

# Singular Value Decomposition

## Linear Algebra, Course 124A, Fall, 2007

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Department of Mathematics & Statistics  
University of Vermont

Approximating  
matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

Bonus Puzzle

Colbert on Equations



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## Approximating matrices with SVD

The basic idea

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## Image approximation (80x60)

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- ▶ Idea: use SVD to approximate images.
- ▶ Interpret elements of matrix  $A$  as color values of an image.
- ▶ Truncate series SVD representation of  $A$ :

$$A = U\Sigma V^T = \sum_{i=1}^r \sigma_i \hat{u}_i \hat{v}_i^T$$

- ▶ Use fact that  $\sigma_1 > \sigma_2 > \dots > \sigma_r > 0$ .
- ▶ For color: approximate 3 matrices (RGB).

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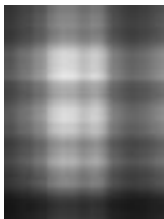
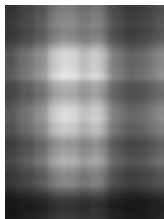
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$$A = \sum_{i=1}^1 \sigma_i \hat{u}_i \hat{v}_i^T$$



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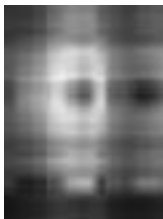
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$$A = \sum_{i=1}^2 \sigma_i \hat{u}_i \hat{v}_i^T$$



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$$A = \sum_{i=1}^3 \sigma_i \hat{u}_i \hat{v}_i^T$$



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$$A = \sum_{i=1}^4 \sigma_i \hat{u}_i \hat{v}_i^T$$



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$$A = \sum_{i=1}^5 \sigma_i \hat{u}_i \hat{v}_i^T$$



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$$A = \sum_{i=1}^6 \sigma_i \hat{u}_i \hat{v}_i^T$$



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$$A = \sum_{i=1}^7 \sigma_i \hat{u}_i \hat{v}_i^T$$



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$$A = \sum_{i=1}^8 \sigma_i \hat{u}_i \hat{v}_i^T$$



# Image approximation (80x60)

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$$A = \sum_{i=1}^9 \sigma_i \hat{u}_i \hat{v}_i^T$$



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$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^T$$



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$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^T$$



# Image approximation (80x60)

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$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^T$$



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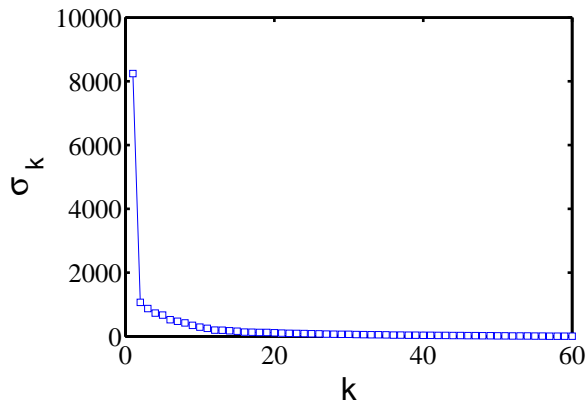
Colbert on Equations

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# Decay of sigma values: Einstein



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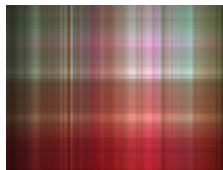
Bonus example 4

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# Image approximation (480x640)

$$A = \sum_{i=1}^1 \sigma_i \hat{u}_i \hat{v}_i^T$$



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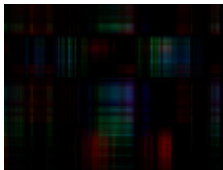
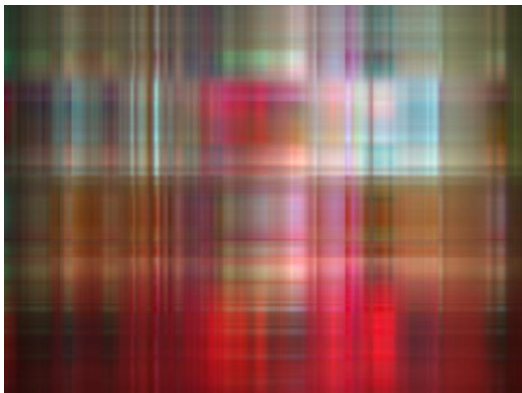
Bonus example 4

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# Image approximation (480x640)

$$A = \sum_{i=1}^2 \sigma_i \hat{u}_i \hat{v}_i^T$$



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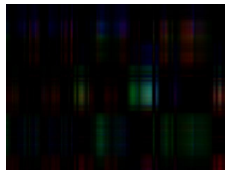
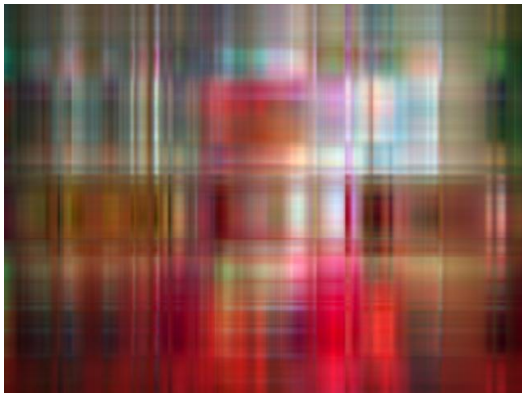
Bonus example 4

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# Image approximation (480x640)

$$A = \sum_{i=1}^3 \sigma_i \hat{u}_i \hat{v}_i^T$$



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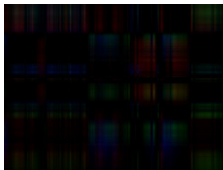
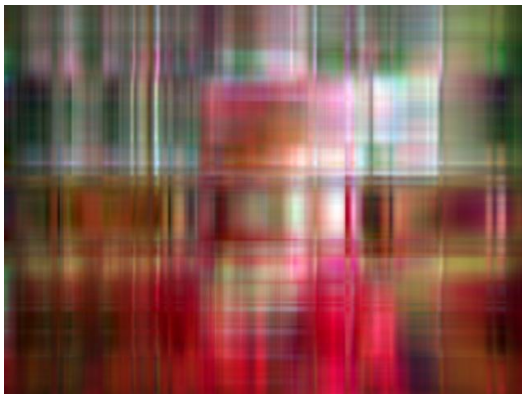
Bonus example 4

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# Image approximation (480x640)

$$A = \sum_{i=1}^4 \sigma_i \hat{u}_i \hat{v}_i^T$$



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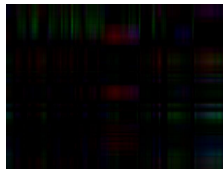
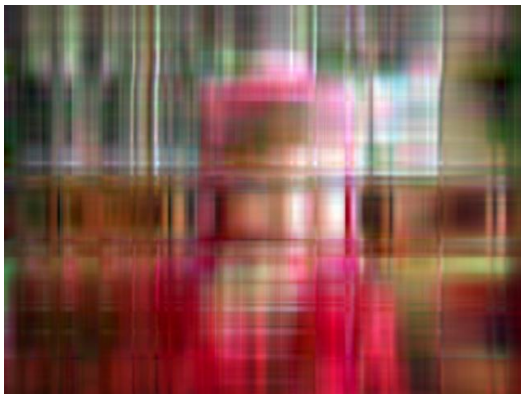
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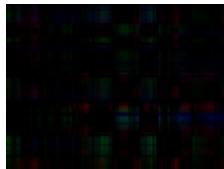
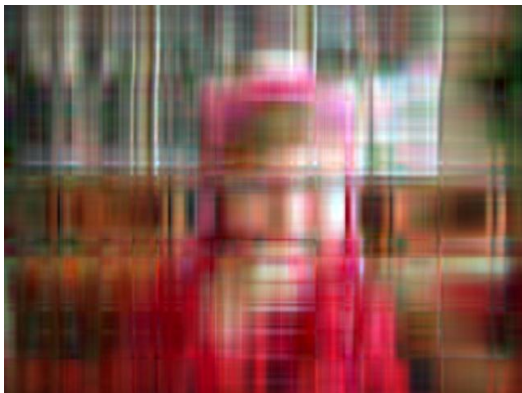
Bonus example 4

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Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^6 \sigma_i \hat{u}_i \hat{v}_i^T$$



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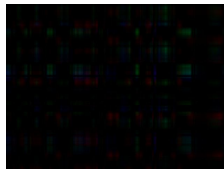
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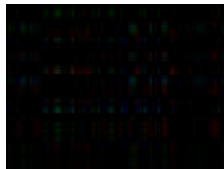
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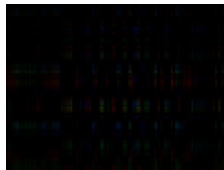
Bonus example 4

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# Image approximation (480x640)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^T$$



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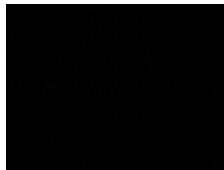
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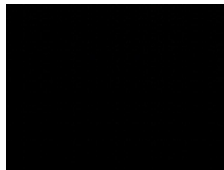
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# Image approximation (480x640)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^T$$



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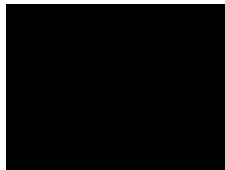
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# Image approximation (480x640)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^T$$



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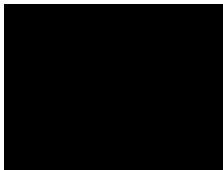
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# Image approximation (480x640)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^T$$



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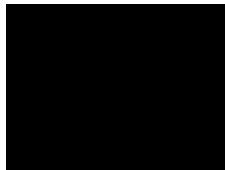
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# Image approximation (480x640)

$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^T$$



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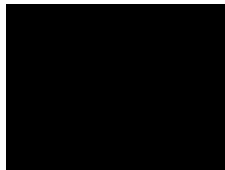
Bonus example 4

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# Image approximation (480x640)

$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^T$$



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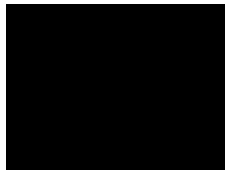
Bonus example 4

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# Image approximation (480x640)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^T$$



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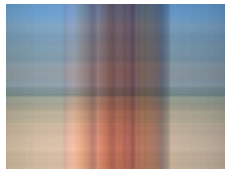
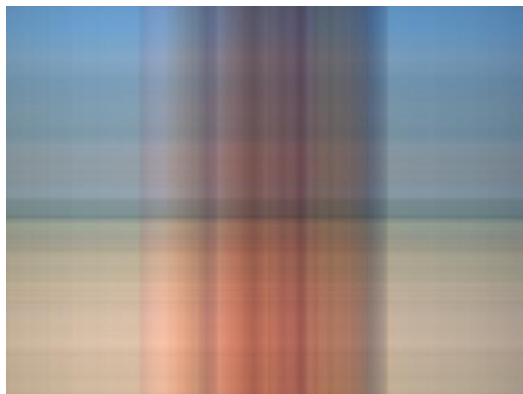
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# Image approximation (480x640)

$$A = \sum_{i=1}^1 \sigma_i \hat{u}_i \hat{v}_i^T$$



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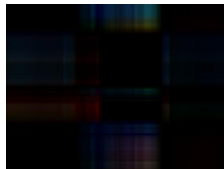
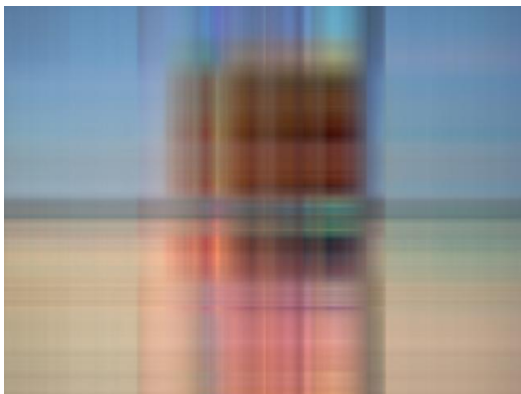
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$$A = \sum_{i=1}^2 \sigma_i \hat{u}_i \hat{v}_i^T$$



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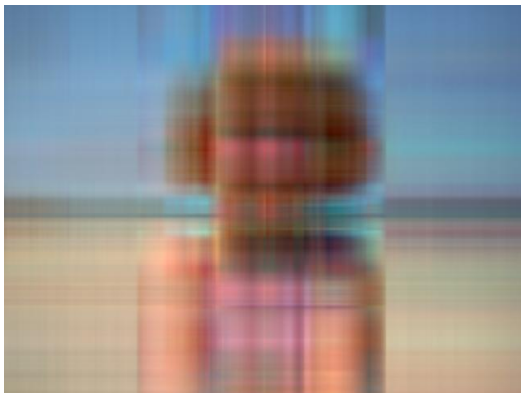
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$$A = \sum_{i=1}^3 \sigma_i \hat{u}_i \hat{v}_i^T$$



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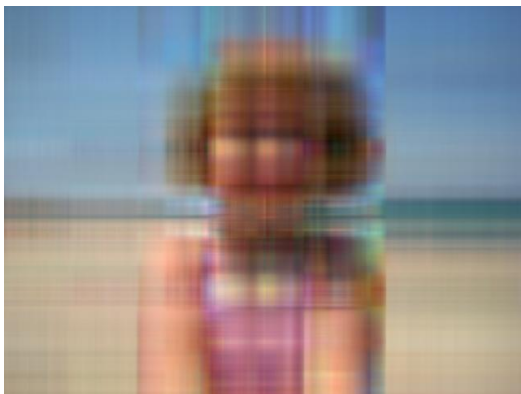
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Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

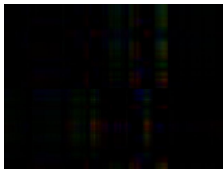
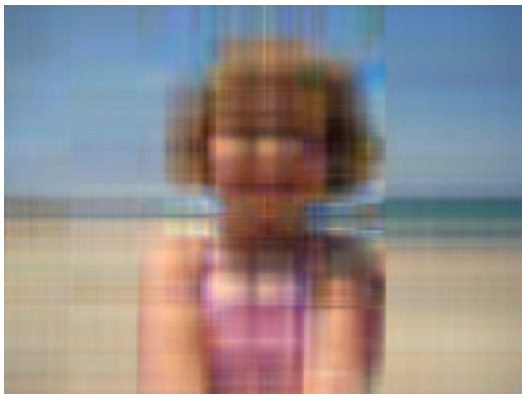
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^5 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

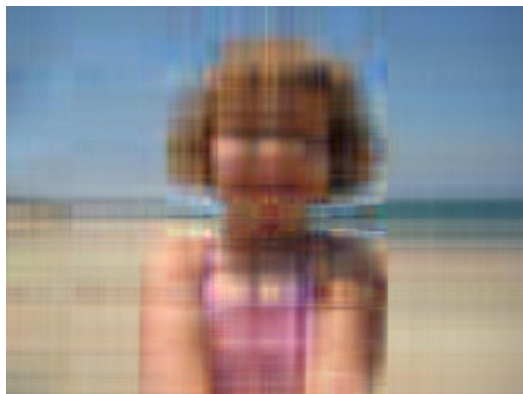
Bonus example 4

Bonus Puzzle

Colbert on Equations

## Image approximation (480x640)

$$A = \sum_{i=1}^6 \sigma_i \hat{u}_i \hat{v}_i^T$$

Approximating  
matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

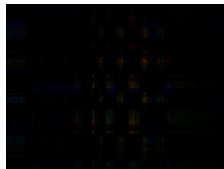
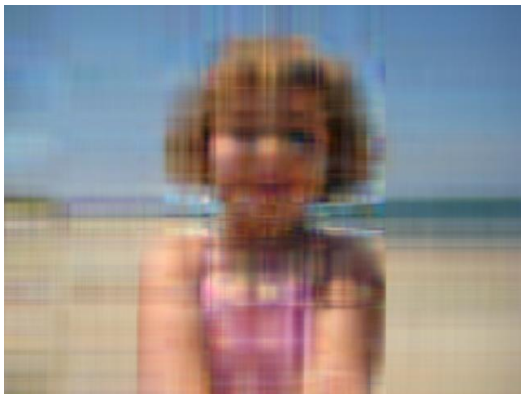
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^7 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

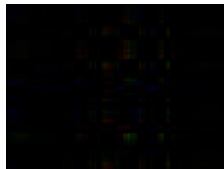
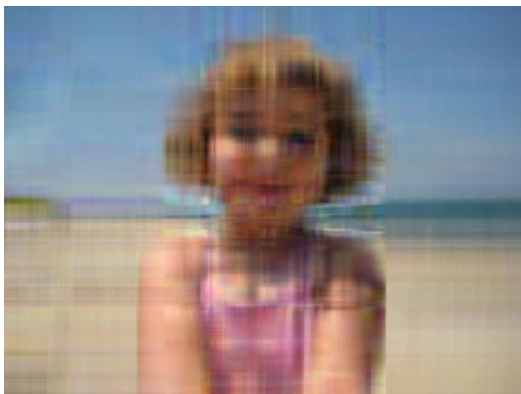
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^8 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

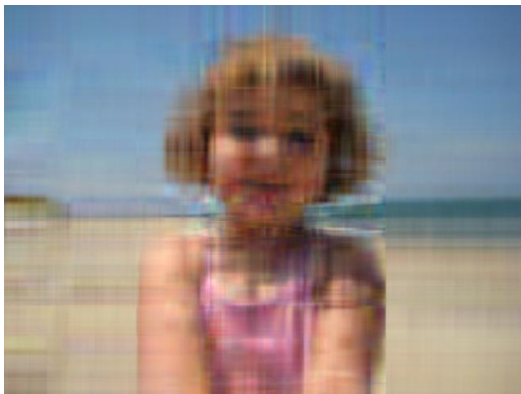
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^9 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

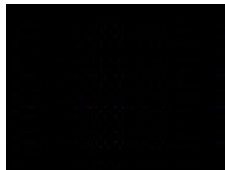
Bonus Puzzle

Colbert on Equations



# Image approximation (480x640)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

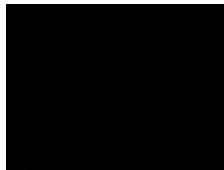
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

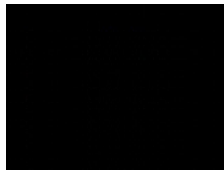
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

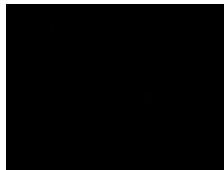
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

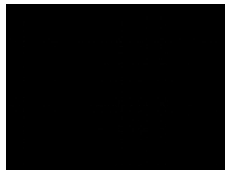
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

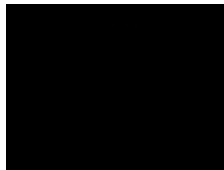
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

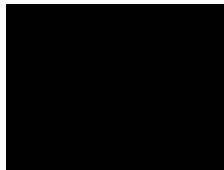
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

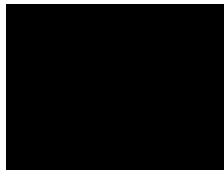
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

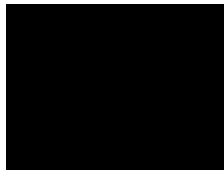
Bonus Puzzle

Colbert on Equations



# Image approximation (480x640)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

Bonus Puzzle

Colbert on Equations

## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

Bonus example 4

Bonus Puzzle

Colbert on Equations

### Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

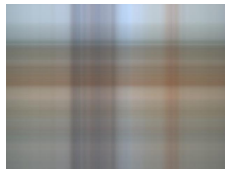
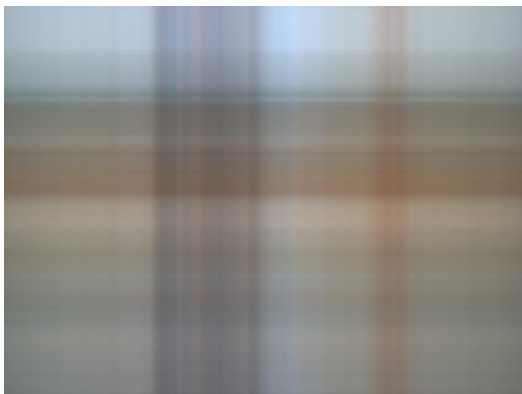
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^1 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

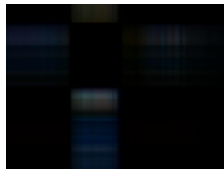
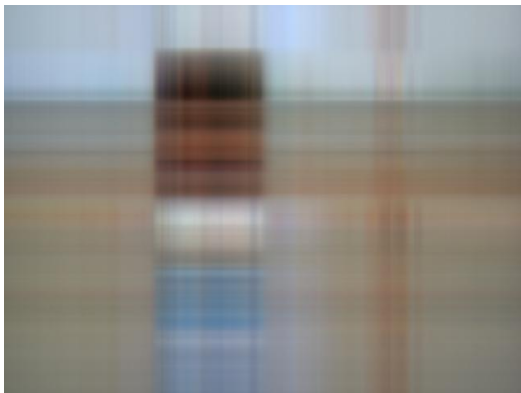
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^2 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

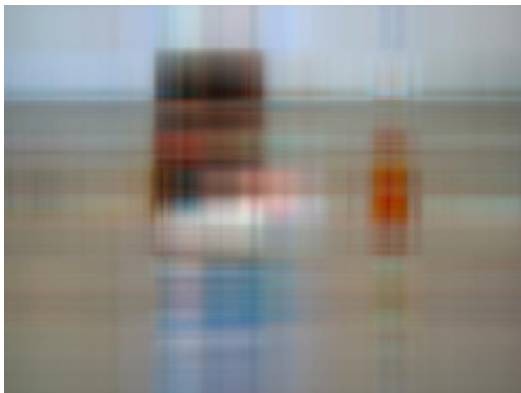
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^3 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

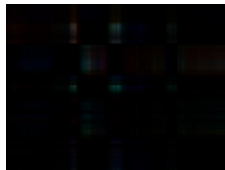
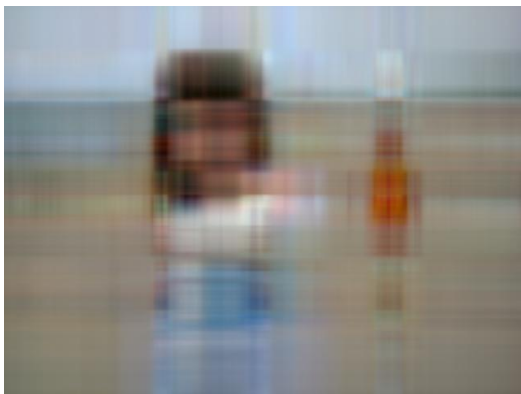
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^4 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

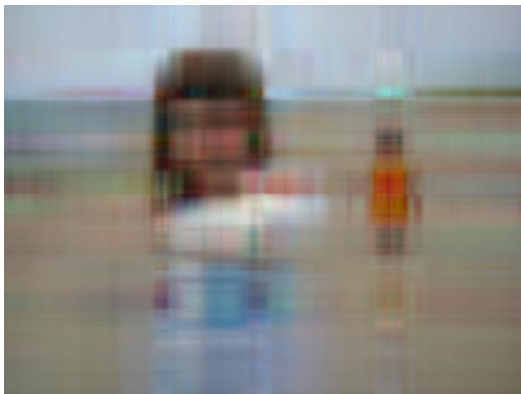
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^5 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

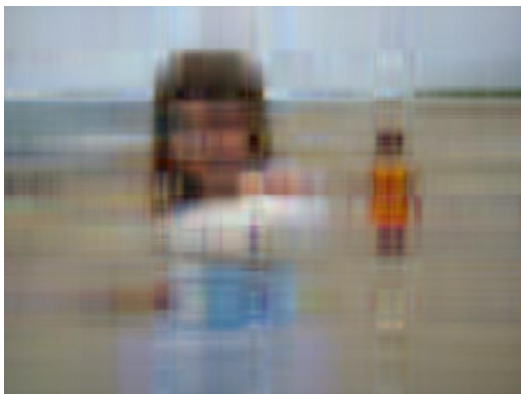
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^6 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

Bonus example 4

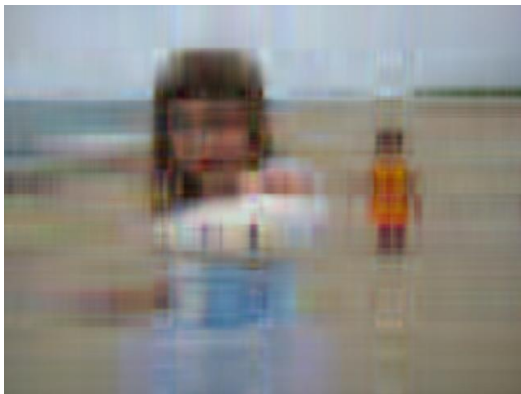
Bonus Puzzle

Colbert on Equations



# Image approximation (480x640)

$$A = \sum_{i=1}^7 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^8 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^9 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

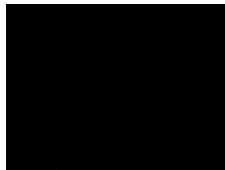
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

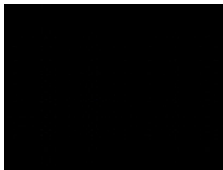
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

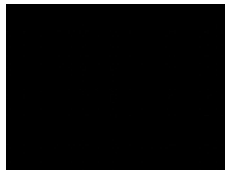
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

Bonus example 4

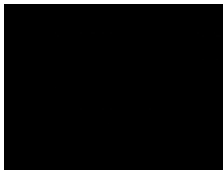
Bonus Puzzle

Colbert on Equations



# Image approximation (480x640)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

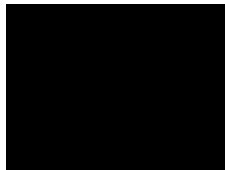
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

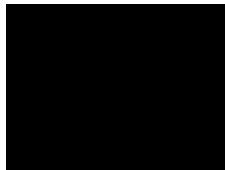
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

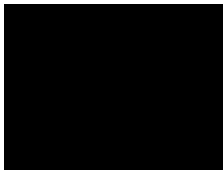
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

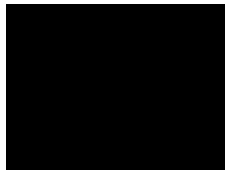
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

**Bonus example 3**

Bonus example 4

Bonus Puzzle

Colbert on Equations

## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

**Bonus example 4**

Bonus Puzzle

Colbert on Equations

### Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

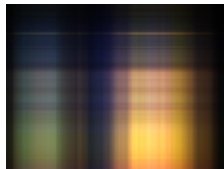
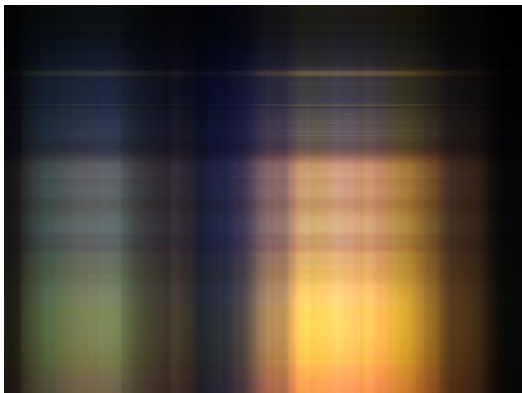
**Bonus example 4**

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^1 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

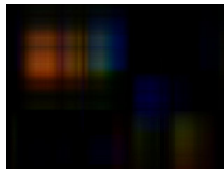
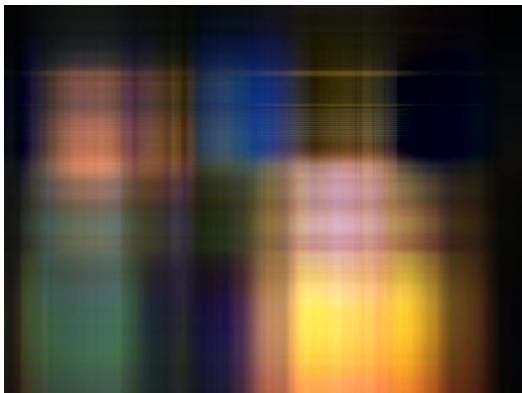
**Bonus example 4**

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^2 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

**Bonus example 4**

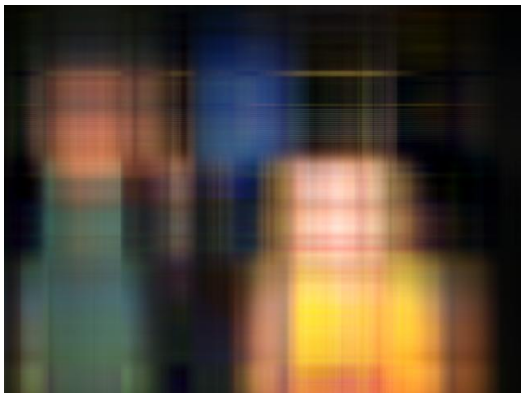
Bonus Puzzle

Colbert on Equations



# Image approximation (480x640)

$$A = \sum_{i=1}^3 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

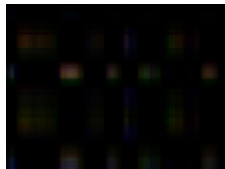
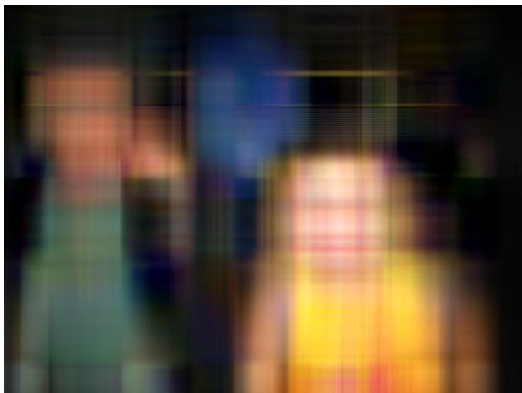
**Bonus example 4**

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^4 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

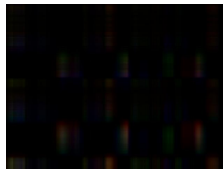
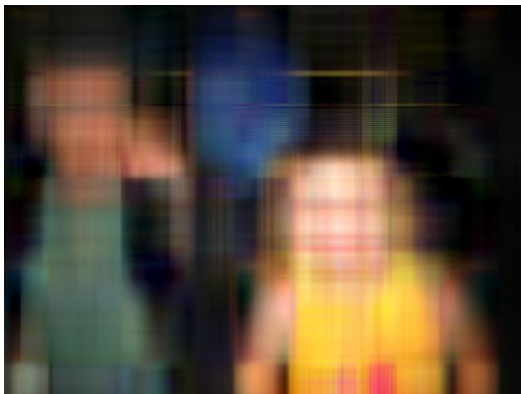
**Bonus example 4**

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^5 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

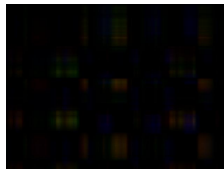
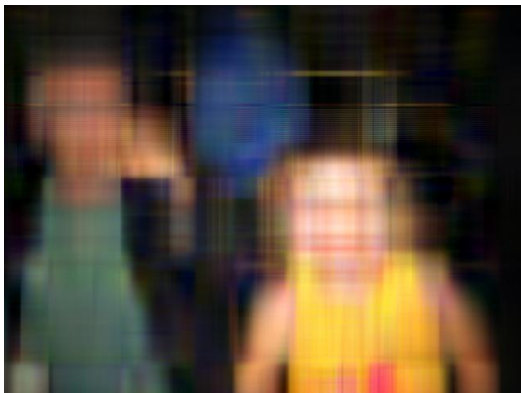
**Bonus example 4**

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^6 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

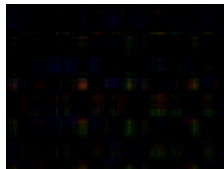
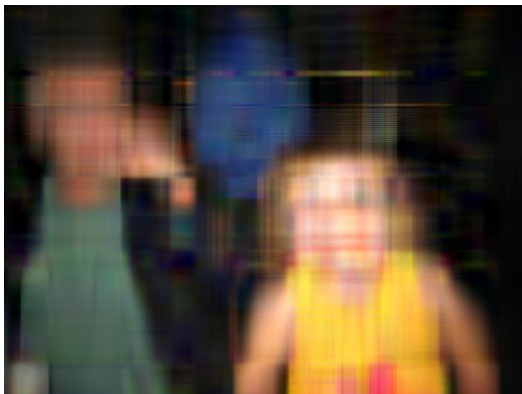
**Bonus example 4**

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^7 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

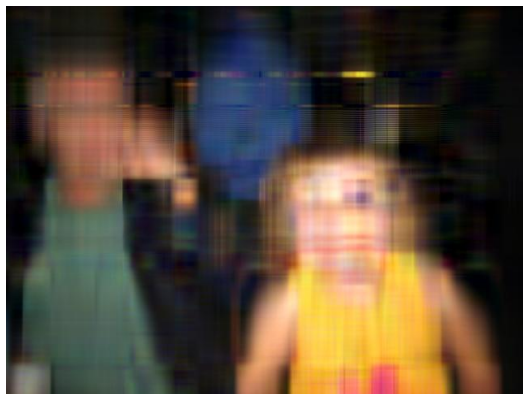
**Bonus example 4**

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^8 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

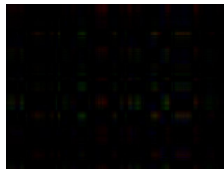
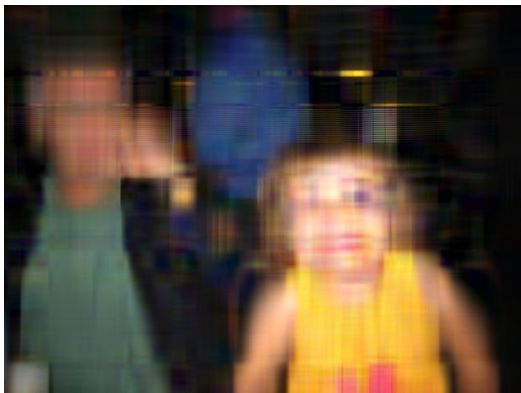
**Bonus example 4**

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^9 \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

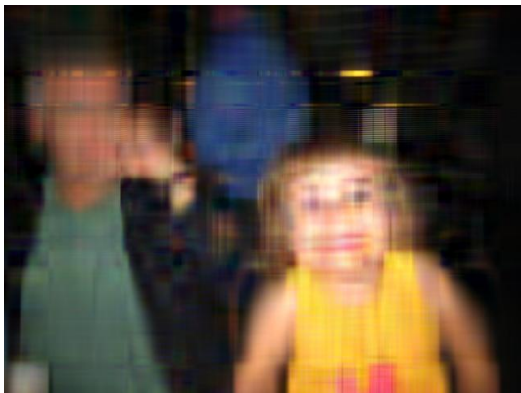
**Bonus example 4**

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

**Bonus example 4**

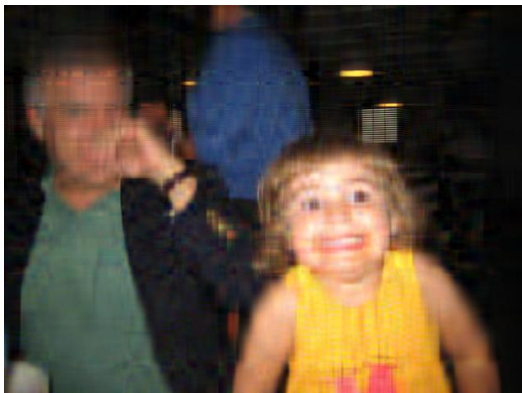
Bonus Puzzle

Colbert on Equations



# Image approximation (480x640)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

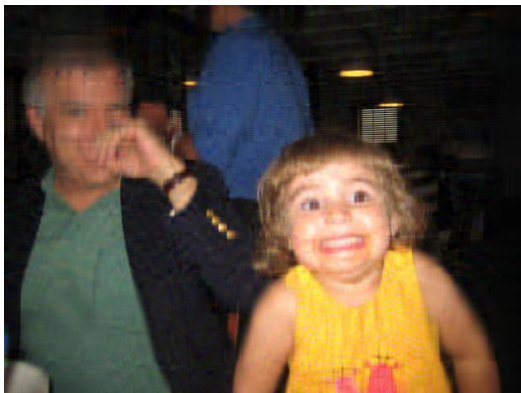
**Bonus example 4**

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

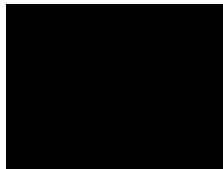
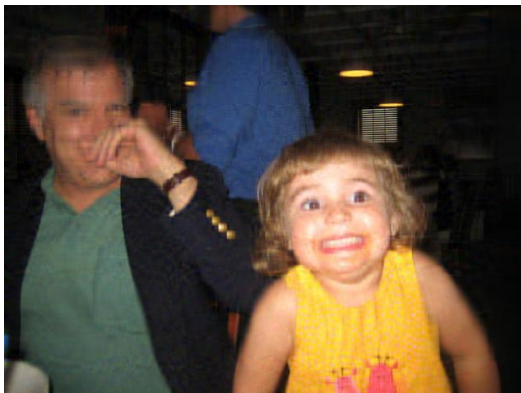
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

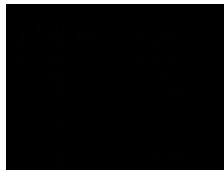
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

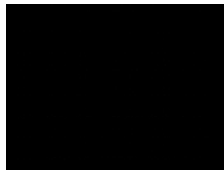
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

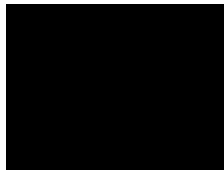
**Bonus example 4**

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

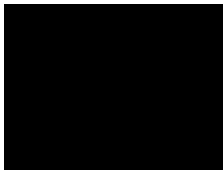
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

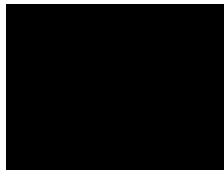
Bonus example 4

Bonus Puzzle

Colbert on Equations

# Image approximation (480x640)

$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

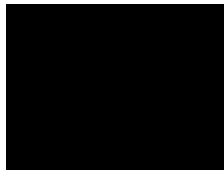
Bonus Puzzle

Colbert on Equations



# Image approximation (480x640)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^T$$



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

Bonus Puzzle

Colbert on Equations

## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

**Bonus Puzzle**

Colbert on Equations

Approximating  
matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

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Bonus example 4

**Bonus Puzzle**

Colbert on Equations

# The bonus puzzle

## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

**Bonus Puzzle**

Colbert on Equations



# The bonus puzzle

## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

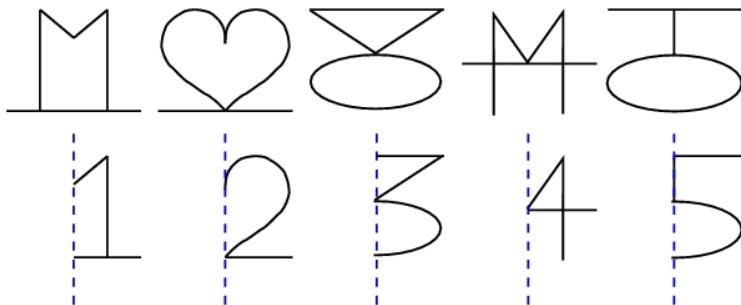
Bonus example 2

Bonus example 3

Bonus example 4

Bonus Puzzle

Colbert on Equations



## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

Bonus Puzzle

Colbert on Equations

### Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

Bonus Puzzle

Colbert on Equations

# The truth about mathematics

## Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 1

Bonus example 2

Bonus example 3

Bonus example 4

Bonus Puzzle

Colbert on Equations

[The Colbert Report on Math](#) (February 2, 2007)