Applications of Random Networks

Complex Networks, Course 295A, Spring, 2008

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Department of Mathematics & Statistics
University of Vermont

Outline

Analysis of real networks

How to build revisited

Motifs

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More on building random networks

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- Degree frequency N_k = observed frequency of degrees for a real network.
- What we now need to do: Create an ensemble of random networks with degree frequency N_k and then compare.

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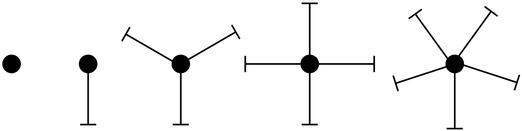
Building random networks: Stubs

Phase 1:

- Idea: start with a soup of unconnected nodes with stubs (half-edges):



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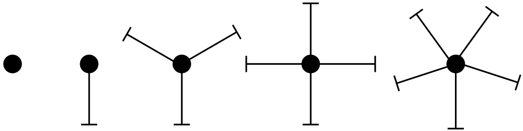

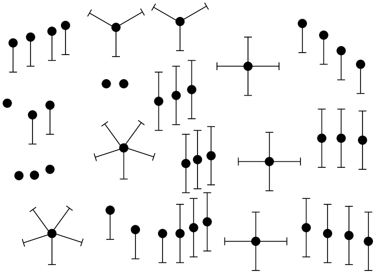

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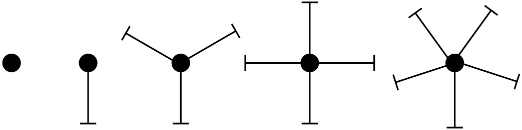

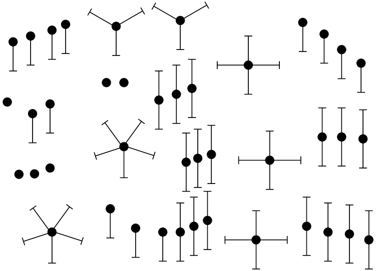

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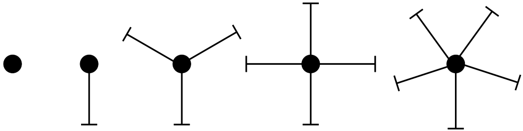

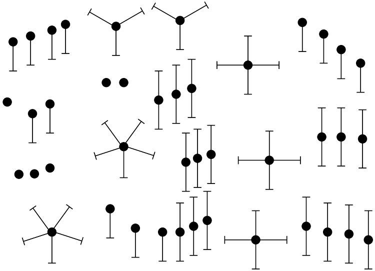

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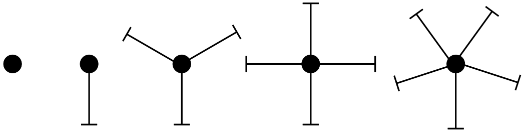

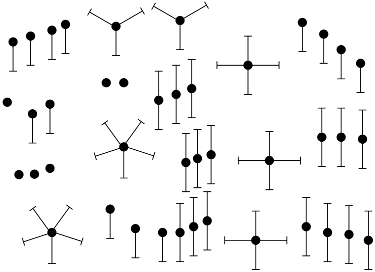

- Randomly select stubs (not nodes!) and connect them.
- Must have an even number of stubs.
- Initially allow self- and repeat connections.

Building random networks: First rewiring

Phase 2:

- Now find any (A) self-loops and (B) repeat edges and randomly rewire them.

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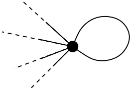

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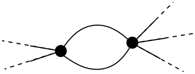

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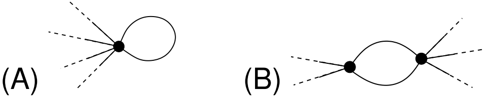

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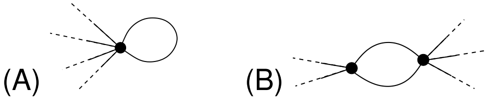

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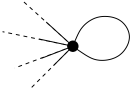

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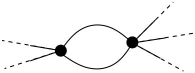

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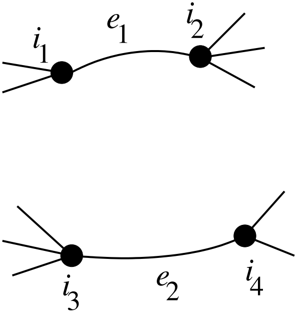
General random rewiring algorithm



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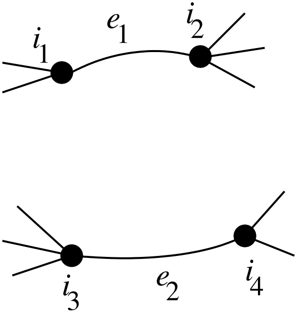

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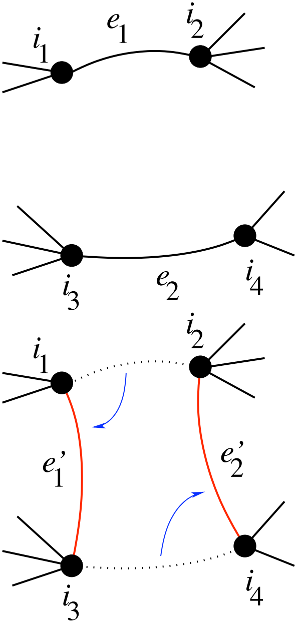

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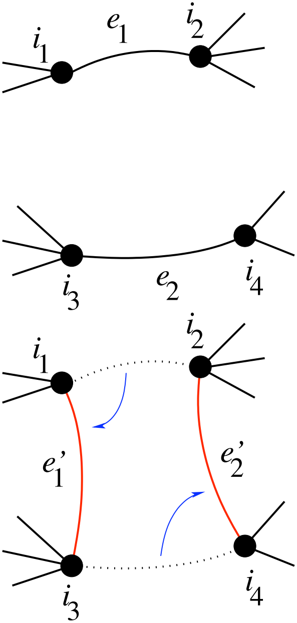

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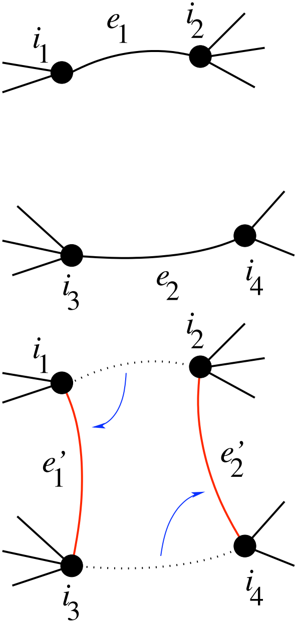

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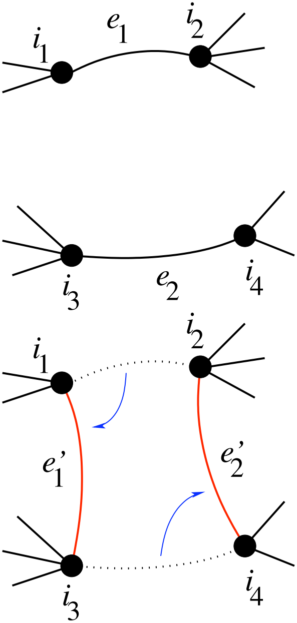

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- Same as finding on/off/on/off 4-cycles. and rotating them.

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- Rule of thumb: # Rewirings $\simeq$ 10 $\times$ # edges [1].

Random sampling

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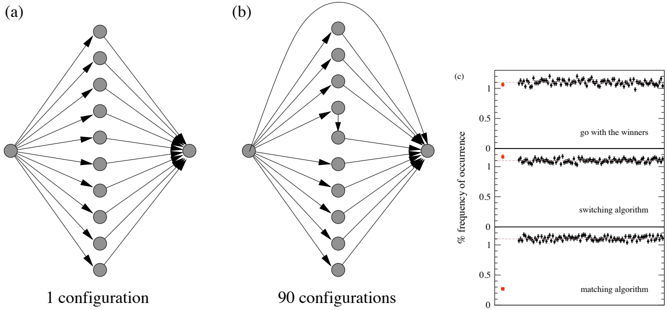
- Example from Milo et al. (2003) [1]:



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- Easy to do exactly numerically since k is discrete.
- Note: not all P_k will always give nodes that can be wired together.

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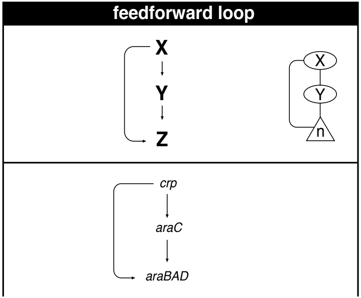
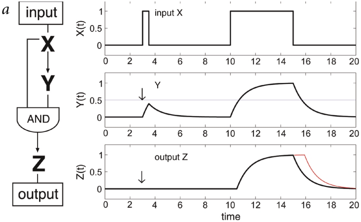
- Used network randomization to produce ensemble of alternate networks with same degree frequency N_k.
- Looked for certain subnetworks (motifs) that appeared more or less often than expected

Network motifs





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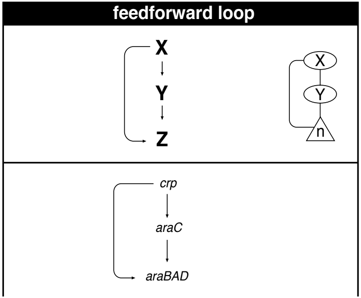

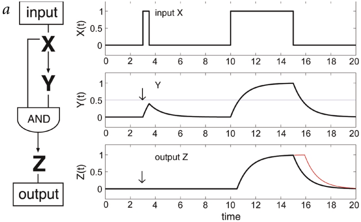

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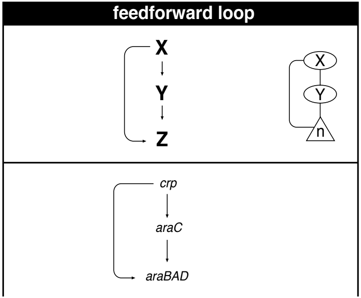

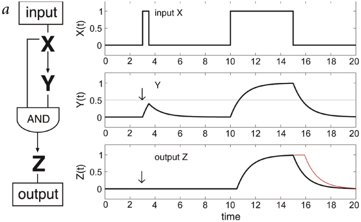

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- Analogy to elevator doors.

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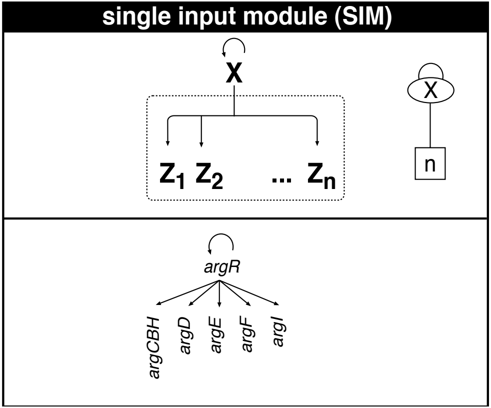

- Master switch.

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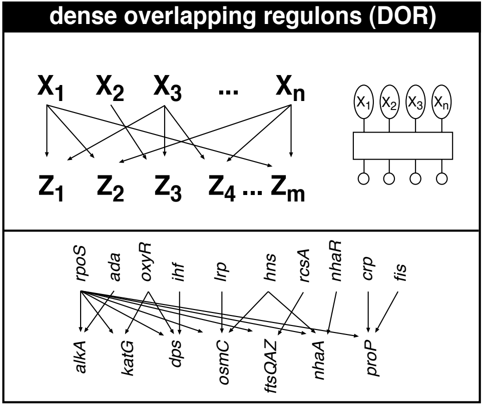

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- For more, see work carried out by Wiggins et al. at Columbia.

References I

R. Milo, N. Kashtan, S. Itzkovitz, M. E. J. Newman, and U. Alon.
On the uniform generation of random graphs with prescribed degree sequences, 2003. pdf (⊞)

S. S. Shen-Orr, R. Milo, S. Mangan, and U. Alon.
Network motifs in the transcriptional regulation network of *Escherichia coli*.
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