Overview of Complex Networks

Complex Networks, Course 295A, Spring, 2008

Prof. Peter Dodds

Department of Mathematics & Statistics
University of Vermont

Outline

Class admin

Outline

Class admin

Basic definitions

Outline

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Books

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Examples of Complex Networks

Outline

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Examples of Complex Networks

Properties of Complex Networks

Modelling Complex Networks

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References

Class Admin

- Office hours: Tuesday 10:45 am–12:30 pm

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- Course outline

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- Projects

Basic definitions

Complex: (Latin = with + fold/weave (com + plex))

Adjective

- Made up of multiple parts; intricate or detailed.
- Not simple or straightforward.

Basic definitions

Complex System—Basic ingredients:

Basic definitions

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- Relationships are nonlinear

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- Relationships contain feedback loops

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- Relationships are nonlinear
- Relationships contain feedback loops
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- Modular (nested)/multiscale structure
- Opaque boundaries
- May produce emergent phenomena

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2. Multiple computers and other devices connected together to share information

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Noun:

1. Any interconnected group or system
2. Multiple computers and other devices connected together to share information

Verb:

1. To interact socially for the purpose of getting connections or personal advancement
2. To connect two or more computers or other computerized devices

Observation

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- Opens door to mathematical and numerical analysis
- Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.

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Nodes = A collection of entities which have properties that are somehow related to each other

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- e.g., people, forks in rivers, proteins, webpages, organisms,...

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- Links may be directed or undirected.

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- Links may be directed or undirected.
- Links may be binary or weighted.

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- Notation: Node i's degree = k_i.
- $k_i = 0,1,2,\ldots$.
- Notation: the average degree of a network = $\langle k \rangle$ (and sometimes as z)

Basic definitions

Adjacency matrix:

- We represent a graph or network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).

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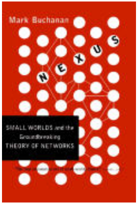
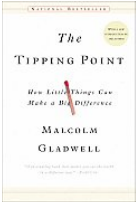
- e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

Books

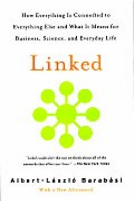


Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

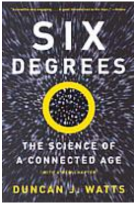


The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell

Books

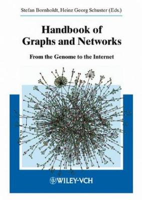


Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási

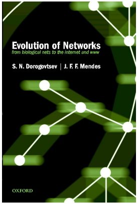


Six Degrees: The Science of a Connected Age—Duncan Watts

Books



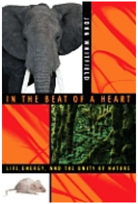
Handbook of Graphs and Networks—editors: Stefan Bornholdt and H. G. Schuster



Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes.

Books

Social Network Analysis—Stanley Wasserman and Kathleen Faust



In the Beat of a Heart: Life, Energy, and the Unity of Nature—John Whitfield

Books

Numerous others:

- Complex Social Networks—F. Vega-Redondo
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung

Examples

What passes for a complex network?

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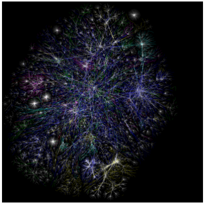
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...

Examples

Physical networks

- River networks



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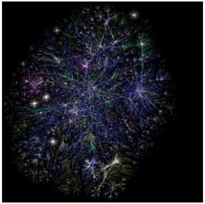
Physical networks

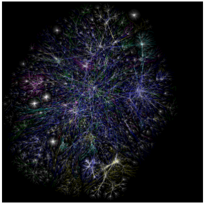
- River networks
- Neural networks

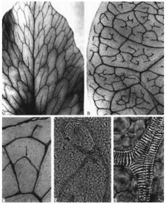


Examples

Physical networks

- River networks
- Neural networks
- Trees and leaves





Examples

Physical networks

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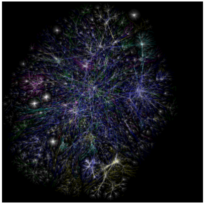
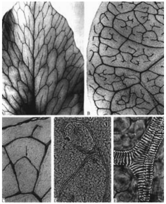
- Trees and leaves
- Blood networks





Examples

Physical networks

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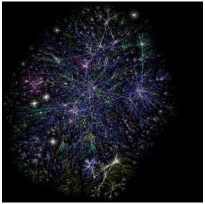
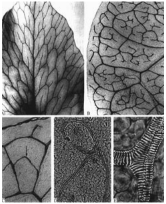
- The Internet





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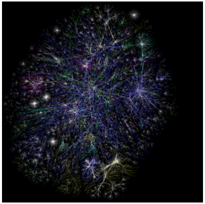

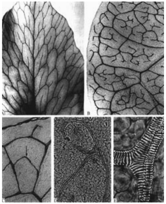

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- Trees and leaves
- Blood networks
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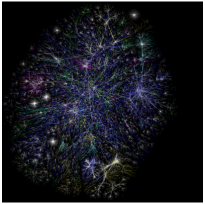
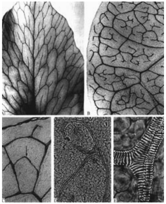
- Road networks
- Power grids





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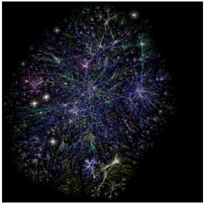
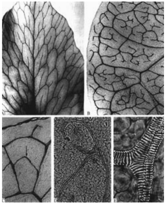


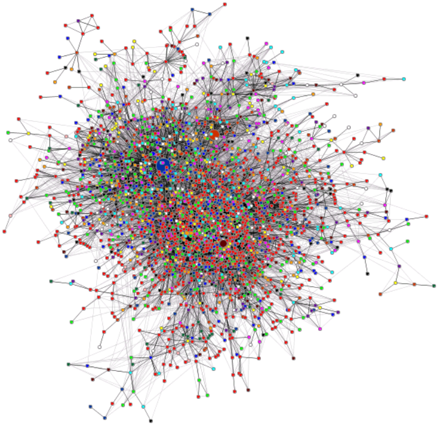


- Distribution (branching) versus redistribution (cyclical)

Examples

Interaction networks

- The Blogosphere



datamining.typepad.com (⊞)

Examples

Interaction networks

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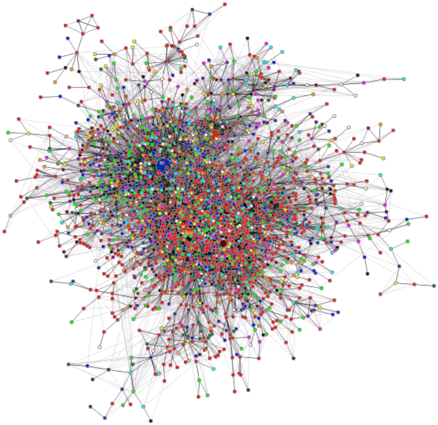
- Biochemical networks



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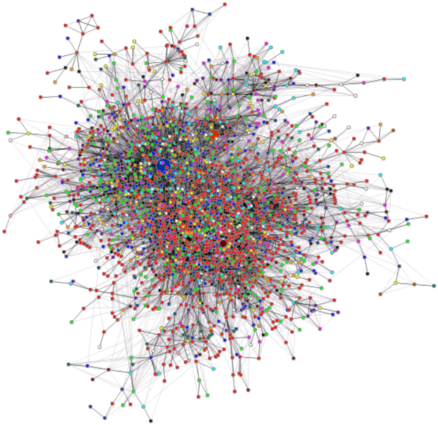
- Food webs: who eats whom



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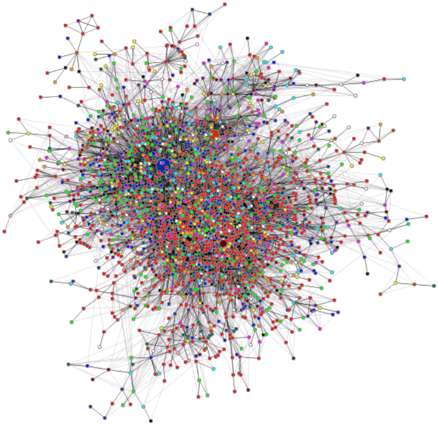

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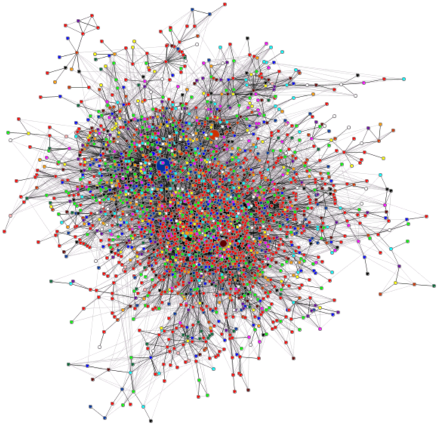

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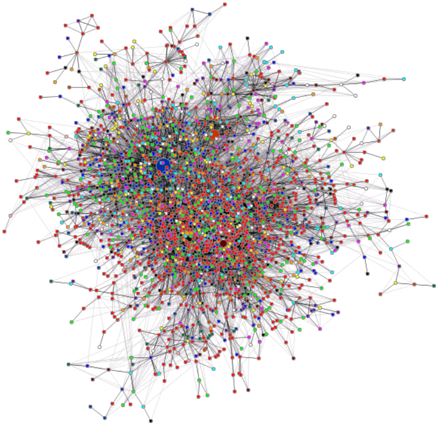
- The Media



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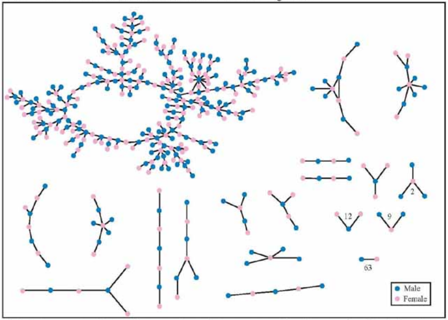
Examples

Interaction networks: social networks

- Snogging

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

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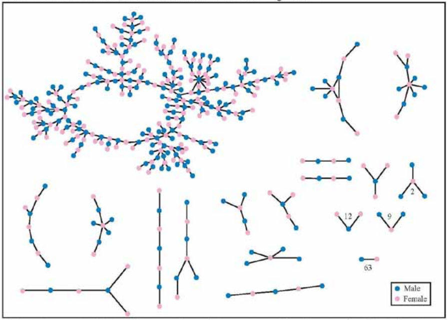

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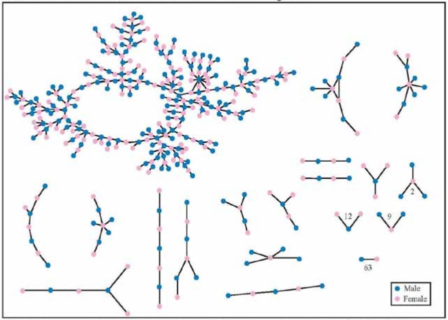

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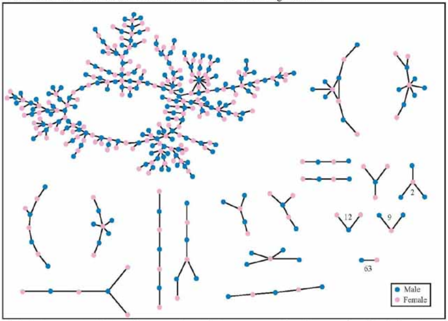

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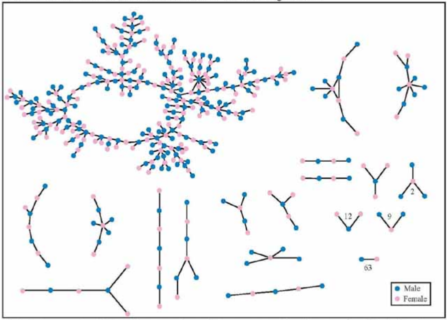

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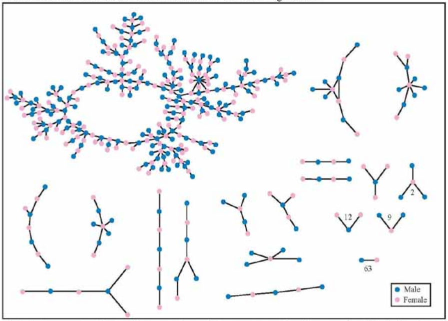

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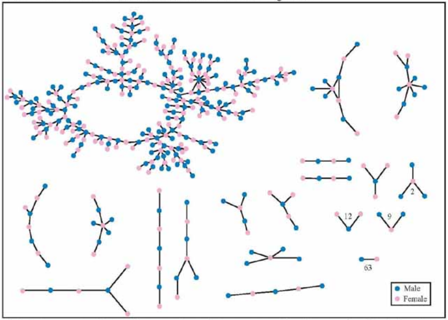

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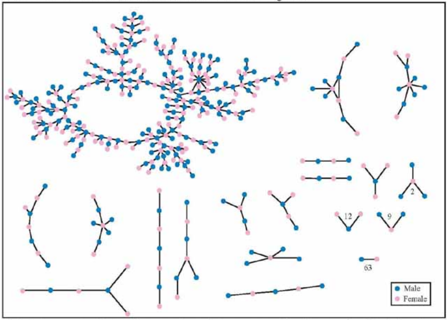

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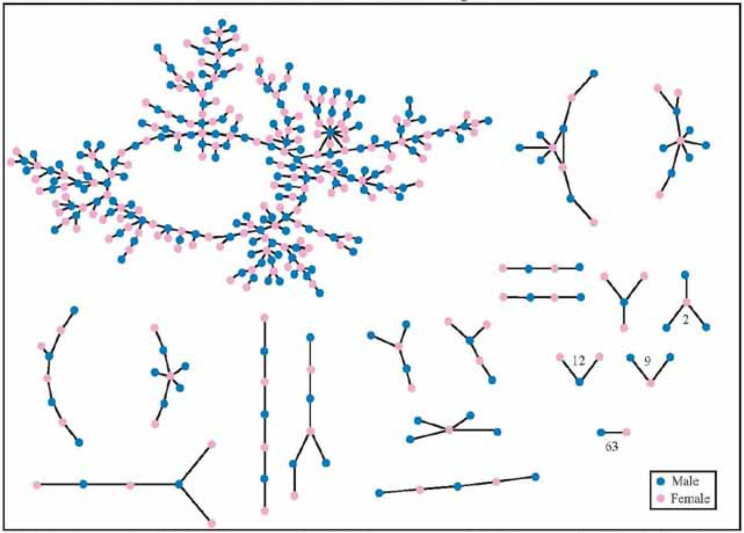

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Relational networks

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- Knowledge/Databases/Ideas

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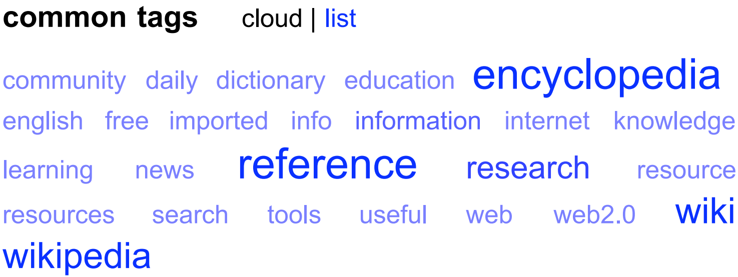
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging:
  del.icio.us (⊞)http://del.icio.usdel.icio.us, flickr (⊞)



Observations

A notable features of large-scale networks:

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- Graphical renderings of complex networks are often just a big mess.

Properties

Some key aspects of real complex networks:

- degree distribution
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
- hierarchical scaling
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- robustness

Properties

Some key aspects of real complex networks:

- degree distribution
- assortativity
- homophily
- clustering
- motifs
- modularity
- concurrency
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- network distances
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- robustness

- + Coevolution of network structure and processes on networks.

Properties

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  Often techological or biological: Internet, WWW, protein interactions, neural networks, food webs.

Clustering

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- In general, $C_1 \neq C_2$.

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- The '3' appears because for each triangle, we have 3 closed triples.

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Properties

5. motifs:

Properties

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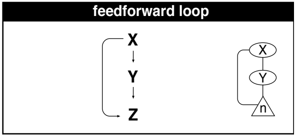
- e.g., Feed Forward Loop:

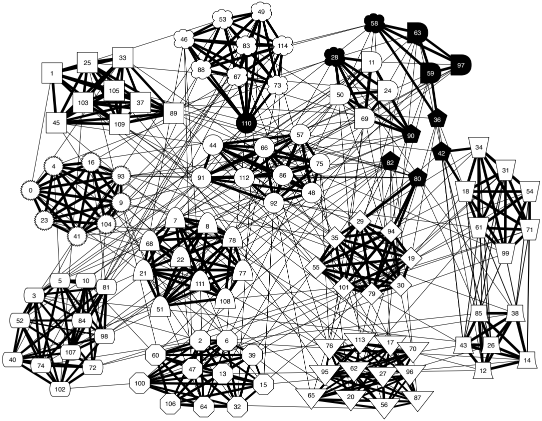
a



Shen-Orr, Uri Alon, *et al.* [9]

Properties

6. modularity—community detection:



Clauset *et al.*, 2006 [4]: NCAA football

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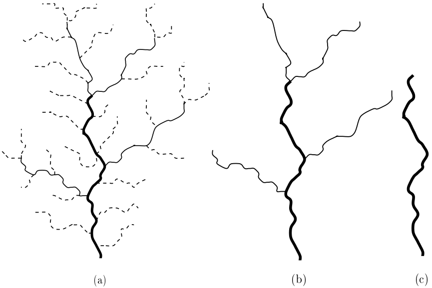
- Kretzschmar and Morris, 1996 [6]

Properties

8. Horton-Strahler ratios:

- Metrics for branching networks:



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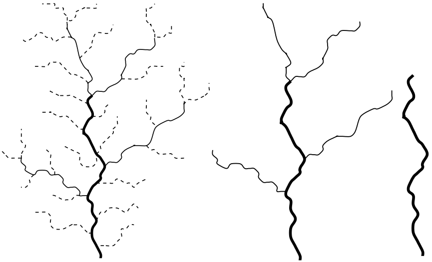

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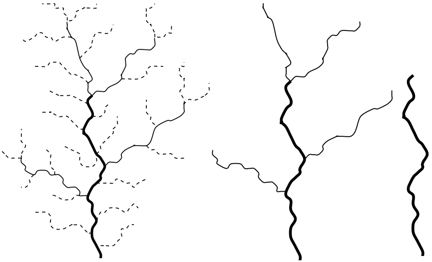

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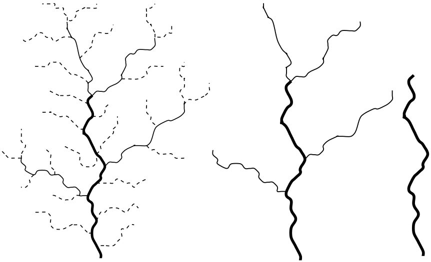

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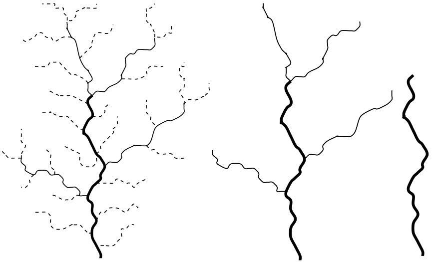
  - Area/Volume: $R_a = \langle a_{\omega+1} \rangle / \langle a_\omega \rangle$



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- closeness $d_{\mathrm{cl}} = [\sum_{ij} {d_{ij}}^{-1} / \binom{n}{2}]^{-1}$:
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- ex 3: Recursive centrality: Hubs and Authorities (Kleinberg [5])

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Some important models:

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- Trickiness: other models generate skewed degree distributions.

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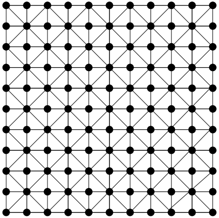

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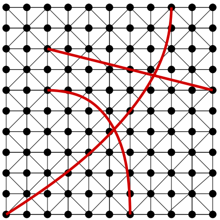

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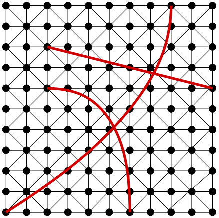

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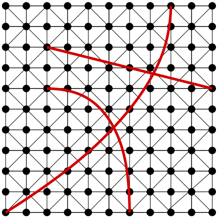
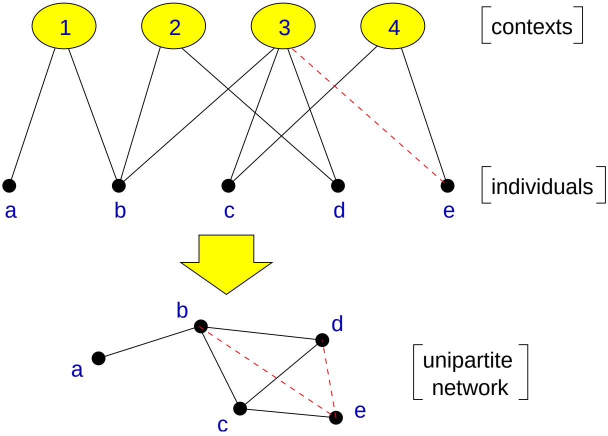
- Number of infectives increases exponentially in time



Models

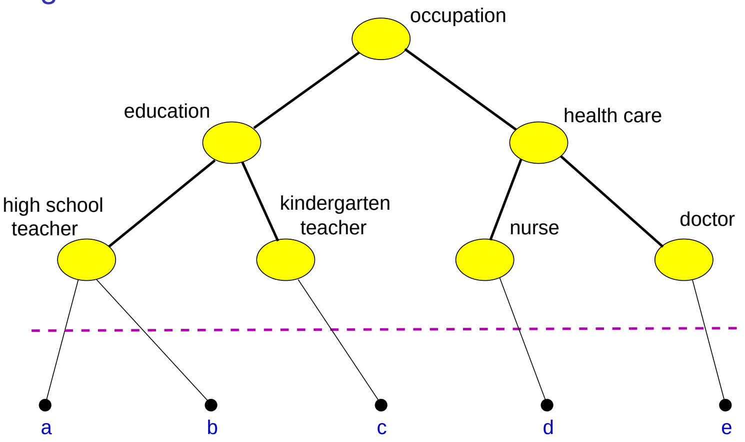
5. generalized affiliation networks



Bipartite affiliation networks: boards and directors, movies and actors.

Models

5. generalized affiliation networks



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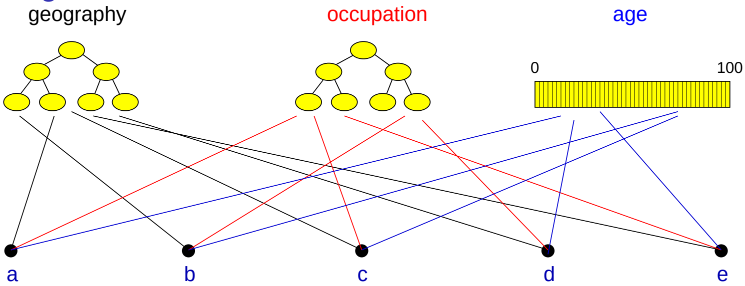

- Blau & Schwartz [2], Simmel [10], Breiger [3], Watts *et al.* [11]

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