

The amusing and excellent law of Benford

Principles of Complex Systems
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Benford's law

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References

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Outline

Benford's law

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Benford's law—The law of first digits

- ▶ First observed by Simon **Newcomb**^[2] in 1881
“Note on the Frequency of Use of the Different Digits in Natural Numbers”
- ▶ Independently discovered by Frank **Benford** in 1938.
- ▶ Newcomb almost always noted but Benford gets the stamp

$$P(\text{first digit} = d) \propto \log_b(d + 1/d)$$

for numbers in base b

Benford's law

Benford's law
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Benford's Law—The law of first digits

Observed for

- ▶ Fundamental constants (electron mass, charge, etc.)
- ▶ Utilities bills
- ▶ Numbers on tax returns
- ▶ Death rates
- ▶ Street addresses
- ▶ Numbers in newspapers

Benford's law

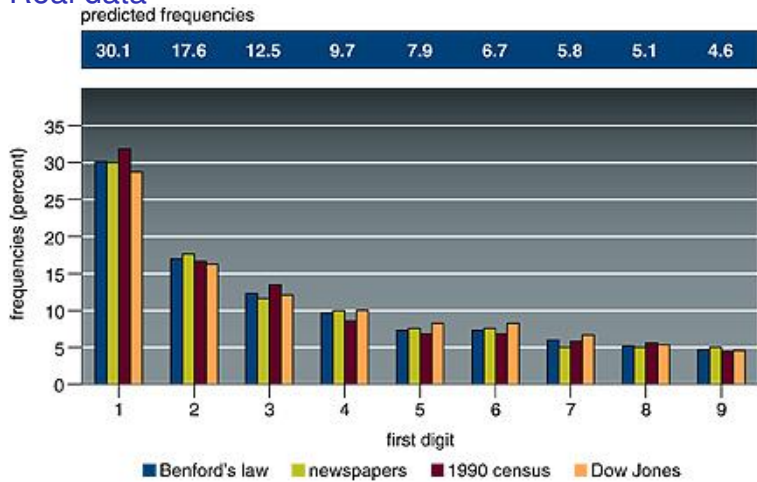
Benford's law
References

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Benford's law

Real data



From 'The First-Digit Phenomenon' by T. P. Hill (1998) [1]

Essential story

- ▶ $P(\text{first digit} = d) \propto \log_b(d + 1/d)$
- ▶ $P(\text{first digit} = d) \propto \log_b\left(\frac{d+1}{d}\right)$
- ▶ $P(\text{first digit} = d) \propto \log_b(d + 1) - \log_b(d)$
- ▶ So numbers are distributed uniformly in log-space:
 $P(\ln x) d(\ln x) \propto 1 \cdot d(\ln x) = x^{-1} dx$
- ▶ Power law distributions at work again... ($\gamma = 1$)

A different Benford

Not to be confused with **Benford's law of controversy**:

- ▶ "Passion is inversely proportional to the amount of real information available."

Gregory Benford, Sci-Fi writer & Astrophysicist

References I

- T. P. Hill.
The first-digit phenomenon.
American Scientist, 86:358–, 1998.
- S. Newcomb.
Note on the frequency of use of the different digits in natural numbers.
American Journal of Mathematics, 4:39–40, 1881.
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