The amusing and excellent law of Benford

Principles of Complex Systems
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Prof. Peter Dodds

Department of Mathematics & Statistics
University of Vermont

Outline

Benford's law

References

Benford's law—The law of first digits

- First observed by Simon Newcomb [2] in 1881 "Note on the Frequency of Use of the Different Digits in Natural Numbers"
- Independently discovered by Frank Benford in 1938.
- Newcomb almost always noted but Benford gets the stamp
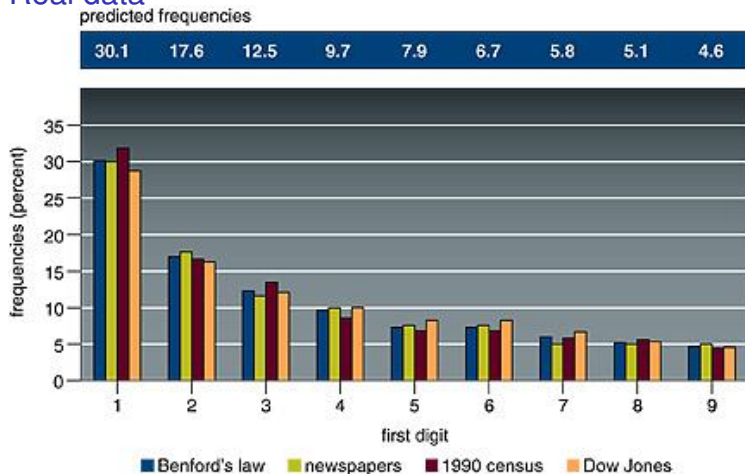-
$$P(\text{first digit} = d) \propto \log_b (d + 1/d)$$

for numbers is base b

Benford's Law—The law of first digits

Observed for

- Fundamental constants (electron mass, charge, etc.)
- Utilities bills
- Numbers on tax returns
- Death rates
- Street addresses
- Numbers in newspapers

Benford's law

Real data

predicted frequencies

| 30.1 | 17.6 | 12.5 | 9.7 | 7.9 | 6.7 | 5.8 | 5.1 | 4.6 |

From 'The First-Digit Phenomenon' by T. P. Hill (1998) [1]

Essential story

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$$P(\text{first digit} = d) \propto \log_b (d + 1/d)$$

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$$P(\text{first digit} = d) \propto \log_b \left(\frac{d + 1}{d} \right)$$

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$$P(\text{first digit} = d) \propto \log_b (d + 1) - \log_b (d)$$

▶ So numbers are distributed uniformly in log-space:

$$P(\ln x)\, \mathrm{d}(\ln x) \propto 1 \cdot \mathrm{d}(\ln x) = x^{-1}\, \mathrm{d}x$$

▶ Power law distributions at work again... ($\gamma = 1$)

A different Benford

Not to be confused with Benford's law of controversy:

- "Passion is inversely proportional to the amount of real information available."

Gregory Benford, Sci-Fi writer & Astrophysicist

References I

📄 T. P. Hill.
The first-digit phenomenon.
American Scientist, 86:358–, 1998.

📄 S. Newcomb.
Note on the frequency of use of the different digits in natural numbers.
American Journal of Mathematics, 4:39–40, 1881.
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