Complex Networks, CSYS/MATH 303—Assignment 7
University of Vermont, Spring 2010

Dispersed: Wednesday, April 7, 2010.
Due: By start of lecture, 10:00 am, Thursday, April 15, 2010.
Some useful reminders:
Instructor: Peter Dodds
Office: 203 Lord House, 16 Colchester Avenue (TR)
E-mail: peter.dodds@uvm.edu
Office hours: 1:00 pm to 2:30 pm, Wednesday @ Farrell, and by appointment
Course website: `http://www.uvm.edu/~pdodds/teaching/courses/2010-01UVM-303/`

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related variant).

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1. Using Gleeson and Calahane's iterative equations below, derive the contagion condition for a vanishing seed by taking the limit $\phi_0 \to 0$ and $t \to \infty$.

$$\phi_{t+1} = \phi_0 + (1-\phi_0)\sum_{k=0}^{\infty} P_k \sum_{j=0}^{k} \binom{k}{j} \theta_t^j (1-\theta_t)^{k-j} \beta_{kj},$$

$$\theta_{t+1} = \phi_0 + (1-\phi_0)\sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} {\theta_t}^j (1-\theta_t)^{k-1-j} \beta_{kj},$$

where $\theta_0 = \phi_0$, and β_{kj} is the probability that a degree k node becomes active when j of its neighbors are active. Recall that by contagion condition, we mean the requirements of a random network for spreading to occur given a specific response function F.

Allow β_{k0} to be arbitrary (i.e., not necessarily 0 as for simple threshold functions).

2. Derive equation 4 in Gleeson and Cahalane (2007) [1]:

$$C_\ell = \sum_{k=\ell+1}^{\infty} \sum_{n=0}^{\ell} \binom{k-1}{\ell} \binom{\ell}{n} (-1)^{\ell+n} \frac{k}{z} P_k F\left(\frac{n}{k}\right).$$

3. (9 pts)

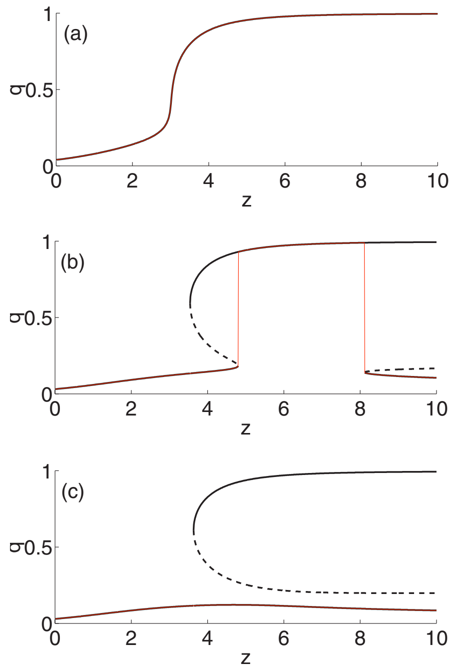
   (a) Derive equation 6 in Gleeson and Cahalane (2007), which is a second order approximation to the cascade condition for non-zero seeds.

(b) Hence reproduce the dashed analytic curve shown in Figure 1 of their paper.

(c) Explain why there are jumps in the cascade window outline that do not occur at reciprocals of the integers.

4. (6 pts)

(a) By solving for the fixed points of $\theta_{t+1} = G(\theta_t; 0)$, reproduce Figure 3 in Gleeson and Cahalane (2007):



(b) Also plot $G(\theta_t; 0)$ for an average threshold $\phi_*(= R)$ of 0.371 for $\langle k \rangle (= z) = 1, 2, 3, \ldots, 10$.

Add the cobweb diagram for a $\phi_0 = 0$ seed (do this by creating a recursive plotting script in matlab, for example).

(c) Discuss how the stable points move with $\langle k \rangle$.

Note: $\phi_* = 0.371$ matches plot (b) in Figure 3.

References

[1] J. P. Gleeson and D. J. Cahalane. Seed size strongly affects cascades on random networks. *Phys. Rev. E*, 75:056103, 2007. pdf (⊞)