Data from our man Zipf

Principles of Complex Systems
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Outline

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George Kingsley Zipf:

In brief:

- Zipf (⊞) (1902–1950) was a linguist at Harvard, specializing in Chinese languages.
- Unusual passion for statistical analysis of texts.
- Studied human behavior much more generally...

Zipf's masterwork:

- "Human Behavior and the Principle of Least Effort"
  Addison-Wesley, 1949
  Cambridge, MA [2]

- Bonus field of study: Glottometrics. (⊞)
- Bonus 'word' word: Glossolalia. (⊞)

Human Behavior/Principle of Least Effort:

From the Preface—

Nearly twenty-five years ago it occurred to me that we might gain considerable insight into the mainsprings of human behavior if we viewed it purely as a natural phenomenon like everything else in the universe, ...

And—

... the expressed purpose of this book is to establish The Principle of Least Effort as the primary principle that governs our entire individual and collective behavior ...

The Principle of Least Effort:

Zipf's framing (p. 1):

"... a person in solving his immediate problems will view these against the background of his probable future problems *as estimated by himself*."

"... he will strive ... to minimize the *total work* that he must expend in solving *both* his immediate problems *and* his probable future problems."

"[he will strive to] minimize the *probable average rate of his work-expenditure*..."

Rampaging research

Within Human Behavior and the Principle of Least Effort:

- City sizes
- # retail stores in cities
- # services (barber shops, beauty parlors, cleaning, ...)
- # people in occupations
- # one-way trips in cars and trucks vs. distance
- # new items by dateline
- weight moved between cities by rail
- # telephone messages between cities
- # people moving vs. distance
- # marriages vs. distance

- Observed general dependency of ‘interactions’ between cities A and B on $P_A P_B / D_{AB}$ where P_A and P_B are population size and D_{AB} is distance between A and B.

Zipfian empirics:

- vocabulary balance: $f \sim r^{-1} \rightarrow r \cdot f \sim$ constant (f = frequency, r = rank).

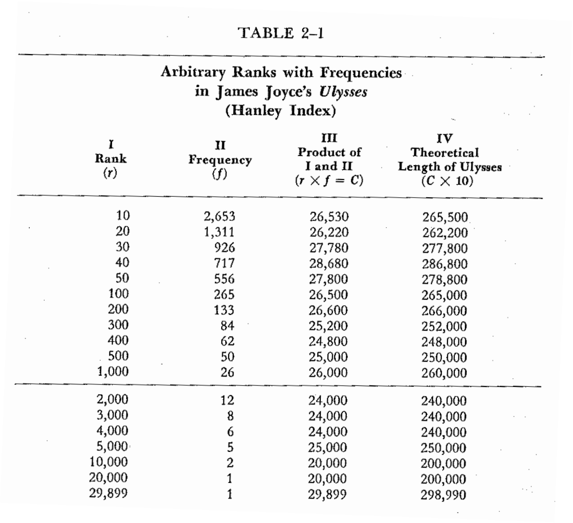


TABLE 2–1

Arbitrary Ranks with Frequencies
in James Joyce's *Ulysses*
(Hanley Index)

I Rank (r)	II Frequency (f)	III Product of I and II ($r \times f = C$)	IV Theoretical Length of Ulysses ($C \times 10$)
10	2,653	26,530	265,500
20	1,311	26,220	262,200
30	926	27,780	277,800
40	717	28,680	286,800
50	556	27,800	278,800
100	265	26,500	265,000
200	133	26,600	266,000
300	84	25,200	252,000
400	62	24,800	248,000
500	50	25,000	250,000
1,000	26	26,000	260,000
2,000	12	24,000	240,000
3,000	8	24,000	240,000
4,000	6	24,000	240,000
5,000	5	25,000	250,000
10,000	2	20,000	200,000
20,000	1	20,000	200,000
29,899	1	29,899	298,990

Zipfian empirics:

- $f \sim r^{-1}$ for word frequency:

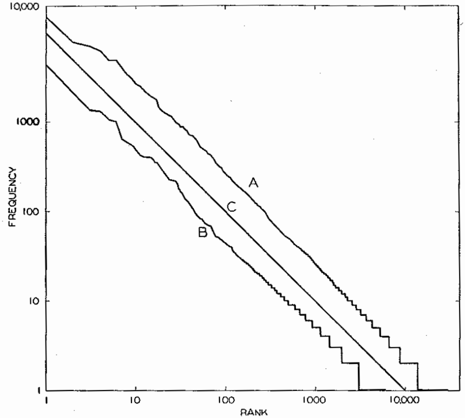


Fig. 2–1. The rank-frequency distribution of words. (A) The James Joyce data; (B) the Eldridge data; (C) ideal curve with slope of negative unity.

Zipf's basic idea:

Forces of Unification and Diversification:

- Easiest for the speaker to use just one word.
  - Encoding is simple but decoding is hard
- Zipf uses the analogy of tools: one tool for all tasks.

- Optimal for listener if all pieces of information correspond to different words (or morphemes).
- Analogy: a specialized tool for every task.
  - Decoding is simple but encoding is hard

- Zipf thereby argues for a tension that should lead to an uneven distribution of word usage.
- No formal theory beyond this...

Zipfian empirics:

- Number of meanings $m_r \propto f_r^{1/2}$ where r is rank and f_r is frequency.

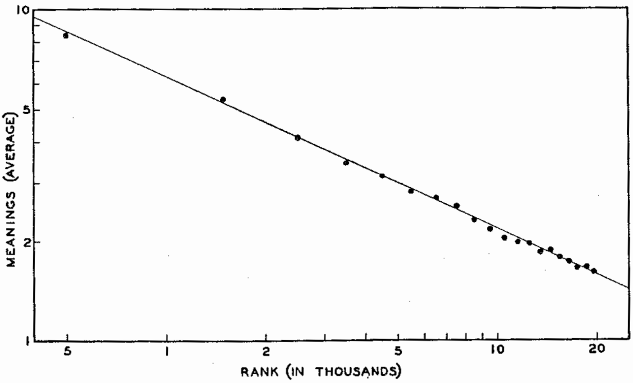


Fig. 2–2. The meaning-frequency distribution of words.

Zipfian empirics:

- Article length in the Encyclopedia Britannica:

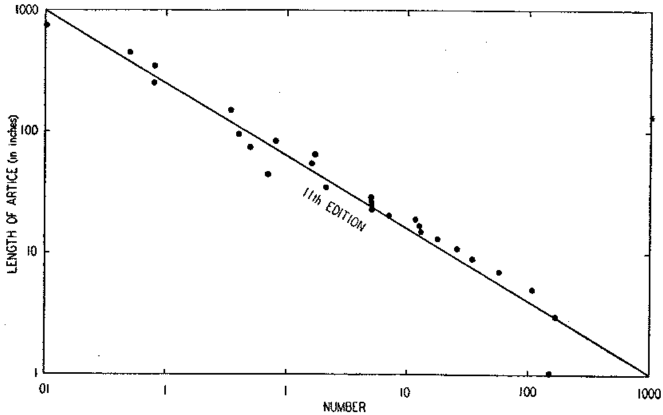


Fig. 5–3. The number of different articles of like length in samples of the 11th edition of the *Encyclopaedia Britannica*. Lengths in inches.

- (?) slope of $-3/5$ corresponds to $\gamma = 5/3$.

Zipfian empirics:

- Population size of districts:

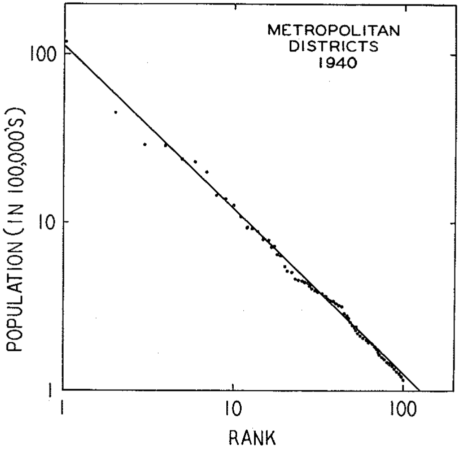


Fig. 9–2. Metropolitan districts. One hundred largest in the U. S. A. in 1940, ranked in the order of decreasing population size.

- $\alpha = 1$ corresponds to $\gamma = 1 + 1/\alpha = 2$.

Zipfian empirics:

- Number of employees in organizations

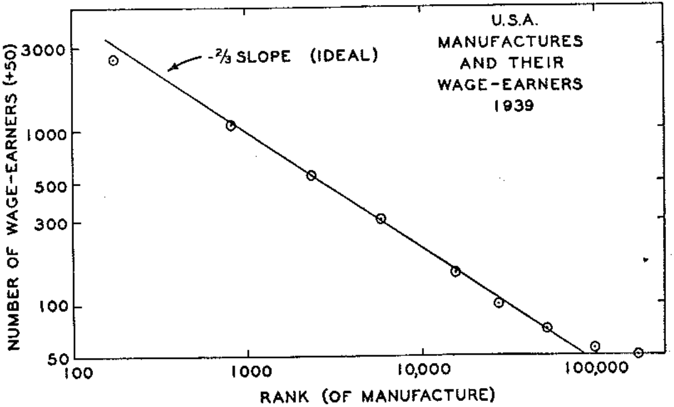


Fig. 9–8. Manufactures and their wage earners in the U. S. A. in 1939, with the manufactures ranked in the order of their decreasing number of wage earners.

- $\alpha = 2/3$ corresponds to $\gamma = 1 + 1/\alpha = 5/2$.

Zipfian empirics:

- # news items as a function of population P_2 of location in the Chicago Tribune
- D = distance, P_1 = Chicago's population
- Solid line = +1 exponent.

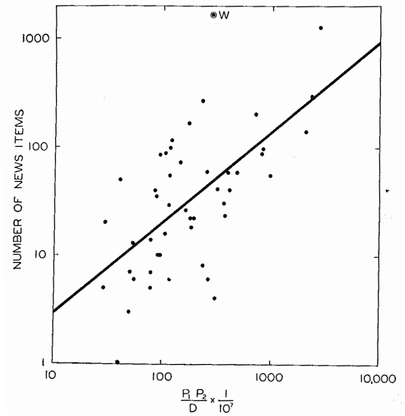


Fig. 9–10. Number of different news items in *The Chicago Tribune* (*W* is the dateline of Washington, D. C.).

Zipfian empirics:

- # obituaries in the New York Times for locations with population P_2.
- D = distance, P_1 = New York's population
- Solid line = +1 exponent.

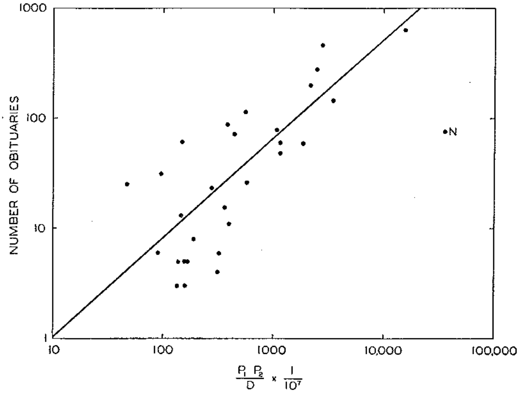


Fig. 9–11. Number of obituaries in *The New York Times* (*N* represents Newark, New Jersey).

Zipfian empirics:

- Movement of stuff between cities
- D = distance, P_1 and P_2 = city populations.
- Solid line = +1 exponent.

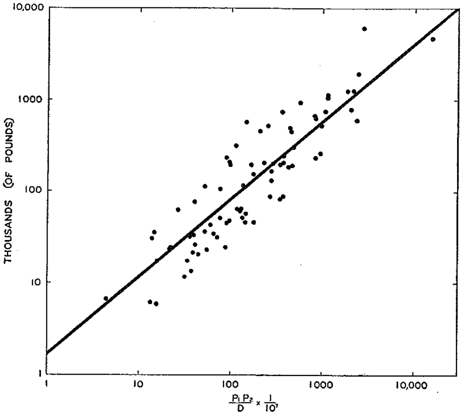


Fig. 9–14. Railway express. The movement by weight (less carload lots) between 13 arbitrary cities in the U. S. A., May 1939.

Zipfian empirics:

- Length of trip versus frequency of trip.
- Solid line = -1/2 exponent corresponds to $\gamma = 2$.

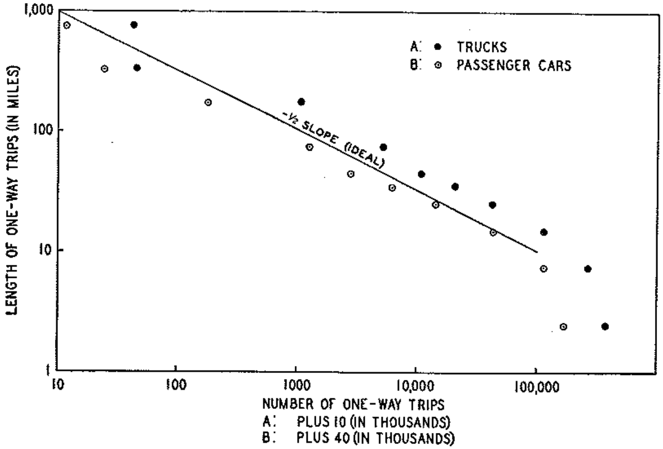


Fig. 9–19. Trucks and passenger cars: the number of one-way trips of like length.

Zipfian empirics:

- The probability of marriage?
- $\gamma = 1$?

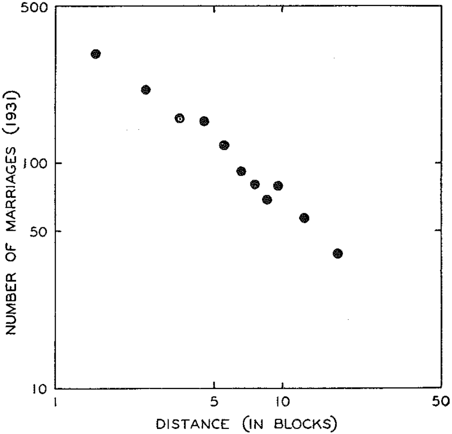
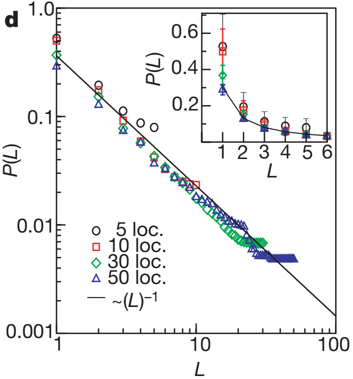


Fig. 9-22. Number of marriage licenses issued to 5,000 pairs of applicants living within Philadelphia in 1931 and separated by varying distances (the data of J. H. S. Bossard).

Recent Zipf action:



- Probability of people being in certain locations follows a Zipfish law...
- From Gonzàlez et al., Nature (2008) “Understanding individual human mobility patterns”[1]

References I

M. C. González, C. A. Hidalgo, and A.-L. Barabási.
Understanding individual human mobility patterns.
Nature, 453:779–782, 2008. pdf (⊞)

G. K. Zipf.
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