More Mechanisms for Generating Power-Law Distributions

Principles of Complex Systems
Course 300, Fall, 2008

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Outline

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Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
Extra
Robustness
HOT theory
Predicting social catastrophe
Self-Organized Criticality
COLD theory
Network robustness
References

Another approach

Benoit Mandelbrot

- Mandelbrot = father of fractals
- Mandelbrot = almond bread
- Derived Zipf's law through optimization [11]
- Idea: Language is efficient
- Communicate as much information as possible for as little cost
- Need measures of information (H) and cost (C)...
- Minimize C/H by varying word frequency
- Recurring theme: what role does optimization play in complex systems?

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- Simon (1955): “On a class of skew distribution functions” [14]
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Outline

Zipfarama via Optimization

Mandelbrot's Assumptions

- Language contains n words: $w_1, w_2, \ldots, w_n$.
- ith word appears with probability p_i
- Words appear randomly according to this distribution (obviously not true...)
- Words = composition of letters is important
- Alphabet contains m letters
- Words are ordered by length (shortest first)

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- Length of word (plus a space)
- Word length was irrelevant for Simon's method

Objection

- Real words don't use all letter sequences

Objections to Objection

- Maybe real words roughly follow this pattern (?)
- Words can be encoded this way
- Na na na-na naaaaa...

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Binary alphabet plus a space symbol

i	1	2	3	4	5	6	7	8
word	1	10	11	100	101	110	111	1000
length	1	2	2	3	3	3	3	4
$1 + \ln_2 i$	1	2	2.58	3	3.32	3.58	3.81	4

- Word length of 2^kth word: $= k + 1$
- Word length of ith word $\simeq 1 + \log_2 i$
- For an alphabet with m letters,
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Total Cost C

- Cost of the ith word: $C_i \simeq 1 + \log_m i$
- Cost of the ith word plus space: $C_i \simeq 1 + \log_m (i+1)$
- Subtract fixed cost: $C'_i = C_i - 1 \simeq \log_m (i+1)$
- Simplify base of logarithm:

$$C'_i \simeq \log_m (i+1) = \frac{\log_e (i+1)}{\log_e m}$$

- Total Cost:

$$C \sim \sum_{i=1}^{n} p_i C'_i \propto \sum_{i=1}^{n} p_i \ln(i+1)$$

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- Use Shannon's Entropy (or Uncertainty):

$$H = -\sum_{i=1}^{n} p_i \log_2 p_i$$

- (allegedly) von Neumann suggested 'entropy'...
- Proportional to average number of bits needed to encode each 'word' based on frequency of occurrence
- $-\log_2 p_i = \log_2 1/p_i$ = minimum number of bits needed to distinguish event i from all others
- If $p_i = 1/2$, need only 1 bit ($log_2 1/p_i = 1$)
- If $p_i = 1/64$, need 6 bits ($log_2 1/p_i = 6$)

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where $g = 1/\ln 2$

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Zipfarama via Optimization

- Minimize

$$F(p_1, p_2, \ldots, p_n) = C/H$$

subject to constraint

$$\sum_{i=1}^{n} p_i = 1$$

- Tension:
(1) Shorter words are cheaper

- (Good) question: how much does choice of C/H as function to minimize affect things?

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Zipfarama via Optimization

- Minimize
$$F(p_1, p_2, \ldots, p_n) = C/H$$
subject to constraint
$$\sum_{i=1}^{n} p_i = 1$$
- Tension:
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(2) Longer words are more informative (rarer)
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Outline

Zipfarama via Optimization

Time for Lagrange Multipliers:

- Minimize

$$\Psi(p_1, p_2, \ldots, p_n) =$$

$$F(p_1, p_2, \ldots, p_n) + \lambda G(p_1, p_2, \ldots, p_n)$$

where

$$F(p_1, p_2, \ldots, p_n) = \frac{C}{H} = \frac{\sum_{i=1}^{n} p_i \ln(i+1)}{-g \sum_{i=1}^{n} p_i \ln p_i}$$

and the constraint function is

$$G(p_1, p_2, \ldots, p_n) = \sum_{i=1}^{n} p_i - 1 = 0$$

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- [Insert assignment problem...]

Zipfarama via Optimization

Some mild suffering leads to:

- $$p_j = e^{-1-\lambda H^2/gC}(j+1)^{-H/gC} \propto (j+1)^{-H/gC}$$
- A power law appears [applause]: $\alpha = H/gC$
- Next: sneakily deduce λ in terms of g, C, and H.
- Find

$$p_j = (j+1)^{-H/gC}$$

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Zipfarama via Optimization

Finding the exponent

- Now use the normalization constraint:

$$1 = \sum_{j=1}^{n} p_j = \sum_{j=1}^{n} (j+1)^{-H/gC} = \sum_{j=1}^{n} (j+1)^{-\alpha}$$

- As $n \to \infty$, we end up with $\zeta(H/gC) = 2$ where ζ is the Riemann Zeta Function
- Gives $\alpha \simeq 1.73$ (> 1, too high)
- If cost function changes ($j + 1 \to j + a$) then exponent is tunable
- Increase a, decrease α

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All told:

- Reasonable approach: Optimization is at work in evolutionary processes
- But optimization can involve many incommensurate elements: monetary cost, robustness, happiness,...
- Mandelbrot's argument is not super convincing
- Exponent depends too much on a loose definition of cost

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More

Reconciling Mandelbrot and Simon

- Mixture of local optimization and randomness
- Numerous efforts...

1. Carlson and Doyle, 1999:
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More

Other mechanisms:

Much argument about whether or not monkeys typing could produce Zipf's law... (Miller, 1957) [12]

Outline

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
Extra
Robustness
HOT theory
Predicting social catastrophe
Self-Organized Criticality
COLD theory
Network robustness
References

Others are also not happy

Krugman and Simon

- "The Self-Organizing Economy" (Paul Krugman, 1995) [10]
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Who needs a hug?

From Berry [4]

- Déjà vu, Mr. Krugman. Been there, done that. The Simon-Ijiri model was introduced to geographers in 1958 as an explanation of city size distributions, the first of many such contributions dealing with the steady states of random growth processes, ...
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Outline

Robustness

- Many complex systems are prone to cascading catastrophic failure:
  - Blackouts
  - Disease outbreaks
  - Wildfires
  - Earthquakes
- But complex systems also show persistent robustness
- Robustness and Failure may be a power-law story...

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- System robustness may result from
  1. Evolutionary processes
  2. Engineering/Design
- Idea: Explore systems optimized to perform under uncertain conditions.
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Robustness

Features of HOT systems: [6]

- High performance and robustness
- Designed/evolved to handle known environmental variability
- Fragile in the face of unpredicted environmental signals
- Highly specialized, low entropy configurations
- Power-law distributions

Robustness

HOT combines things we've seen:

- Variable transformation
- Constrained optimization
- Need power law transformation between variables: $(Y = X^{-\alpha})$
- MIWO is good: Mild In, Wild Out
- X has a characteristic size but Y does not

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Robustness

Forest fire example [6]

- Square $N \times N$ grid
- Sites contain a tree with probability ρ = density
- Sites are empty with probability $1 - \rho$
- Fires start at location according to some distribution P_{ij}
- Fires spread from tree to tree
- Connected clusters of trees burn completely
- Empty sites block fire
- Best case scenario:
  Maximize average # trees left intact

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- Build a forest by adding one tree at a time
- Test D ways of adding one tree
- D = design parameter
- Average over P_{ij} = spark probability
- $D = 1$: random addition
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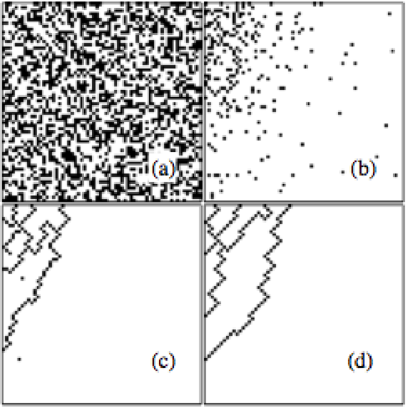
HOT Forests



[6]

$N = 64$

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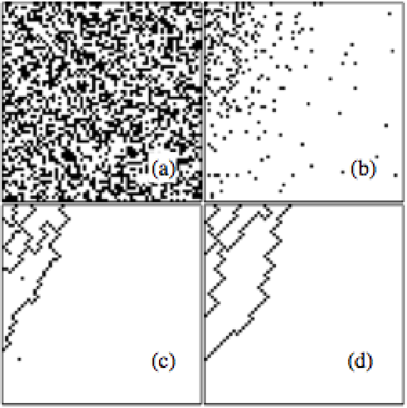
P_{ij} has a Gaussian decay

HOT Forests



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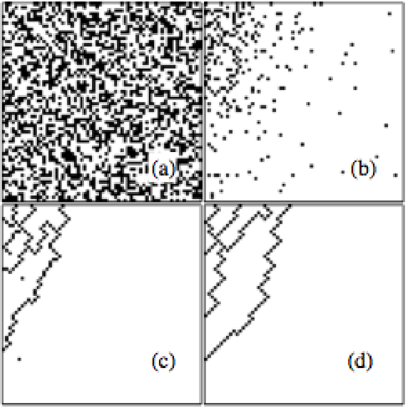

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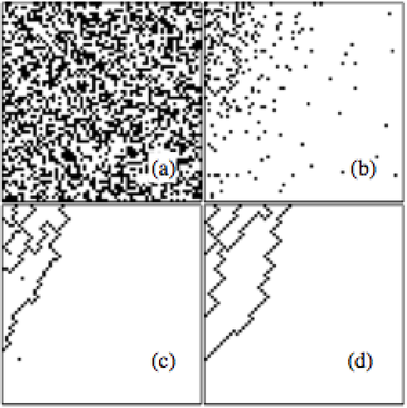
Optimized forests do well on average but rare extreme events occur

HOT Forests



[6]

$N = 64$

(a) $D = 1$
(b) $D = 2$
(c) $D = N$
(d) $D = N^2$

P_{ij} has a Gaussian decay

Optimized forests do well on average (robustness) but rare extreme events occur (fragility)

Random Forests

$D = 1$: Random forests = Percolation

- Randomly add trees
- Below critical density ρ_c, no fires take off
- Above critical density ρ_c, percolating cluster of trees burns
- Only at ρ_c, the critical density, is there a power-law distribution of tree cluster sizes
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- Highly structured
- Power law distribution of tree cluster sizes for $\rho > \rho_c$
- No specialness of ρ_c
- Forest states are tolerant
- Uncertainty is okay if well characterized
- If P_{ij} is characterized poorly, failure becomes highly likely

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The abstract story:

- Given $y_i = x_i^{-\alpha}$, $i = 1, \ldots, N$
- Design system to minimize $\langle y \rangle$ subject to a constraint on the x_i
- Minimize cost:

$$C = \sum_{i=1}^{N} Pr(y_i) y_i$$

Subject to $\sum_{i=1}^{N} x_i = \text{constant}$

- Drag out the Lagrange Multipliers, battle away and find:

$$p_i \propto y_i^{-\gamma}$$

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Optimal fire walls in d dimensions:

- Two costs:
  1. Expected size of fire

$$C_{\text{fire}} \propto \sum_{i=1}^{N} (p_i a_i) a_i = \sum_{i=1}^{N} p_i a_i^2$$

  a_i = area of ith region
  p_i = average probability of fire at site in ith region

  2. Cost of building and maintaining firewalls

$$C_{\text{firewalls}} \propto \sum_{i=1}^{N} a_i^{1/2}$$

  In d dimensions, 1/2 is replaced by $(d-1)/d$

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- Minimize C_{fire} given $C_{\text{firewalls}} = \text{constant}$.
- $$\frac{\partial}{\partial a_j}\left(C_{\text{fire}} - \lambda C_{\text{firewalls}}\right) = 0$$
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Basic idea of designed tolerance

- Build more firewalls in areas where sparks are likely
- Small connected regions in high-danger areas
- Large connected regions in low-danger areas
- Routinely see many small outbreaks (robust)
- Rarely see large outbreaks (fragile)
- Sensitive to changes in the environment (P_{ij})

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Jon Stewart:

"You just bummed the @*!# out of me."

wildbluffmedia.com

- From the Daily Show (⊞) (September 18, 2007)
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Outline

Avalanches on Sand and Rice

SOC theory

SOC = Self-Organized Criticality

- Idea: natural dissipative systems exist at ‘critical states’
- Analogy: Ising model with temperature somehow self-tuning
- Power-law distributions of sizes and frequencies arise ‘for free’
- Introduced in 1987 by Bak, Tang, and Weisenfeld [3, 2, 9]:
  “Self-organized criticality - an explanation of 1/f noise”
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HOT versus SOC

- Both produce power laws
- Optimization versus self-tuning
- HOT systems viable over a wide range of high densities
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- HOT systems produce specialized structures
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- Optimization versus self-tuning
- HOT systems viable over a wide range of high densities
- SOC systems have one special density
- HOT systems produce specialized structures
- SOC systems produce generic structures

Robustness

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Outline

COLD forests

Avoidance of large-scale failures

- Constrained Optimization with Limited Deviations [13]
- Weight cost of larges losses more strongly
- Increases average cluster of trees...
- ... but reduces chances of catastrophe
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Cutoffs

Aside:

- Power law distributions often have an exponential cutoff
- $$P(x) \sim x^{-\gamma} \exp^{-x/x_c}$$
- where x_c is the approximate cutoff scale.

Outline

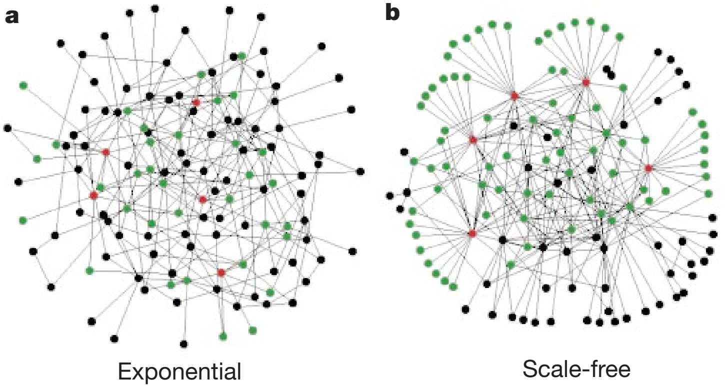
Robustness

And we've already seen this...

- network robustness.
- Albert et al., Nature, 2000:
  "Error and attack tolerance of complex networks" [1]

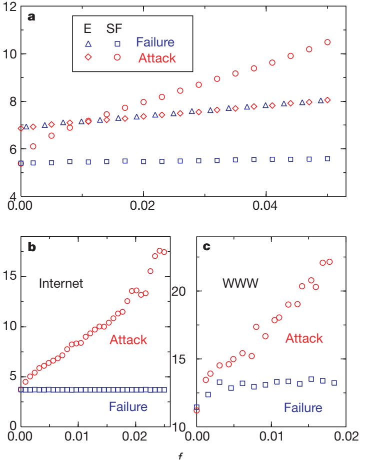
Robustness

- Standard random networks (Erdös-Rényi) versus Scale-free networks



from Albert et al., 2000 [1]

Robustness



from Albert et al., 2000 [1]

- Plots of network diameter as a function of fraction of nodes removed
- Erdös-Rényi versus scale-free networks
- blue symbols = random removal
- red symbols = targeted removal (most connected first)

Robustness

- Scale-free networks are thus robust to random failures yet fragile to targeted ones.
- All very reasonable: Hubs are a big deal.
- But: next issue is whether hubs are vulnerable or not.
- Representing all webpages as the same size node is obviously a stretch (e.g., google vs. a random person's webpage)
- Most connected nodes are either:
  1. Physically larger nodes that may be harder to 'target'
  2. or subnetworks of smaller, normal-sized nodes.
- Need to explore cost of various targeting schemes.

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