

Overview of Complex Networks

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Something of a plan:

- ▶ **Lecture 1:** Overview; Background; Basic models
- ▶ **Lecture 2:** Random networks; Scale-free networks
- ▶ **Lecture 3:** Transportation; Contagion
- ▶ **Lecture 4:** Discovering structure

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Exciting details regarding these slides:

- ▶ Three versions (all in pdf):
 1. Presentation,
 2. Flat Presentation,
 3. Handout (2x2).
- ▶ Presentation versions are **navigable** and hyperlinks are **clickable**.
- ▶ Web links look like this (田).
- ▶ References in slides link to full citation at end. ^[1]
- ▶ Citations contain links to papers in pdf (if available).
- ▶ 50 hours of lectures → 5 hours.
- ▶ Brought to you by a concoction of \LaTeX , Beamer, and perl.

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Complex System—Some ingredients:

- ▶ Distributed system of many interrelated parts
- ▶ No centralized control
- ▶ Nonlinear relationships
- ▶ Existence of feedback loops
- ▶ Complex systems are open (out of equilibrium)
- ▶ Presence of Memory
- ▶ Modular (nested)/multiscale structure
- ▶ Opaque boundaries
- ▶ Emergence—‘More is Different’^[1]



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Basic definitions

Complex: (Latin = with + fold/weave (com + plex))

Adjective

- ▶ Made up of multiple parts; intricate or detailed.
- ▶ Not simple or straightforward.



net•work |'net,wərək|

noun

- 1 an arrangement of intersecting horizontal and vertical lines.
 - a complex system of roads, railroads, or other transportation routes : *a network of railroads.*
- 2 a group or system of interconnected people or things : *a trade network.*
 - a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
 - a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*
 - a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
 - a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*

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Thesaurus deliciousness:

network

noun

- 1** *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- 2** *a network of lanes* MAZE, labyrinth, warren, tangle.
- 3** *a network of friends* SYSTEM, complex, nexus, web, webwork.

From Keith Briggs's excellent
etymological investigation: (田)

- ▶ Opus reticulatum:
- ▶ A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network>]

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Ancestry:

First known use: Geneva Bible, 1560

‘And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).’

From the OED via Briggs:

- ▶ 1658—: reticulate structures in animals
- ▶ 1839—: rivers and canals
- ▶ 1869—: railways
- ▶ 1883—: distribution network of electrical cables
- ▶ 1914—: wireless broadcasting networks

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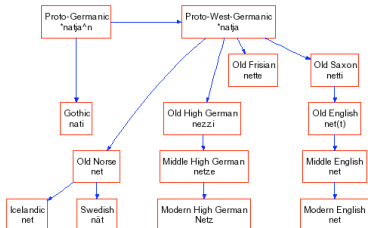
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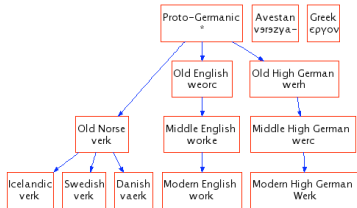
Ancestry:

Net and Work are venerable old words:

- ▶ **'Net'** first used to mean spider web (King Ælfréd, 888).
- ▶ **'Work'** appear to have long meant purposeful action.



The network of Germanic 'net' words



The network of 'work' words

- ▶ **'Network'** = something built based on the idea of natural, flexible lattice or web.
- ▶ c.f., ironwork, stonework, fretwork.

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Key Observation:

- ▶ Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.
- ▶ Opens door to mathematical and numerical analysis.
- ▶ Dominant approach of last decade of a **theoretical-physics/stat-mechish** flavor.
- ▶ Mindboggling amount of work published on complex networks since 1998...
- ▶ ... largely due to your typical theoretical physicist:



- ▶ *Piranha physicus*
- ▶ Hunt in packs.
- ▶ Feast on new and interesting ideas (see chaos, cellular automata, ...)

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Popularity (according to ISI)

“Collective dynamics of ‘small-world’ networks” [19]

- ▶ Watts and Strogatz
Nature, 1998
- ▶ ≈ 3752 citations (as of June 5, 2009)
- ▶ Over 1100 citations in 2008 alone.

“Emergence of scaling in random networks” [2]

- ▶ Barabási and Albert
Science, 1999
- ▶ ≈ 3860 citations (as of June 5, 2009)
- ▶ Over 1100 citations in 2008 alone.

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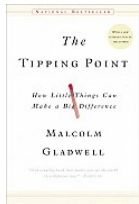
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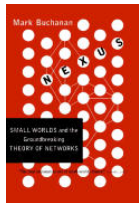
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Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [8]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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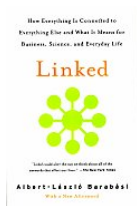
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Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age^[18]—Duncan Watts

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Numerous others:

- ▶ [Complex Social Networks](#)—F. Vega-Redondo^[17]
- ▶ [Fractal River Basins: Chance and Self-Organization](#)—I. Rodríguez-Iturbe and A. Rinaldo^[14]
- ▶ [Random Graph Dynamics](#)—R. Durrett
- ▶ [Scale-Free Networks](#)—Guido Caldarelli
- ▶ [Evolution and Structure of the Internet: A Statistical Physics Approach](#)—Romu Pastor-Satorras and Alessandro Vespignani
- ▶ [Complex Graphs and Networks](#)—Fan Chung
- ▶ [Social Network Analysis](#)—Stanley Wasserman and Kathleen Faust
- ▶ [Handbook of Graphs and Networks](#)—Eds: Stefan Bornholdt and H. G. Schuster^[4]
- ▶ [Evolution of Networks](#)—S. N. Dorogovtsev and J. F. F. Mendes^[7]

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More observations

- ▶ But surely **networks aren't new**...
- ▶ Graph theory is well established...
- ▶ Study of social networks started in the 1930's...
- ▶ So why all this 'new' research on networks?
- ▶ **Answer: Oodles of Easily Accessible Data.**
- ▶ We can now inform (alas) our theories with a much more measurable reality.*
- ▶ A worthy goal: establish **mechanistic explanations.**

** If this is upsetting, maybe string theory is for you...*

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More observations

- ▶ **Web-scale** data sets can be overly **exciting**.

Witness:

- ▶ The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) (田)
- ▶ “The Unreasonable Effectiveness of Data,” Halevy et al. [9].

But:

- ▶ For scientists, description is only part of the battle.
- ▶ We still need to **understand**.

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Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

- ▶ e.g., people, forks in rivers, proteins, webpages, organisms,...

Links = Connections between nodes

- ▶ **Links** may be directed or undirected.
- ▶ **Links** may be binary or weighted.

Other spiffing words: vertices and edges.

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Node degree = Number of links per node

- ▶ Notation: Node i 's degree = k_i .
- ▶ $k_i = 0, 1, 2, \dots$
- ▶ Notation: the average degree of a network = $\langle k \rangle$
(and sometimes z)
- ▶ Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

- ▶ Defn: \mathcal{N}_i = the set of i 's k_i neighbors

Super Basic definitions

Adjacency matrix:

- ▶ We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j) .
- ▶ e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

- ▶ (n.b., for numerical work, we always use sparse matrices.)

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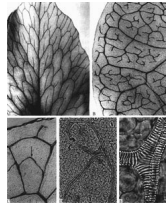
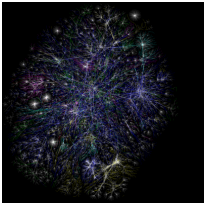
So what passes for a complex network?

- ▶ Complex networks are **large** (in node number)
- ▶ Complex networks are **sparse** (low edge to node ratio)
- ▶ Complex networks are usually **dynamic** and **evolving**
- ▶ Complex networks can be social, economic, natural, informational, abstract, ...

Examples

Physical networks

- ▶ River networks
- ▶ Neural networks
- ▶ Trees and leaves
- ▶ Blood networks
- ▶ The Internet
- ▶ Road networks
- ▶ Power grids



- ▶ **Distribution** (branching) vs. **redistribution** (cyclical)

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Examples

Interaction networks

- ▶ The Blogosphere
- ▶ Biochemical networks
- ▶ Gene-protein networks
- ▶ Food webs: who eats whom
- ▶ The World Wide Web (?)
- ▶ Airline networks
- ▶ Call networks (AT&T)
- ▶ The Media
- ▶ Paper citations



datamining.typepad.com (田)

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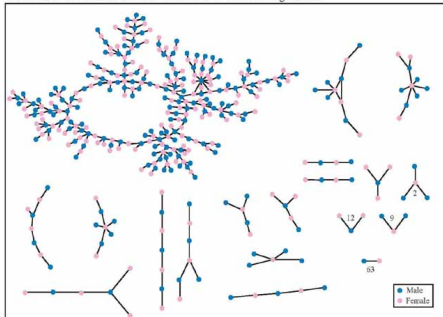
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Examples

Interaction networks: social networks

- ▶ Snogging
- ▶ Friendships
- ▶ Acquaintances
- ▶ Boards and directors
- ▶ Organizations
- ▶ myspace.com (田),
- ▶ facebook.com (田)

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

- ▶ 'Remotely sensed' by: email activity, instant messaging, phone logs (***cough***).

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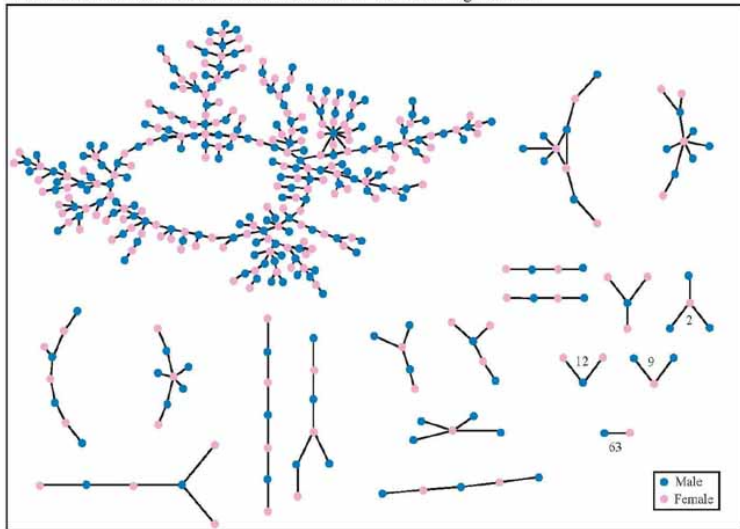
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Examples

Relational networks

- ▶ Consumer purchases
(Wal-Mart: ≈ 1 petabyte = 10^{15} bytes)
- ▶ Thesauri: Networks of words generated by meanings
- ▶ Knowledge/Databases/Ideas
- ▶ Metadata—Tagging: del.icio.us (田), [flickr](http://flickr.com) (田)

common tags cloud | [list](#)

community daily dictionary education **encyclopedia**
 english free imported info information internet knowledge
 learning news **reference** research resource
 resources search tools useful web web2.0 **wiki**
wikipedia

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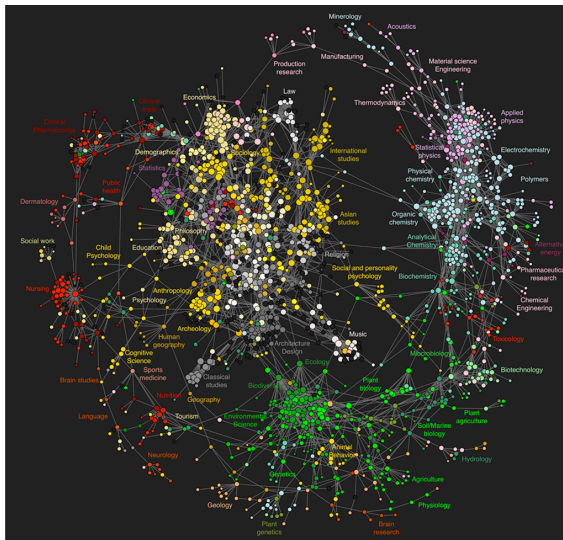
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Clickworthy Science:

Overview



Bollen et al. [3]

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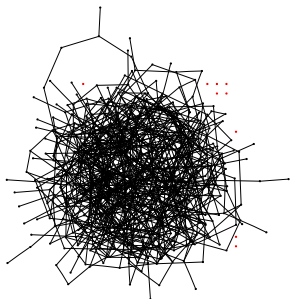
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A notable feature of large-scale networks:

- ▶ Graphical renderings are often just a big mess.



⇐ Typical hairball

- ▶ number of nodes $N = 500$
 - ▶ number of edges $m = 1000$
 - ▶ average degree $\langle k \rangle = 4$
- ▶ And even when renderings somehow look good:
 “That is a very graphic analogy which aids understanding wonderfully while being, strictly speaking, wrong in every possible way”
 said Ponder [Stibbons] — *Making Money*, T. Pratchett.
- ▶ We need to extract **digestible, meaningful aspects**.

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Some key features of real complex networks:

- ▶ Degree distribution
 - ▶ Assortativity
 - ▶ Homophily
 - ▶ Clustering
 - ▶ Motifs
 - ▶ Modularity
 - ▶ Concurrency
 - ▶ Hierarchical scaling
 - ▶ Network distances
 - ▶ Centrality
 - ▶ Efficiency
 - ▶ Robustness
- ▶ Coevolution of network **structure** and **processes** on networks.

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1. Degree distribution P_k

- ▶ P_k is the probability that a randomly selected node has degree k
- ▶ **Big deal:** Form of P_k key to network's behavior
- ▶ **ex 1:** Erdős-Rényi random networks have a Poisson distribution:

$$P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$$

- ▶ **ex 2:** “Scale-free” networks: $P_k \propto k^{-\gamma} \Rightarrow$ ‘hubs’
- ▶ We'll come back to this business soon...

2. Assortativity/3. Homophily:

- ▶ Social networks: Homophily (☒) = birds of a feather
- ▶ e.g., degree is standard property for sorting: measure degree-degree correlations.
- ▶ **Assortative** network: ^[12] similar degree nodes connecting to each other.
 - ▶ Often *social*: company directors, coauthors, actors.
- ▶ **Disassortative** network: high degree nodes connecting to low degree nodes.
 - ▶ Often *techological* or *biological*: Internet, WWW, protein interactions, neural networks, food webs.

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4. Clustering:

- ▶ Your friends tend to know each other.
- ▶ Two measures:

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i \text{ due to Watts \& Strogatz }^{[19]}$$

$$C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}} \text{ due to Newman }^{[13]}$$

- ▶ C_1 is the **average fraction** of **pairs of neighbors** who are **connected**.
- ▶ Interpret C_2 as probability two of a node's friends know each other.

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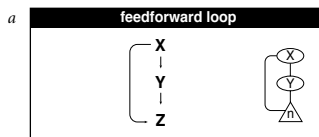
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5. Motifs:

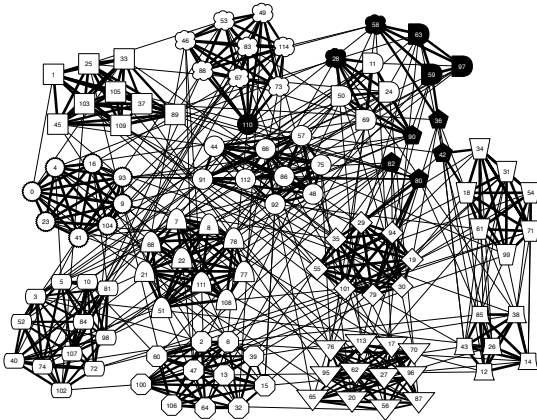
- ▶ Small, recurring functional subnetworks
- ▶ e.g., Feed Forward Loop:



Shen-Orr, Uri Alon, *et al.* [15]

Properties

6. modularity:



Clauset *et al.*, 2006 ^[5]: NCAA football

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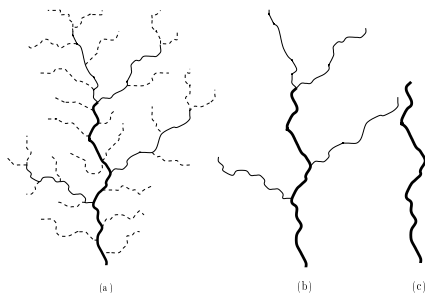
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7. Concurrency:

- ▶ Transmission of a contagious element only occurs during contact^[11]
- ▶ Rather obvious but easily missed in a simple model
- ▶ Dynamic property—static networks are not enough
- ▶ Knowledge of previous contacts crucial
- ▶ **Beware** cumulated network data!

8. Horton-Strahler stream ordering:

- ▶ Metrics for branching networks:
 - ▶ Method for ordering streams hierarchically
 - ▶ Reveals fractal nature of natural branching networks
 - ▶ Hierarchy is not pure but mixed (Tokunaga) [16, 6].
 - ▶ Major examples: rivers and blood networks.



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9. Network distances:

(a) shortest path length d_{ij} :

- ▶ Fewest number of steps between nodes i and j .
- ▶ (Also called the chemical distance between i and j .)

(b) average path length $\langle d_{ij} \rangle$:

- ▶ Average shortest path length in whole network.
- ▶ Good algorithms exist for calculation.
- ▶ Weighted links can be accommodated.

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9. Network distances:

(c) Network diameter d_{\max} :

- ▶ Maximum shortest path length in network.

(d) Closeness $d_{cl} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$:

- ▶ Average 'distance' between any two nodes.
- ▶ Closeness handles disconnected networks ($d_{ij} = \infty$)
- ▶ $d_{cl} = \infty$ only when all nodes are isolated.

10. Centrality:

- ▶ Many such measures of a node's 'importance.'
- ▶ **ex 1:** Degree centrality: k_i .
- ▶ **ex 2:** Node i 's betweenness
= fraction of shortest paths that pass through i .
- ▶ **ex 3:** Edge ℓ 's betweenness
= fraction of shortest paths that travel along ℓ .
- ▶ **ex 4:** Recursive centrality: Hubs and Authorities (Jon Kleinberg^[10])

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Overview Key Points:

- ▶ The field of complex networks came into existence in the late 1990s.
- ▶ Explosion of papers and interest since 1998/99.
- ▶ Hardened up much thinking about complex systems.
- ▶ Specific focus on networks that are **large-scale**, **sparse**, **natural** or **man-made**, **evolving** and **dynamic**, and (crucially) **measurable**.
- ▶ Three main (blurred) categories:
 1. **Physical** (e.g., river networks),
 2. **Interactional** (e.g., social networks),
 3. **Abstract** (e.g., thesauri).

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Nutshell:

Overview Key Points (cont.):

- ▶ Obvious connections with the vast extant field of graph theory.
- ▶ But focus on dynamics is more of a physics/stat-mech/comp-sci flavor.
- ▶ Two main areas of focus:
 1. **Description:** Characterizing very large networks
 2. **Explanation:** Micro story \Rightarrow Macro features
- ▶ Some essential structural aspects are understood: degree distribution, clustering, assortativity, group structure, overall structure,...
- ▶ Still much work to be done, especially with respect to dynamics...

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