Complex Networks, CSYS/MATH 303—Assignment 8
University of Vermont, Spring 2010

Dispersed: Thursday, April 15, 2010.
Due: By start of lecture, 10:00 am, Thursday, April 29, 2010.
Some useful reminders:
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Office hours: 1:00 pm to 2:30 pm, Wednesday @ Farrell, and by appointment
Course website: `http://www.uvm.edu/~pdodds/teaching/courses/2010-01UVM-303/`

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related variant).

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1. (9 pts) Consider a family of undirected random networks with degree distribution

$$P_k = c\delta_{k1} + (1-c)\delta_{k3}$$

where δ_{ij} is the Kronecker delta function where c is a constant to be determined below. Also allow nodes to be correlated according to the following node-node mixing probability:

$$E = [e_{ij}] = \left[\begin{array}{cc} e_{00} & e_{02} \\ e_{20} & e_{22} \end{array} \right] = \frac{1}{4} \left[\begin{array}{cc} (1+r) & (1-r) \\ (1-r) & (1+r) \end{array} \right]$$

where e_{ij} is the probability that a randomly chosen edge connects a node of degree $i+1$ an a node of degree $j+1$, and only the non-zero values of E are shown.

(a) Determine c so that purely disassortative networks are achievable if r is tuned to -1.

(b) Analytically determine the size of the giant component as a function of r.

(c) Determine the size of the largest component containing only degree 3 nodes as a function of r.

Hint: allow degree 3 nodes to be always vulnerable ($\beta_{3i} = 1$ for $i = 0$, 1, 2, and 3) and degree 1 nodes to be never vulnerable ($\beta_{1i} = 0$ for $i = 0$ and 1).

2. Spreading on assortative networks: Starting from

$$\theta_{j,t+1} = G_j(\vec{\theta}_t) = \phi_0 + (1-\phi_0)\times$$

$$\sum_{k=1}^{\infty} \frac{e_{j-1,k-1}}{R_{j-1}} \sum_{i=0}^{k-1} \binom{k-1}{i} \theta_{k,t}^{i} (1-\theta_{k,t})^{k-1-i} \beta_{ki}.$$

show the matrix for which we must have the largest eigenvalue greater than 1 for spreading to occur is

$$\frac{\partial G_j(\vec{0})}{\partial \theta_{k,t}} = \frac{e_{j-1,k-1}}{R_{j-1}} (k-1)(\beta_{k1} - \beta_{k0}).$$

3. Show that for uncorrelated networks, i.e, when $e_{jk} = R_j R_k$, the above condition collapses to the standard condition

$$\sum_{k=1}^{\infty} (k-1) \frac{k P_k}{\langle k \rangle} (\beta_{k1} - \beta_{k0}) > 1.$$