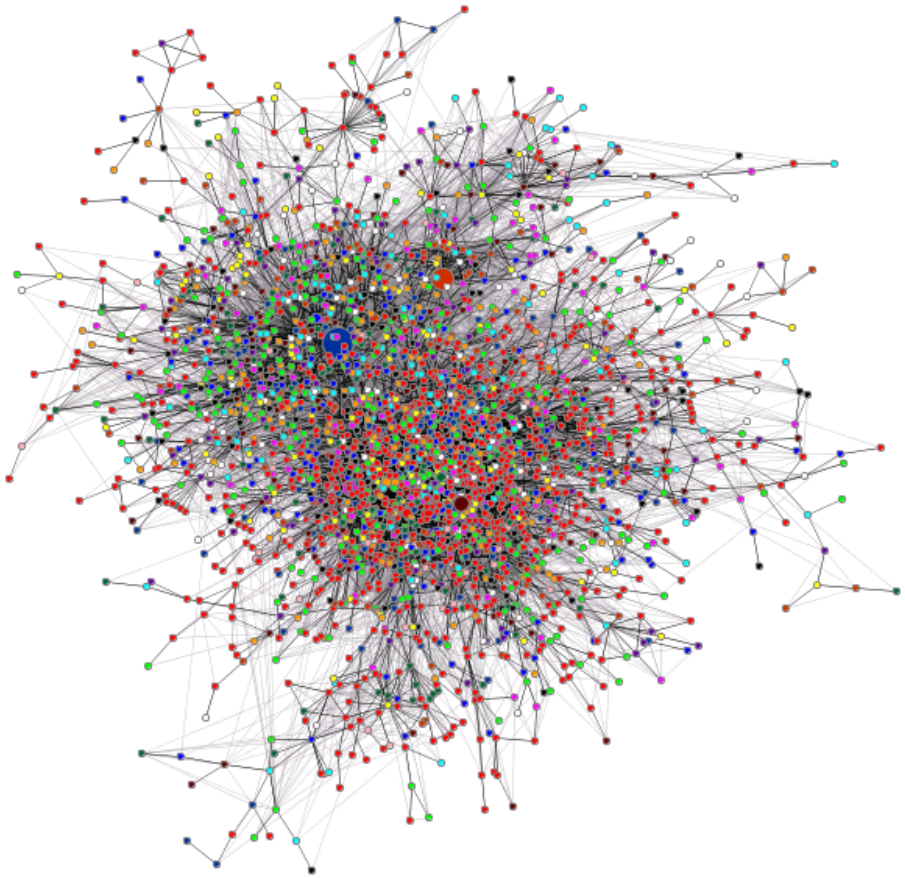

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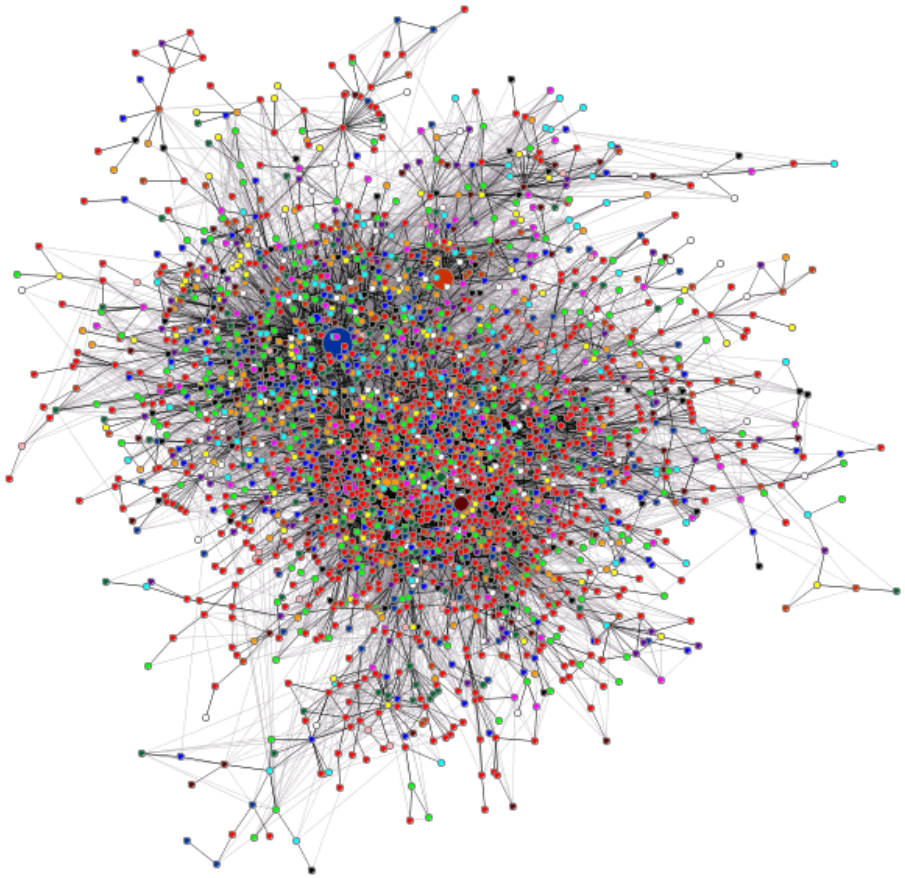

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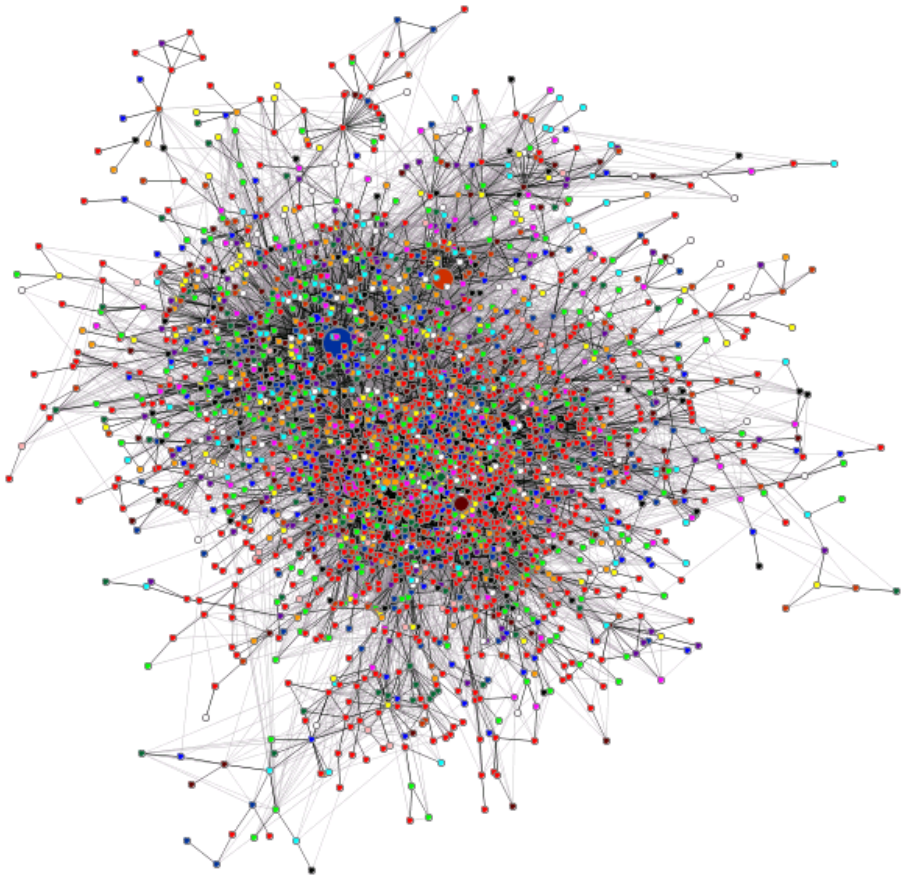

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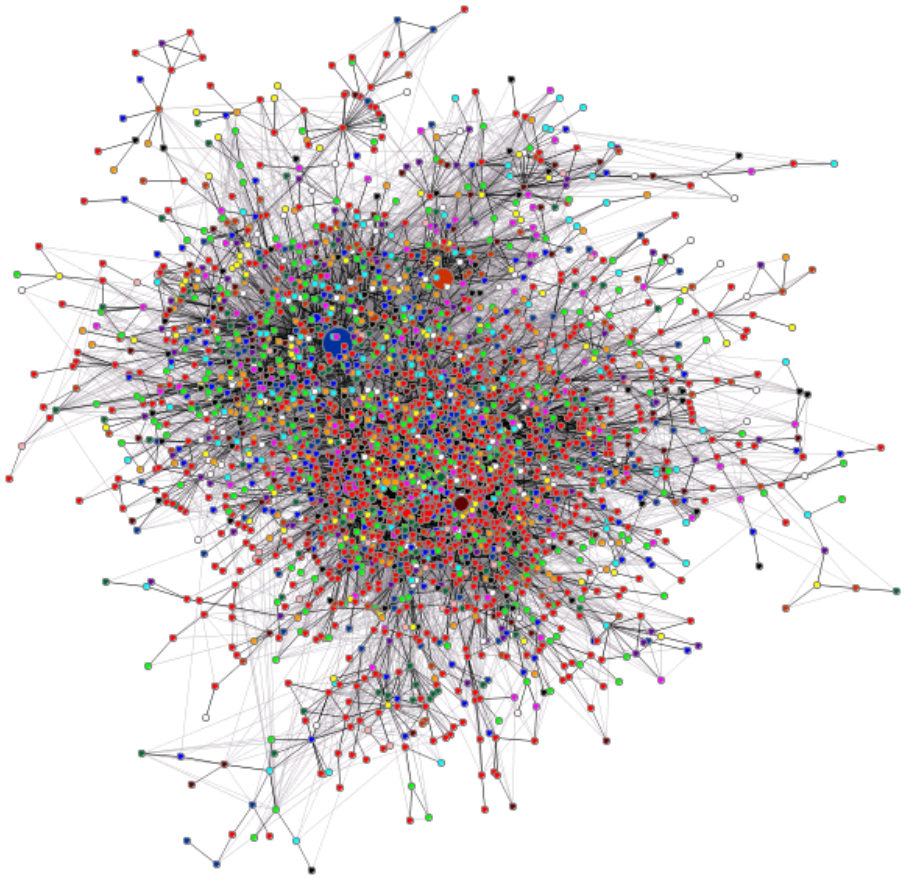

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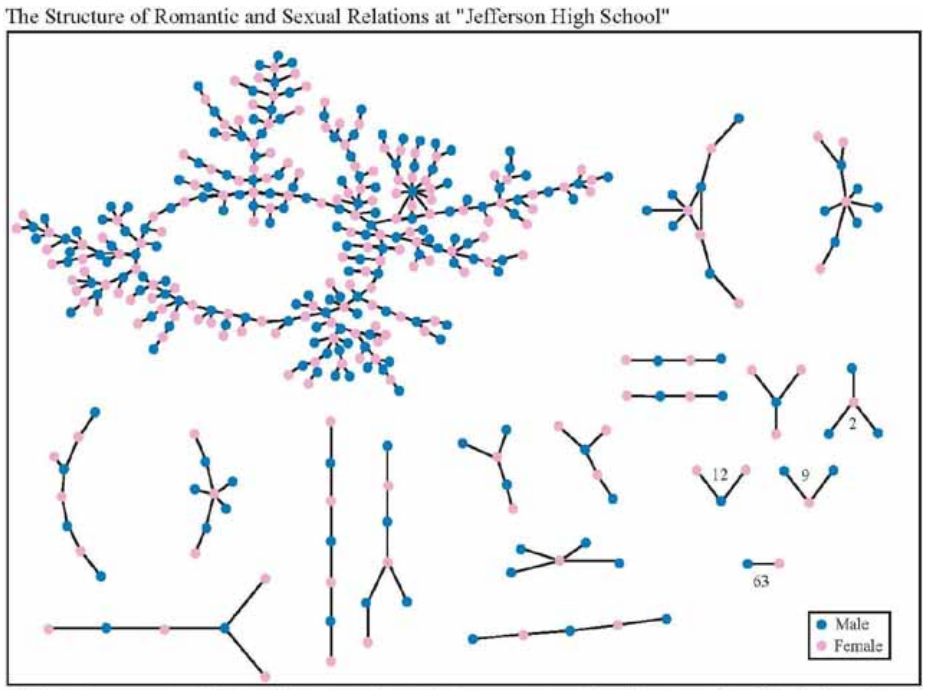
- Boards and directors
- Organizations
- myspace.com (⊞), facebook.com (⊞)



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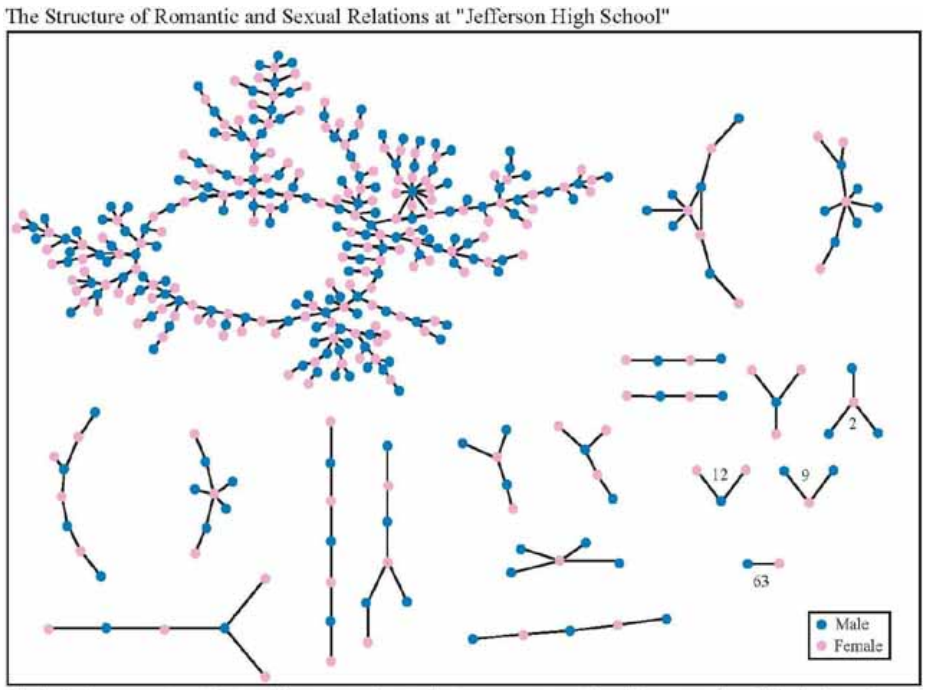

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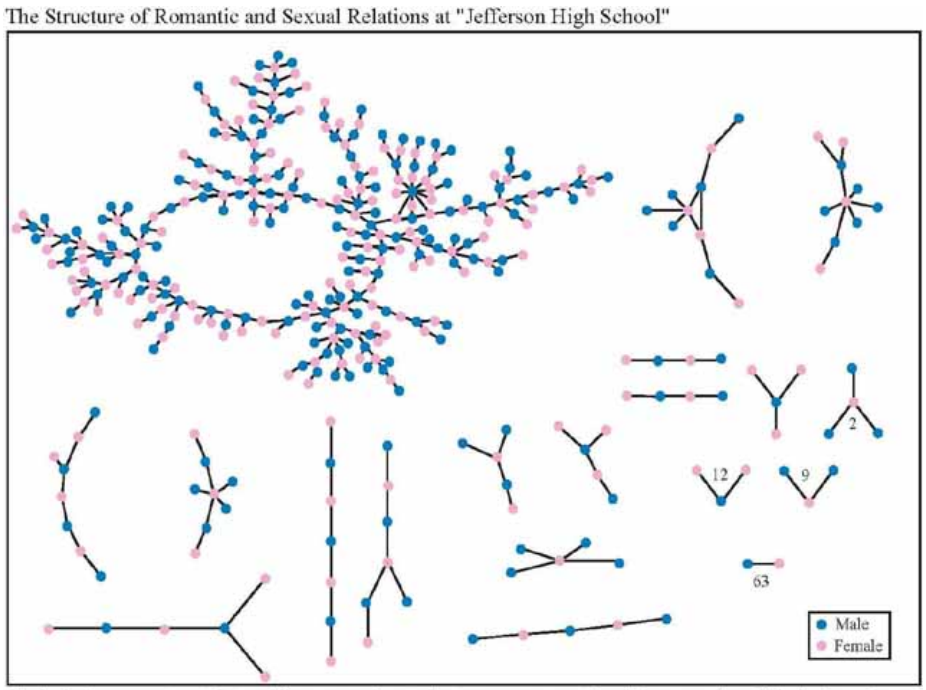

Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

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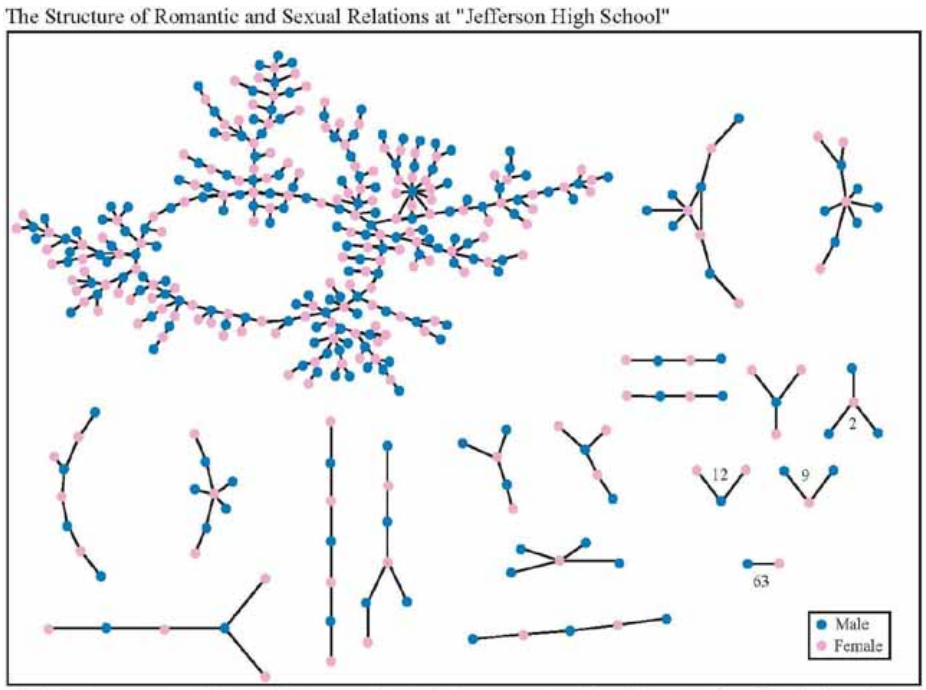

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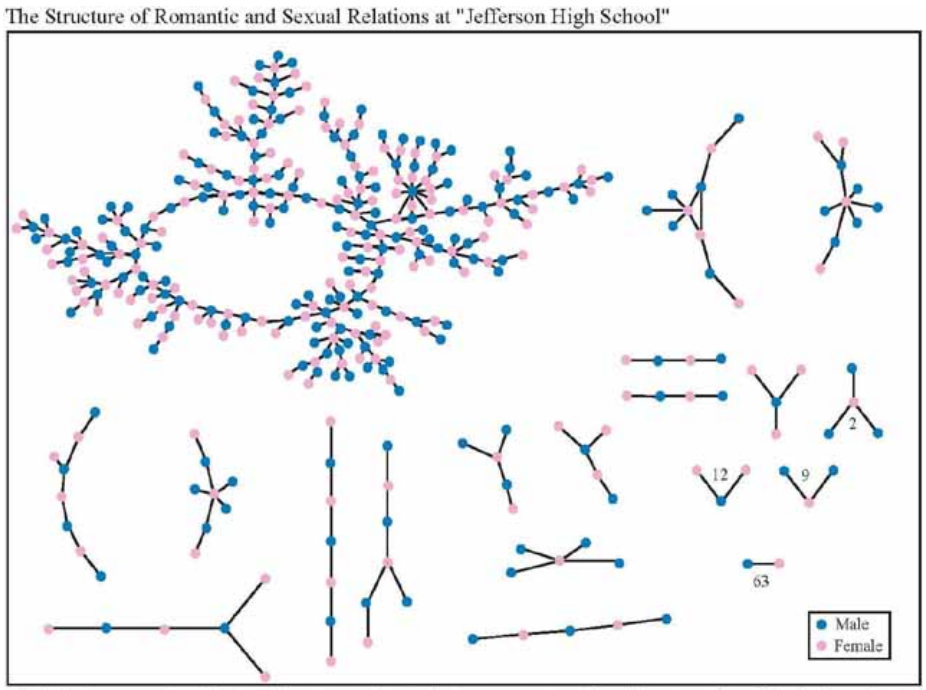

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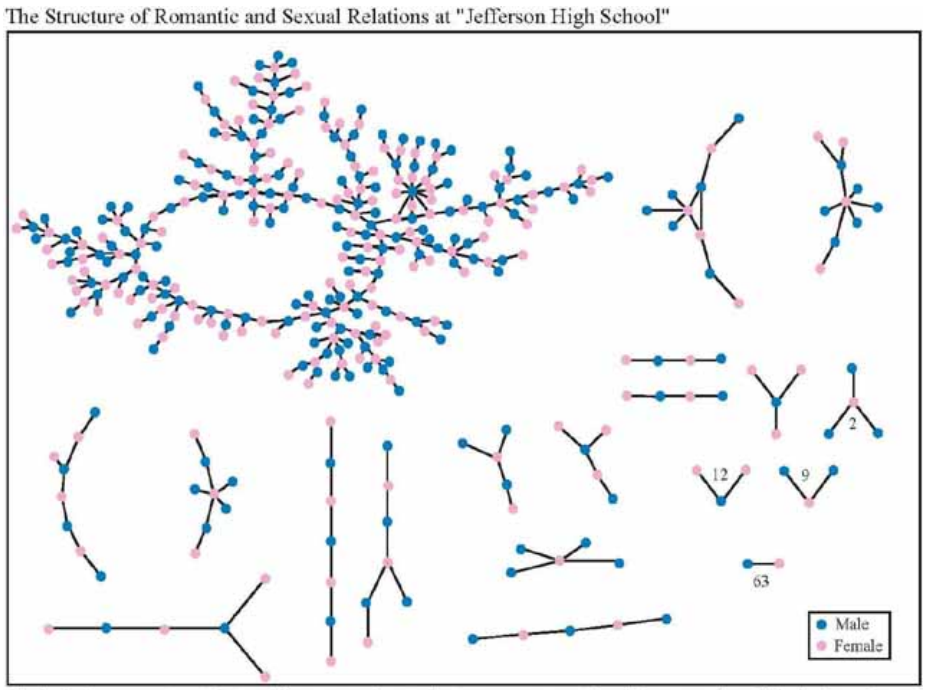

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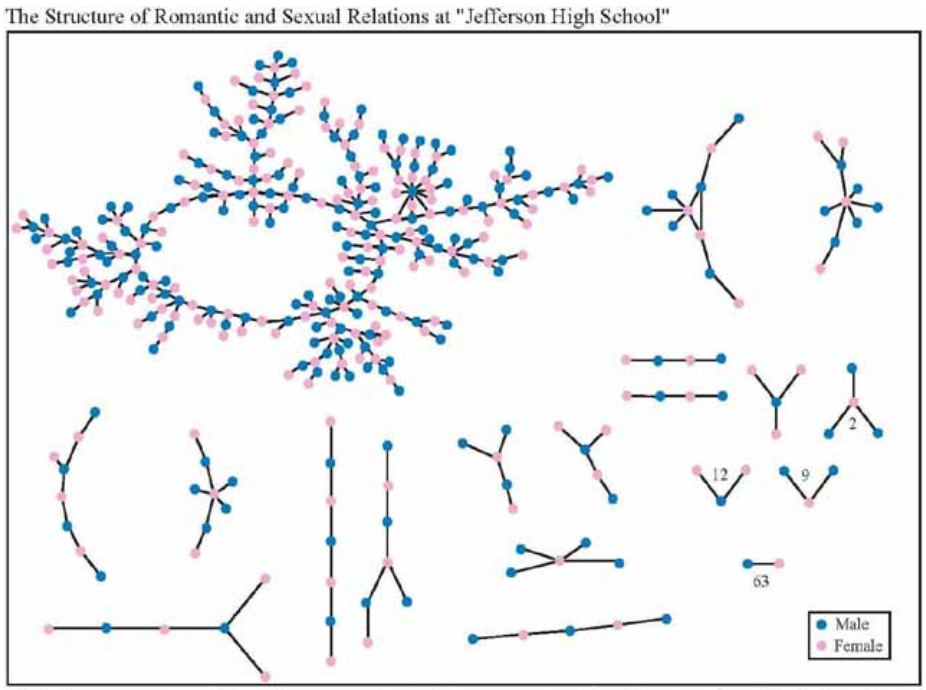

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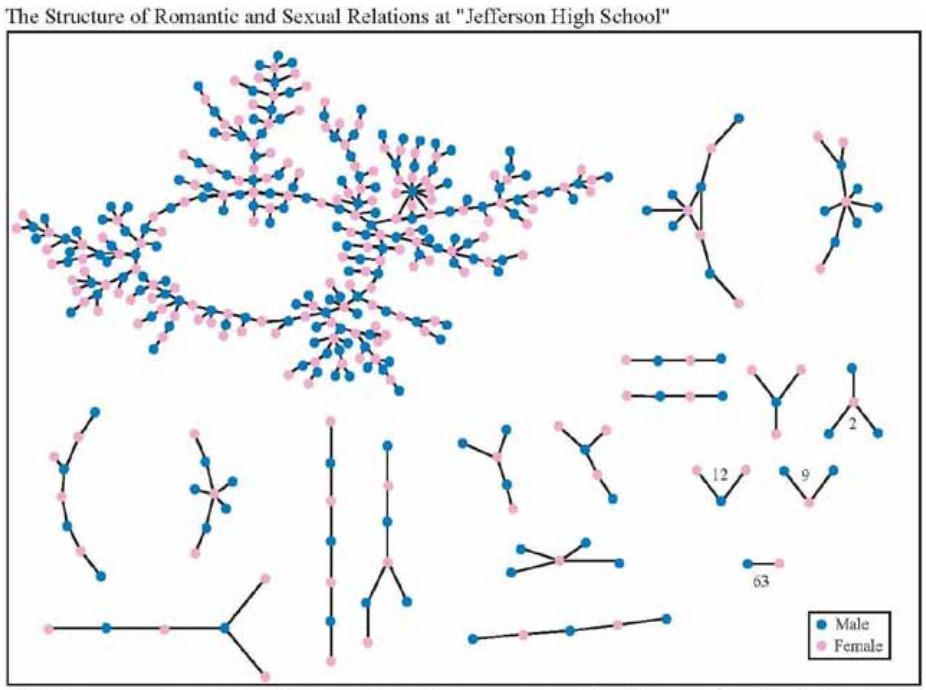

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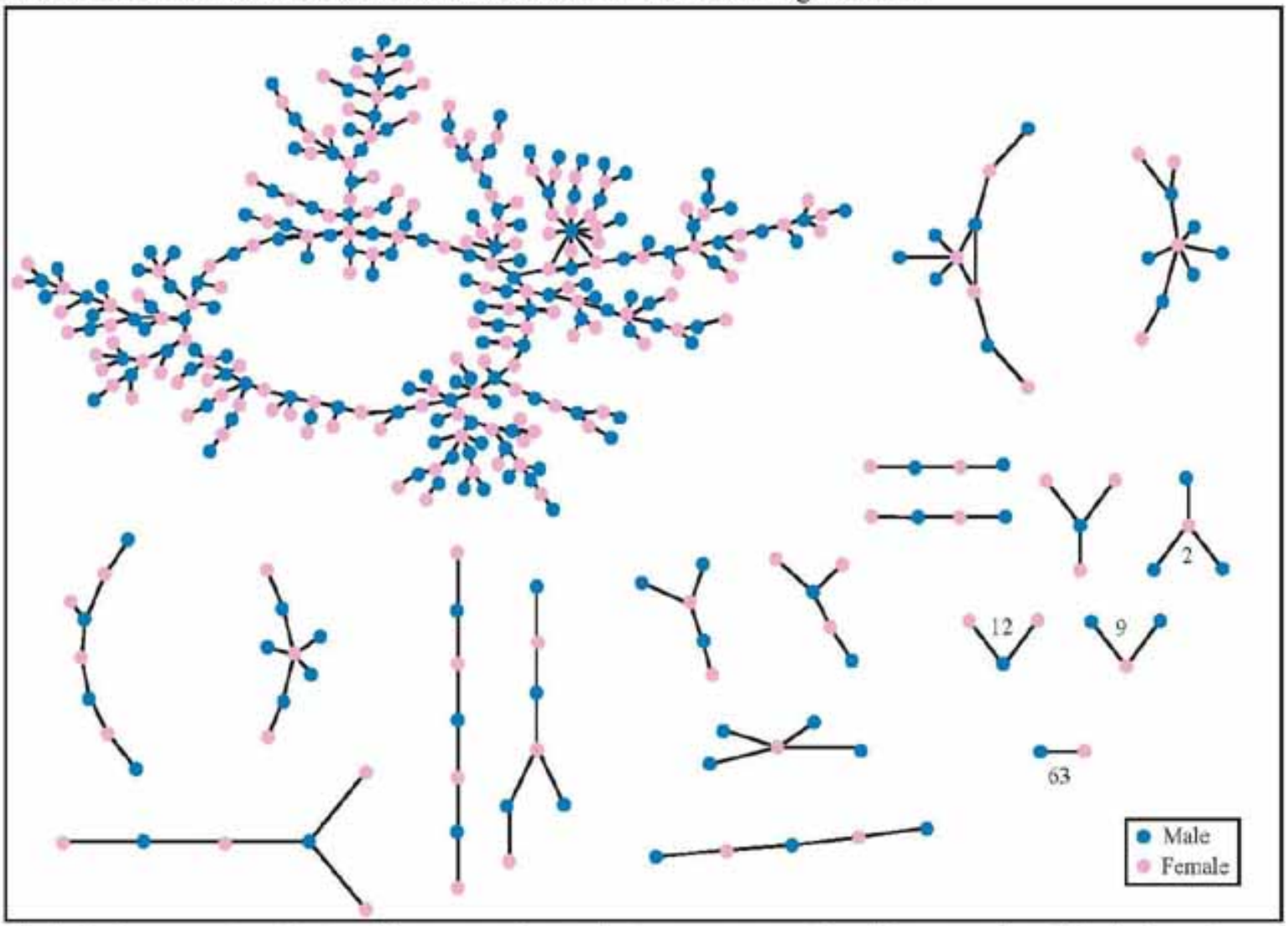
Examples

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

Examples

Relational networks

- Consumer purchases

- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: del.icio.us (⊞), flickr (⊞)

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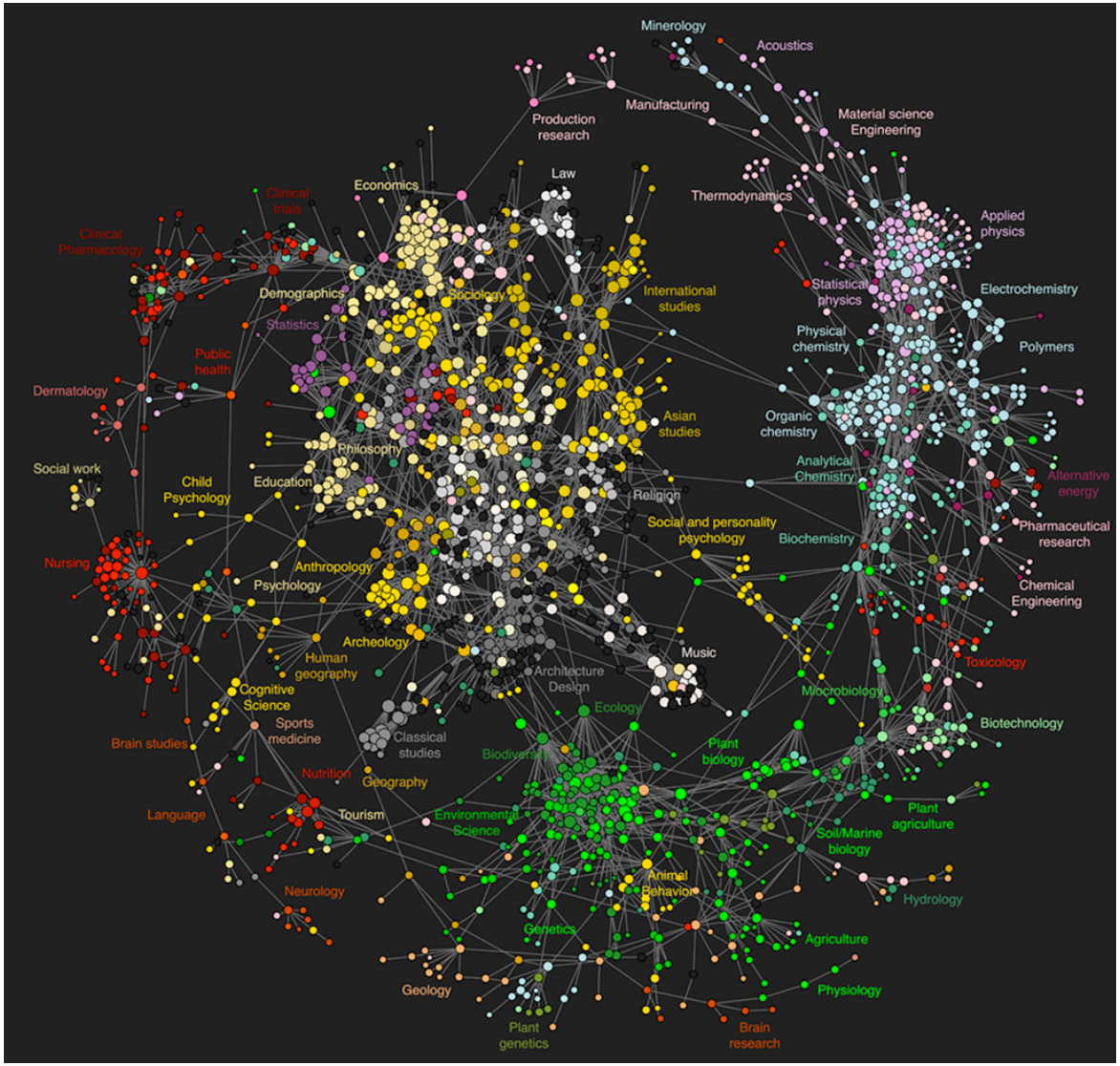
common tags cloud | list

community daily dictionary education **encyclopedia**
english free imported info information internet knowledge
learning news **reference** research resource
resources search tools useful web web2.0 **wiki**
wikipedia

Clickworthy Science:



Bollen et al. [4]

A notable feature of large-scale networks:

- Graphical renderings are often just a big mess.
- And even when renderings somehow look good:
- We need to extract digestible, meaningful aspects.

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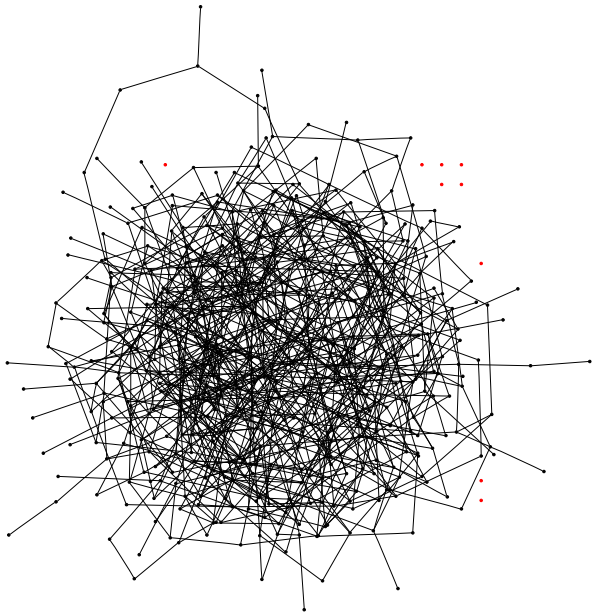

⇐ Typical hairball

- number of nodes $N = 500$
- number of edges $m = 1000$
- average degree $\langle k \rangle = 4$

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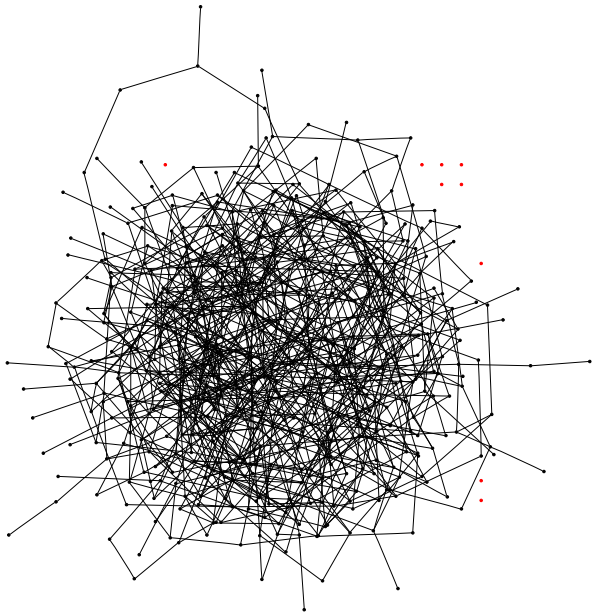

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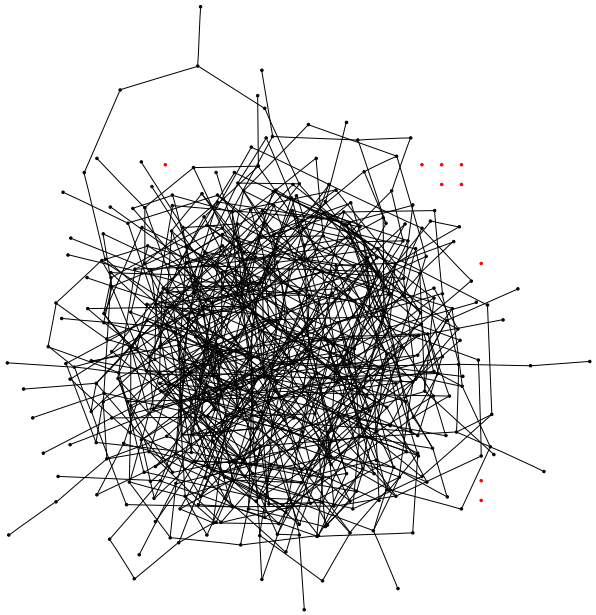

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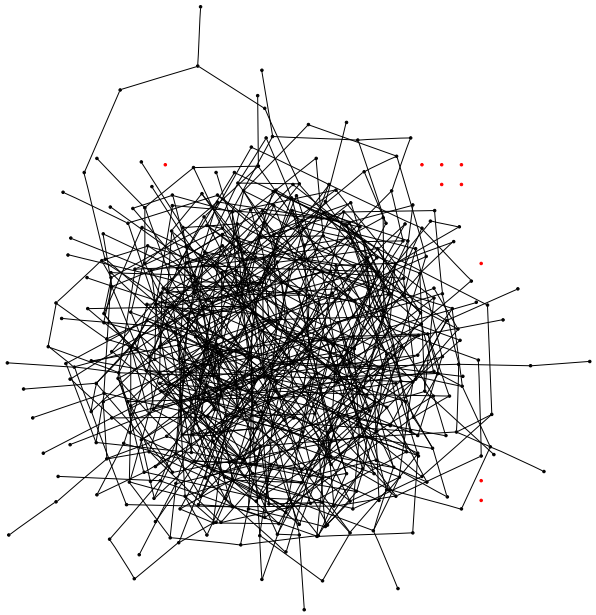

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Properties

Some key aspects of real complex networks:

- degree distribution
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
- hierarchical scaling
- network distances
- centrality
- efficiency
- robustness

- + Coevolution of network structure and processes on networks.

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- P_k is the probability that a randomly selected node has degree k
- k = node degree = number of connections
- ex 1: Erdős-Rényi random networks:

$$P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$$

- Distribution is Poisson

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- Erdős-Rényi random networks are a *mathematical construct*.
- 'Scale-free' networks are growing networks that form according to a plausible mechanism.
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2. assortativity/3. homophily:

- Social networks: Homophily (⊞) = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network:[13] similar degree nodes connecting to each other.
- Disassortative network: high degree nodes connecting to low degree nodes.

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  Often techological or biological: Internet, WWW, protein interactions, neural networks, food webs.

Clustering

4. clustering:

- Your friends tend to know each other.
- Two measures:
  1. Watts & Strogatz [21]

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

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- C_1 is the average fraction of pairs of neighbors who are connected.
- Fraction of pairs of neighbors who are connected is

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  where k_i is node i's degree, and $\mathcal{N}_i$ is the set of i's neighbors.
- Averaging over all nodes, we have

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- C_2 is a useful and often preferred variant
- In general, $C_1 \neq C_2$.
- C_1 is a global average of a local ratio.
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Triples and triangles

- Nodes i_1, i_2, and i_3 form a triple around i_1 if i_1 is connected to i_2 and i_3.
- Nodes i_1, i_2, and i_3 form a triangle if each pair of nodes is connected
- The definition

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$

  measures the fraction of closed triples
- Social Network Analysis (SNA): fraction of transitive triples.
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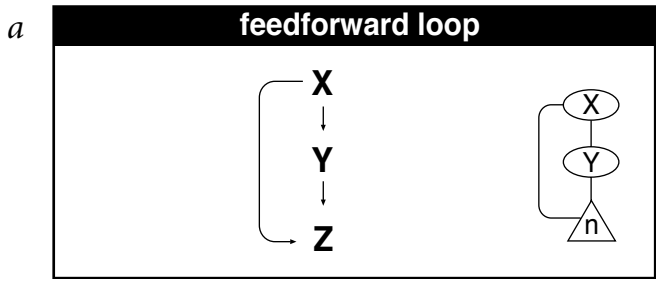

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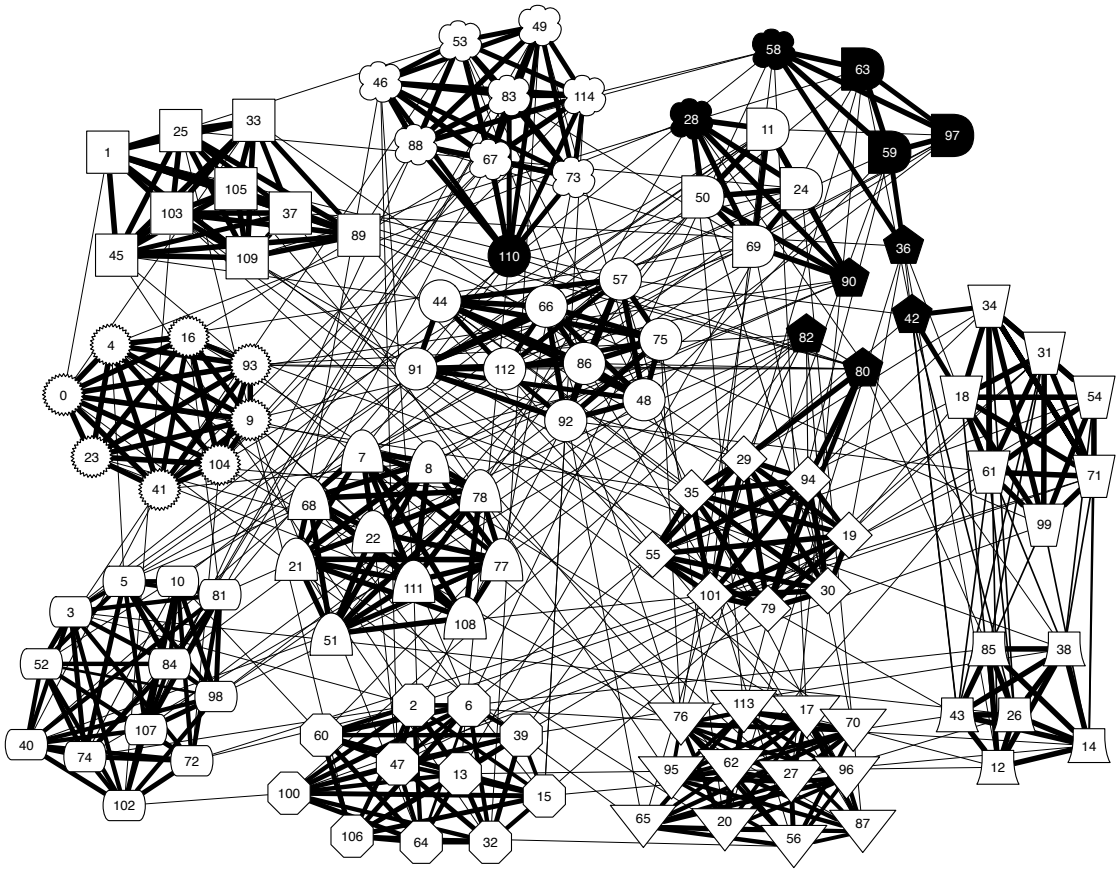
6. modularity and structure/community detection:



Clauset *et al.*, 2006 [7]: NCAA football

Properties

7. concurrency:

- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple model
- dynamic property—static networks are not enough
- knowledge of previous contacts crucial
- beware cumulated network data
- Kretzschmar and Morris, 1996 [12]

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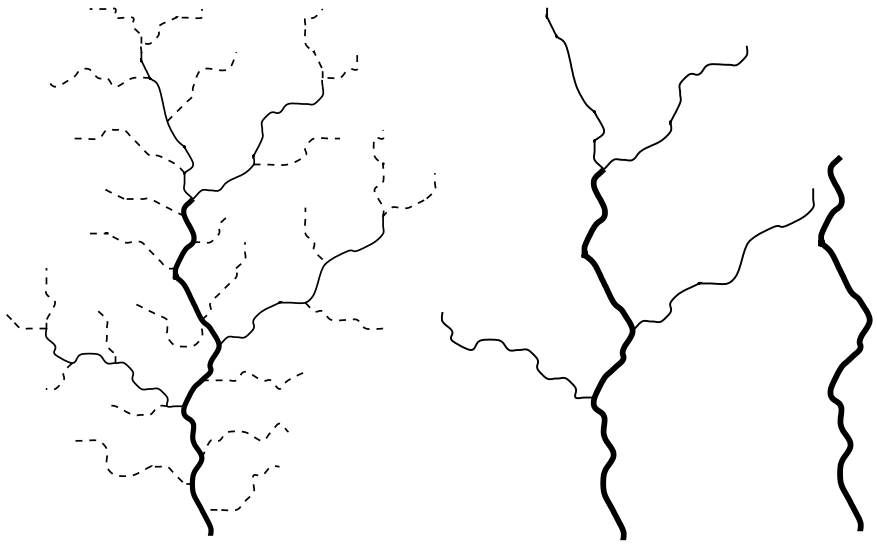
8. Horton-Strahler ratios:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Number: $R_n = N_\omega / N_{\omega+1}$
  - Segment length: $R_l = \langle l_{\omega+1} \rangle / \langle l_\omega \rangle$
  - Area/Volume: $R_a = \langle a_{\omega+1} \rangle / \langle a_\omega \rangle$



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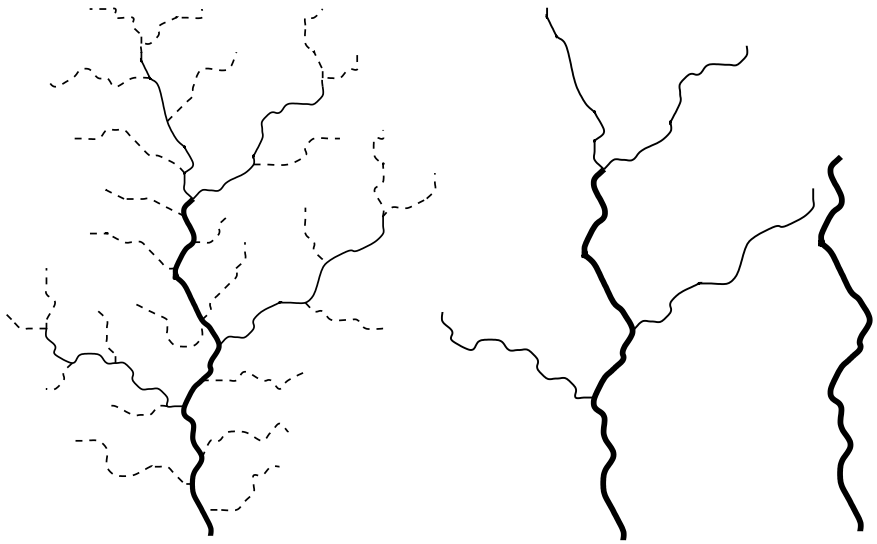

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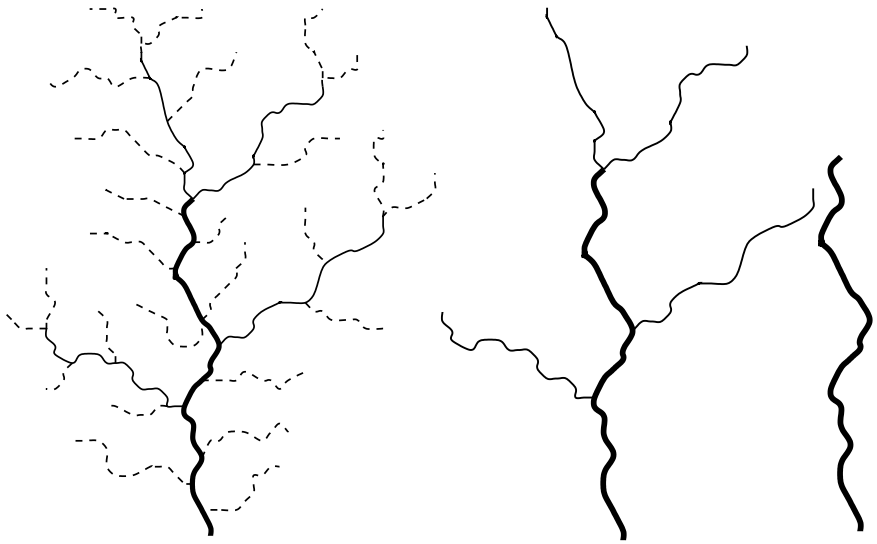

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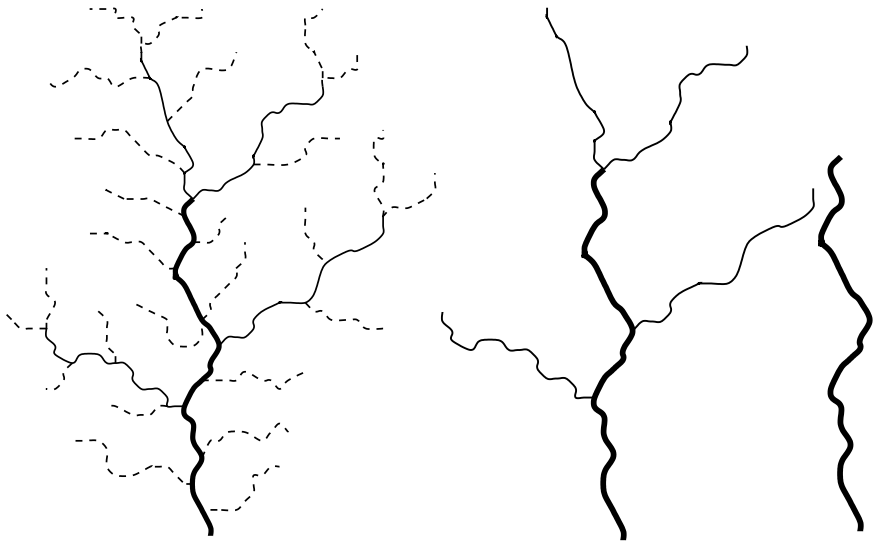

Properties

8. Horton-Strahler ratios:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
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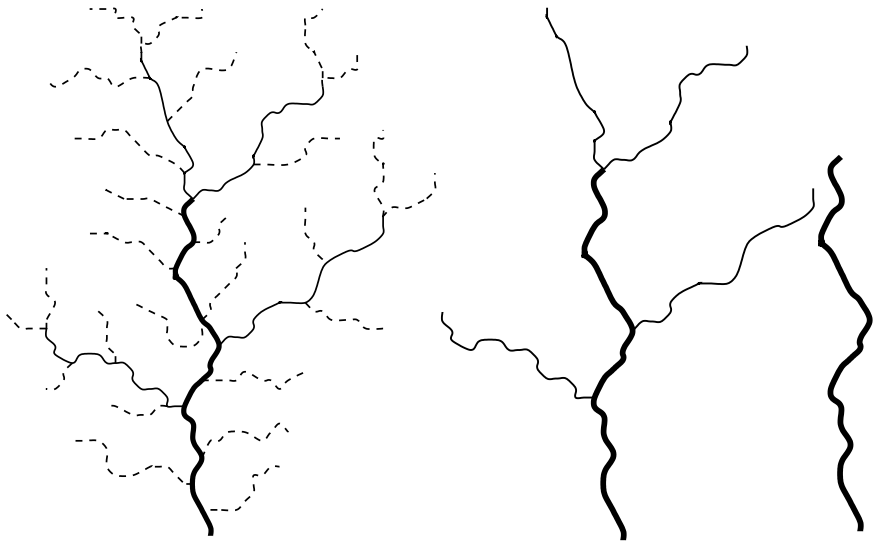

(a) (b) (c)

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9. network distances:

(a) shortest path length d_{ij}:

- Fewest number of steps between nodes i and j.
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2. scale-free networks (⊞) (covered in 300)
3. small-world networks (⊞) (covered in 300)
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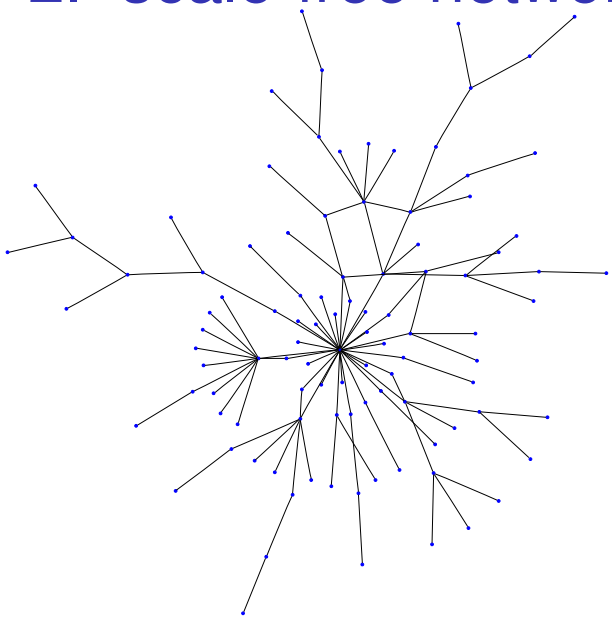

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$N = 150$

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- Generative model
- Preferential attachment model with growth:
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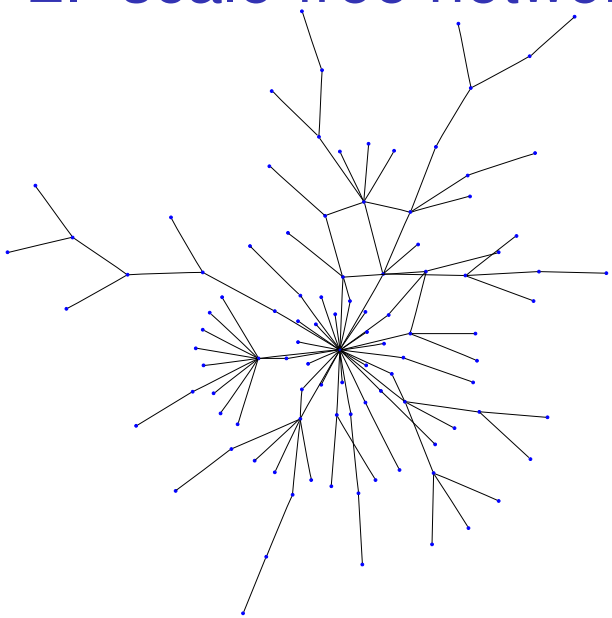

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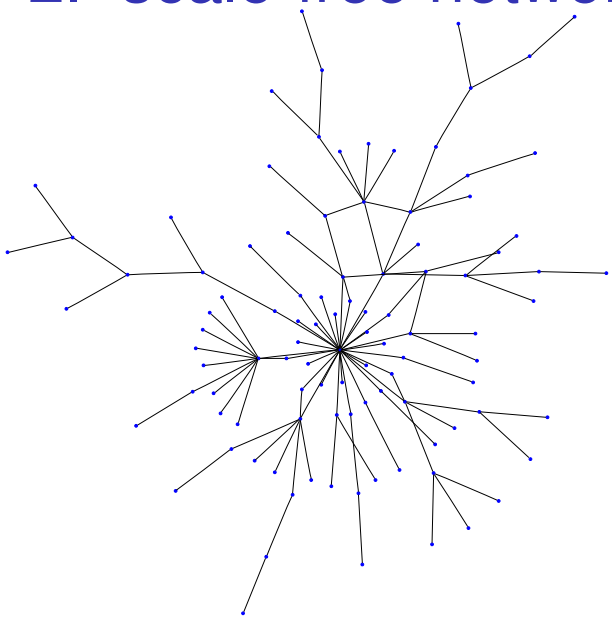

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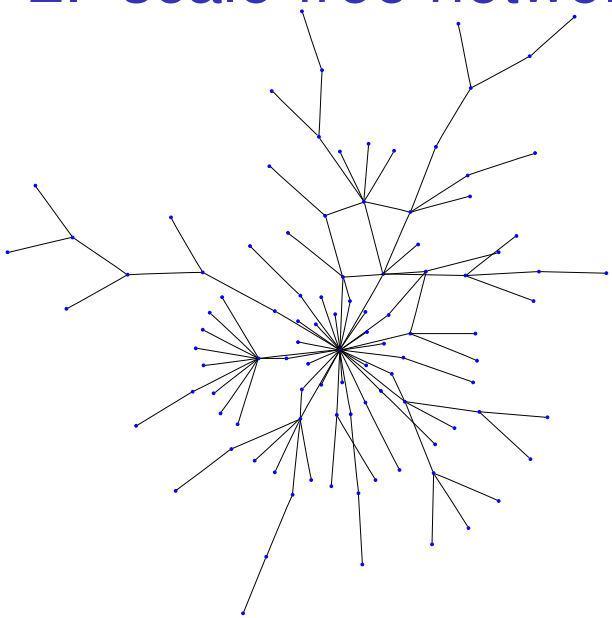

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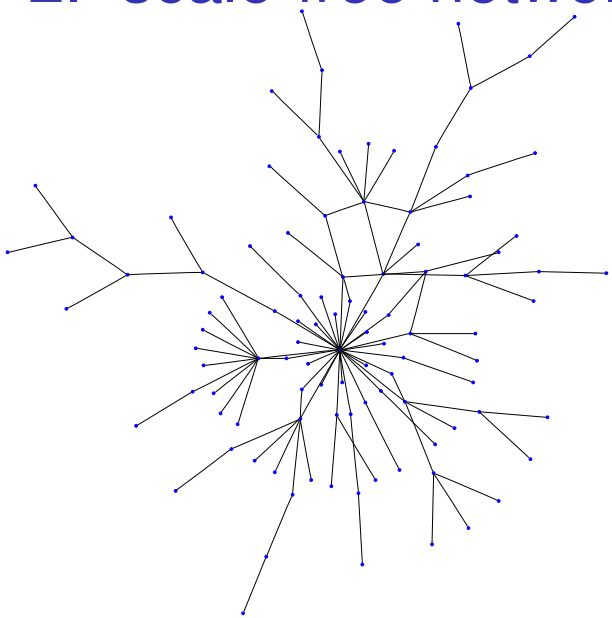

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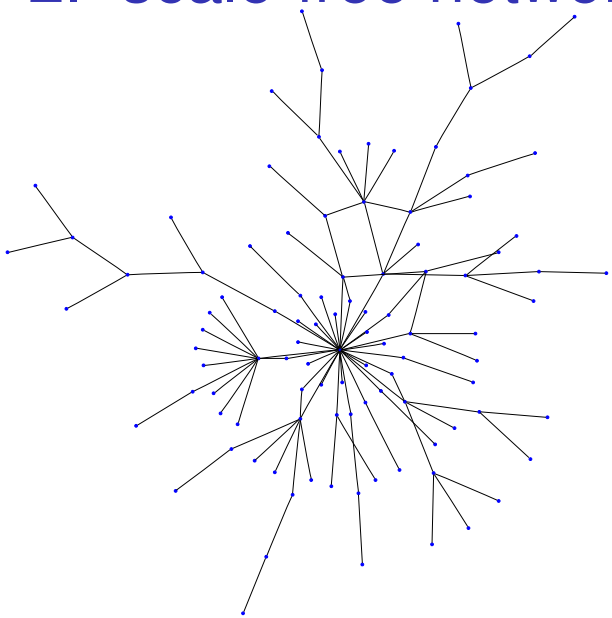

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- Introduced by Watts and Strogatz [21]
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- global randomness (shortcuts).
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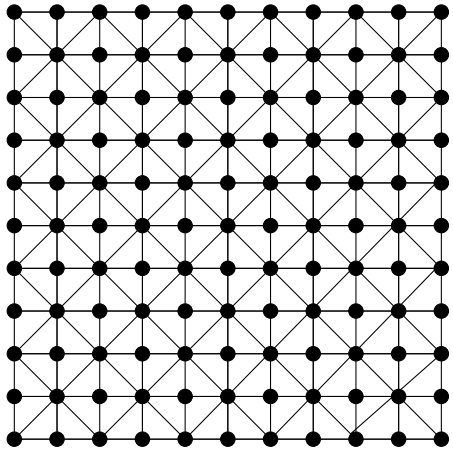

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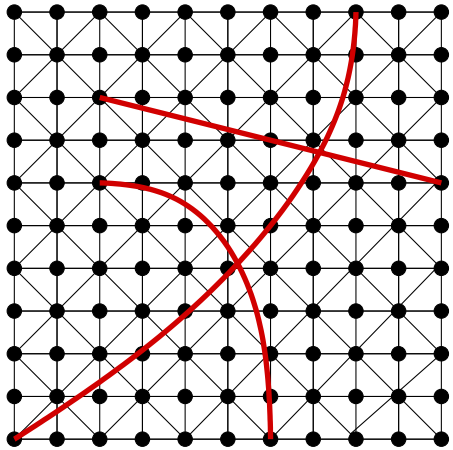

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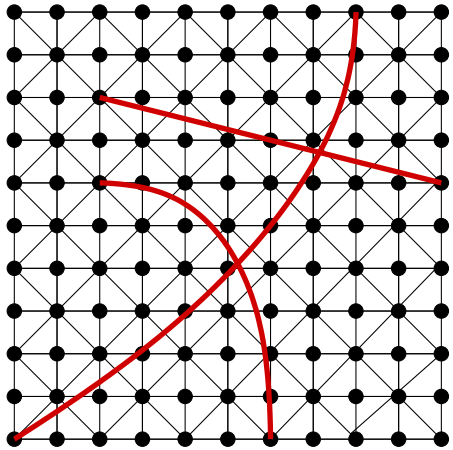

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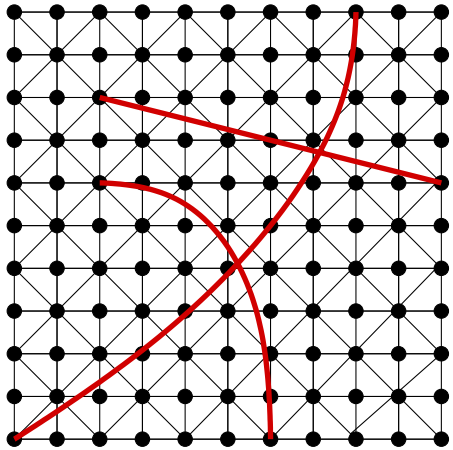

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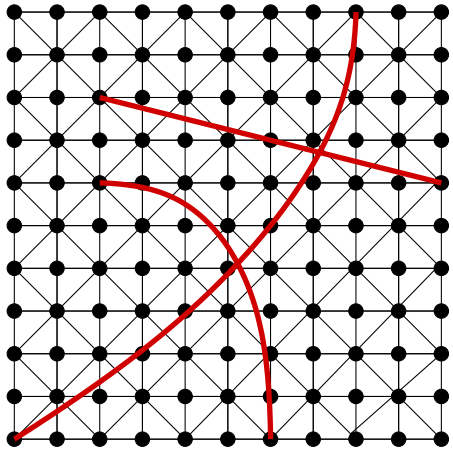

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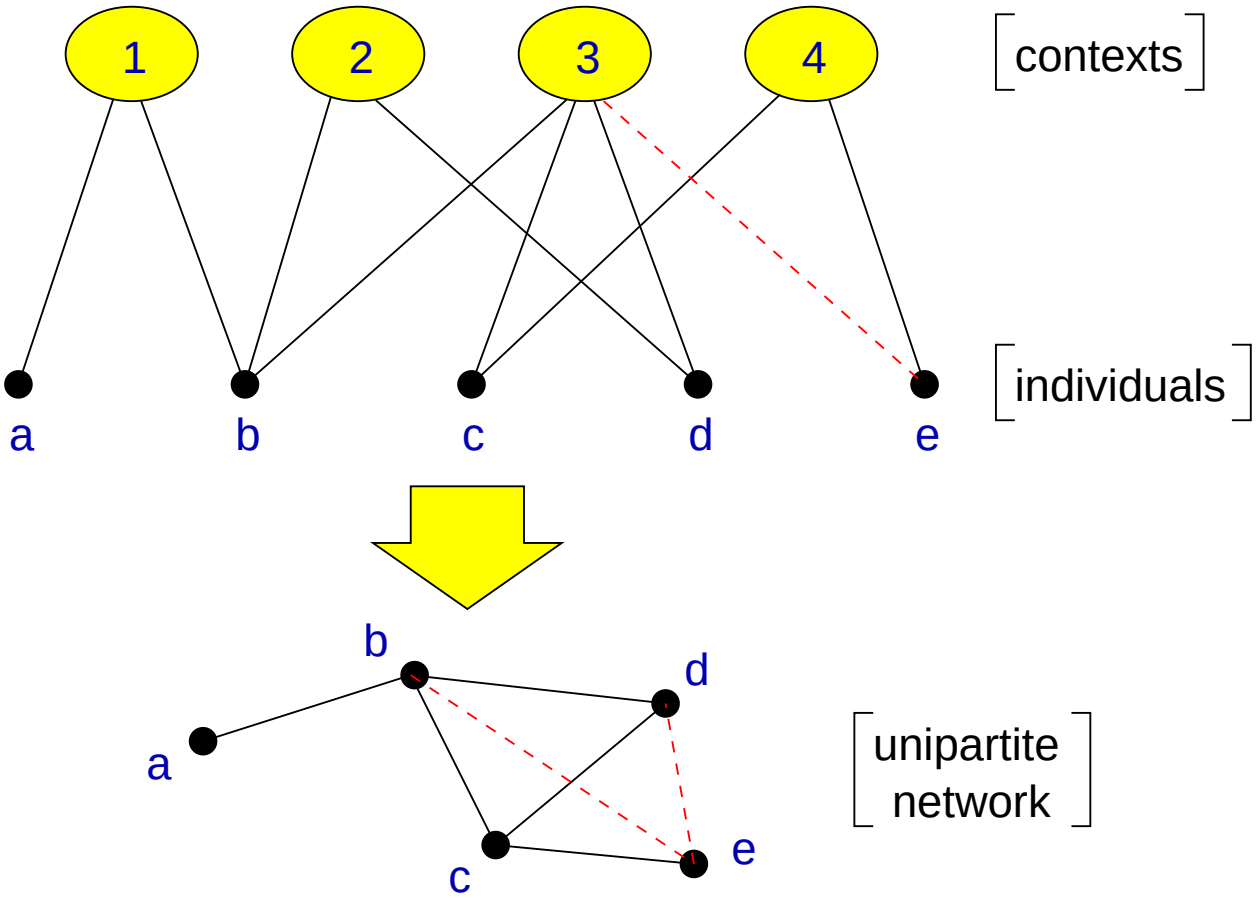

Bipartite affiliation networks: boards and directors, movies and actors.

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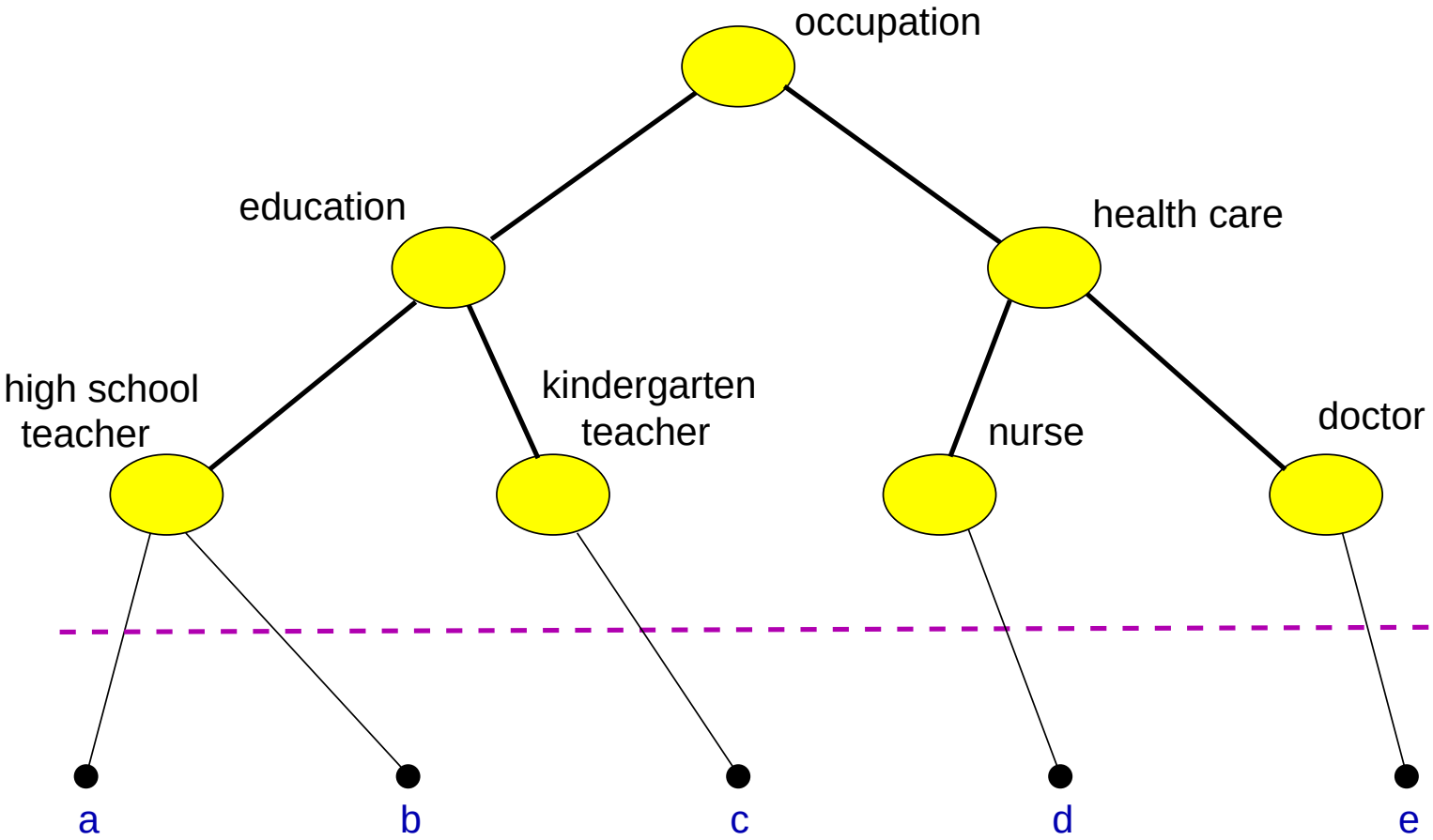

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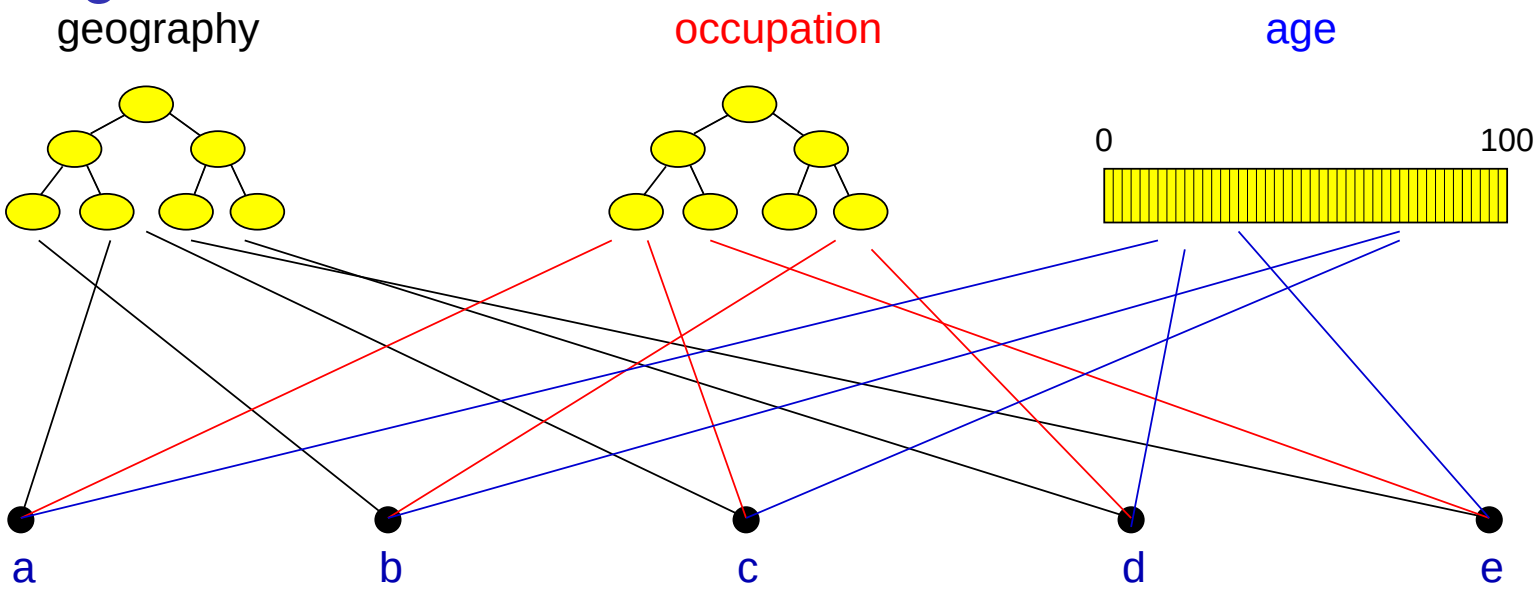

- Blau & Schwartz [3], Simmel [17], Breiger [6], Watts *et al.* [20]

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- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
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[2] A.-L. Barabási and R. Albert.
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