Complex Networks, CSYS/MATH 303—Assignment 4
University of Vermont, Spring 2009

Dispersed: Wednesday, February 25, 2009.
Due: By start of lecture, 10:00 am, Thursday, March 20, 2009.
Some useful reminders:
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Office hours: 2:30 pm to 3:30 pm, Tuesday & 11:30 am to 12:30 pm Thursday
Course website: `http://www.uvm.edu/~pdodds/teaching/courses/2009-01UVM-303/`

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

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1. Consider the simple spreading mechanism on generalized random networks for which each link has a probability $\beta \le 1$ of successfully transmitting a disease.

   We assume that this transmission probability is tested only once: either a link will or will not be able to send an infection one way or the other (this is a bond percolation problem). We'll call these edges 'active.'

   Denote the degree distribution of the network as P_k and the corresponding generating function as F_P. In class, we wrote down the probability that a node has k active edges as

   $$P'_k = \beta^k \sum_{i=k}^{\infty} \binom{i}{k} (1-\beta)^{i-k} P_i.$$

   (a) Given a random network with degree distribution P_k, find $F_{P'}$, the generating function for P'_k, in terms of F_P.

   (b) Find the generating function for R'_k, the analogous version of R_k, the probability that a random friend has k other friends.

2. (a) For standard random networks, use your results for Q2 to find the critical value of $\langle k \rangle$ above which global spreading occurs. Also find an expression connecting the three quantities β, the average degree $\langle k \rangle$, and the size of the giant component S'_1.

   (b) What is the slope of the S'_1 curve near the critical point for ER networks?

(c) Using whichever method you find most exciting, plot how S_1' depends on $\langle k \rangle$ for $\beta = 1$, $\beta = 0.8$, and $\beta = 0.5$.

3. (a) Prove that if random variables U and V are distributed over the non-negative integers according to P_u and P_v, then the generating function for the random variable $W = U + V$ is

$$F_W(x) = F_U(x)F_V(x).$$

(b) Using the your result in part (a), argue that if

$$W = \sum_{i=1}^{V} U_i$$

where $U_i \stackrel{d}{=} U$ then

$$F_W(x) = F_V(F_U(x)).$$

Hint: write down the generating function of $\sum_{i=1}^{n} U_i$ in terms of $F(U)$.