

Chapter 2: Lecture 7

Linear Algebra, Course 124C, Spring, 2009

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Basics:

Sections covered on first midterm:

- ▶ Chapter 1 and Chapter 2 (Sections 2.1–2.7)
- ▶ Chapter 2 is our focus
- ▶ Knowledge of Chapter 1 as needed for Chapter 2 = solving $A\vec{x} = \vec{b}$.
- ▶ Want ‘understanding’ and ‘doing’ abilities.

Stuff to know:

Row, Column, & Matrix Pictures of Linear Systems ($A\vec{x} = \vec{b}$)

- ▶ What dimensions of A mean:
 - ▶ m = number of equations
 - ▶ n = number of unknowns (x_1, x_2, \dots)
- ▶ How to draw the row and column pictures.
- ▶ Be able to identify row picture (e.g., as representing 2 planes in 3-d).
- ▶ How to convert between the three pictures.

Solving $A\vec{x} = \vec{b}$ by elimination

Review for Exam 1

Solve four equivalent ways:

1. Simultaneous equations (snore)
2. Row operations on augmented matrix
 - ▶ Systematically transform $A\vec{x} = \vec{b}$ into $U\vec{x} = \vec{c}$
 - ▶ Solve by back substitution
3. Row operations with E_{ij} and P_{ij} matrices
4. Factor A as $A = LU$
 - ▶ Solve two triangular systems by forward and back substitution
 - ▶ First $L\vec{c} = \vec{b}$ then $U\vec{x} = \vec{c}$.

Understand number of solutions business:

- ▶ 0, 1, or ∞ : why, when, ...

Frame 4/7

Stuff to know:

Review for Exam 1

More on $A = LU$:

- ▶ Be able to find the pivots of A (they live in U)
- ▶ Understand how elimination matrices (E_{ij} 's) are constructed from multipliers (l_{ij} 's)
- ▶ Understand how L is made up of inverses of elimination matrices
 - ▶ e.g.: $L = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} A$.
- ▶ Understand how L is made up of the l_{ij} multipliers.
- ▶ Understand how inverses of elimination matrices are simply related to elimination matrices.

Stuff to know:

Matrix algebra

- ▶ Understand basic matrix algebra
- ▶ Understand matrix multiplication
- ▶ Understand multiplication order matters
- ▶ Understand $AB = BA$ is rarely true

Inverses

- ▶ Understand identity matrix I
- ▶ Understand $AA^{-1} = A^{-1}A = I$
- ▶ Find A^{-1} with Gauss-Jordan elimination
- ▶ Perform row reduction on augmented matrix $[A | I]$.
- ▶ Understand that finding A^{-1} solves $A\vec{x} = \vec{b}$ but is often prohibitively expensive to do.
- ▶ $(AB)^{-1} = B^{-1}A^{-1}$

Stuff to know:

Review for Exam 1

Transposes

- ▶ Definition: flip entries across main diagonal
- ▶ $A = A^T$: A is symmetric
- ▶ Important property: $(AB)^T = B^T A^T$

Extra pieces:

- ▶ If $A\vec{x} = \vec{0}$ has a non-zero solution, A has no inverse
- ▶ If $A\vec{x} = \vec{0}$ has a non-zero solution, then $A\vec{x} = \vec{b}$ always has infinitely many solutions.
- ▶ $(A^{-1})^T = (A^T)^{-1}$

Frame 7/7