Complex Networks, CSYS/MATH 303—Assignment 1
University of Vermont, Spring 2009

Dispersed: Tuesday, January 20, 2009.
Due: By start of lecture, 10:00 am, Tuesday, February 3 (the Teletherm), 2009.
Some useful reminders:
Instructor: Peter Dodds
Office: 203 Lord House, 16 Colchester Avenue (TR)
Other office: Farrell Hall, 210 Colchester Ave (MWF)
E-mail: peter.dodds@uvm.edu
Office phone: (802) 656-3089 (email is better...)
Office hours: 2:30 pm to 3:30 pm, Tuesday & 11:30 am to 12:30 pm Thursday
Course website: `http://www.uvm.edu/~pdodds/teaching/courses/2009-01UVM-303/`

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

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1. Tokunaga's law is statistical but we can consider a rigid version. Take $T_1 = 2$ and $R_T = 2$ and draw an example network of order $\Omega = 4$ with these parameters.

2. Tokunaga's law implies Horton's laws:

   In lectures, we establish the following:

   $$n_\omega = \underbrace{2\, n_{\omega+1}}_{\text{generation}} + \sum_{\omega'=\omega+1}^{\Omega} \underbrace{T_{\omega'-\omega}\, n_{\omega'}}_{\text{absorption}}$$

   From here, derive Horton's law for stream numbers: $n_\omega / n_{\omega+1} = R_n$, where $R_n > 1$ and is independent of ω, and find R_n in terms of Tokunaga's two parameters T_1 and R_T.

3. Show $R_s = R_\ell$. In other words show that Horton's law of stream segments matches that of main stream lengths.

4. Show $R_n = R_a$ by using Tokunaga's law to find the average area of an order ω basin, $\bar{a}_\omega$, in terms of the average area of basins of order 1 to $\omega - 1$.

   (In lectures, we use Horton's laws to roughly demonstrate this result.)

5. For river networks, basin areas are distributed according to $P(a) \propto a^{-\tau}$.

   Determine the exponent τ in terms of the Horton ratios R_n and R_s.