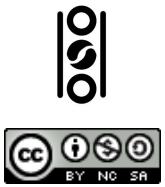
Singular Value Decomposition

Linear Alegebra, Course 124A, Fall, 2007

Prof. Peter Dodds

Department of Mathematics & Statistics
University of Vermont



Outline

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Image approximation (80x60)

- Idea: use SVD to approximate images.
- Interpret elements of matrix A as color values of an image.
- Truncate series SVD representation of A:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

- Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.
- For color: approximate 3 matrices (RGB).

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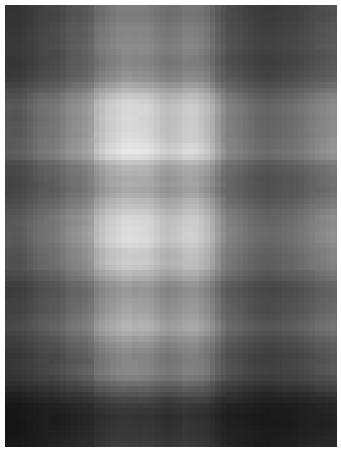


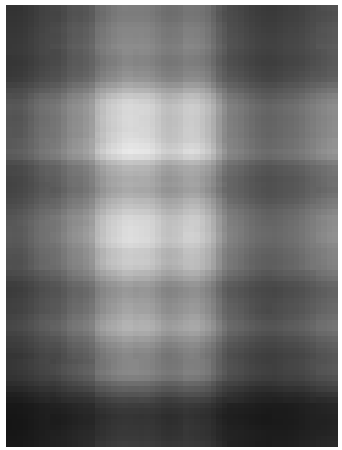
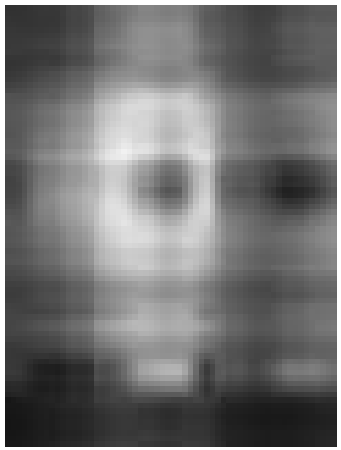


Image approximation (80x60)

$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



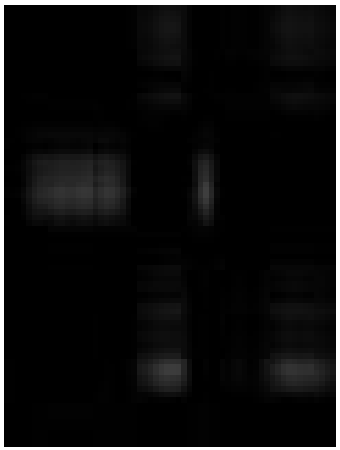
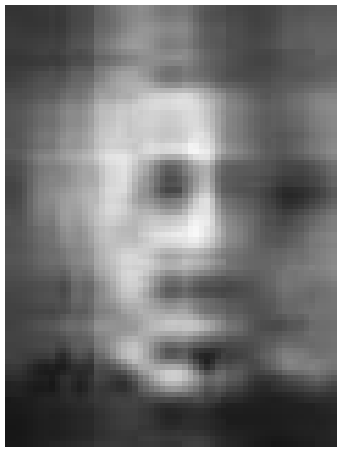
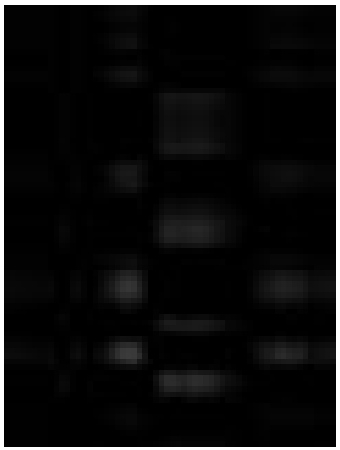


Image approximation (80x60)

$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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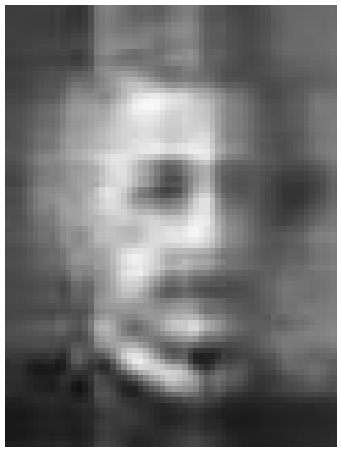
$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



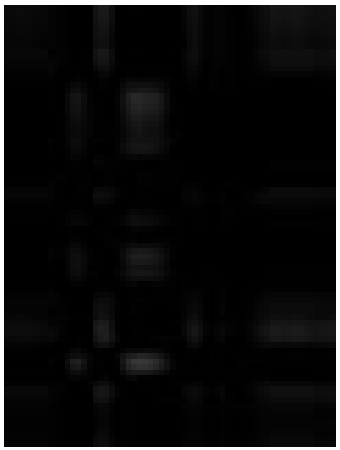


Image approximation (80x60)

$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

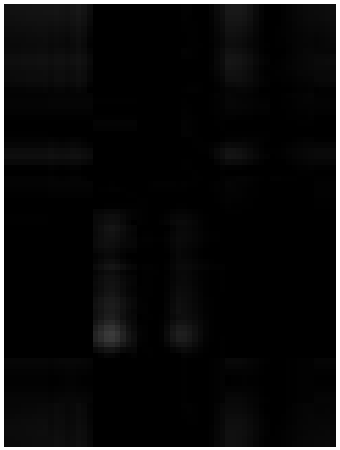


Image approximation (80x60)

$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

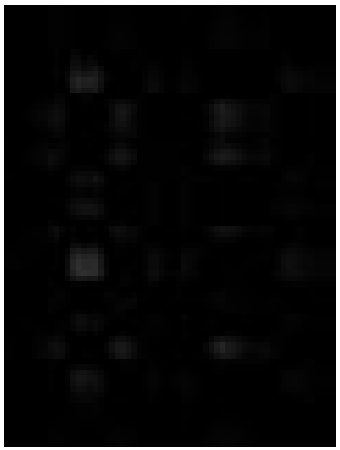


Image approximation (80x60)

$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

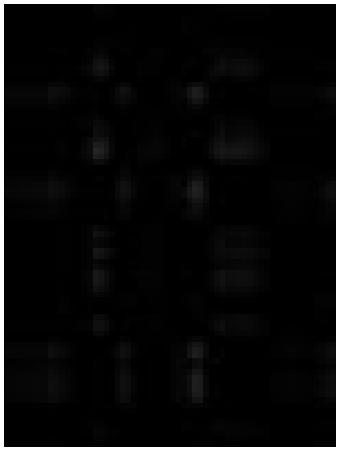


Image approximation (80x60)

$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

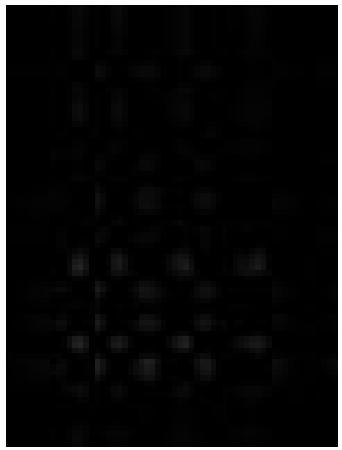


Image approximation (80x60)

$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

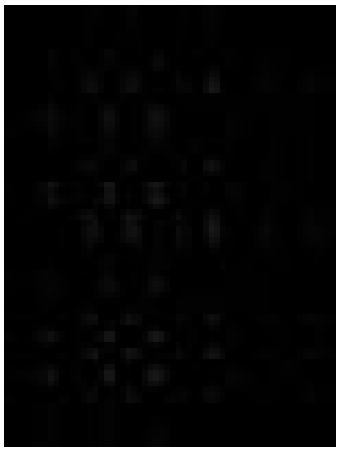
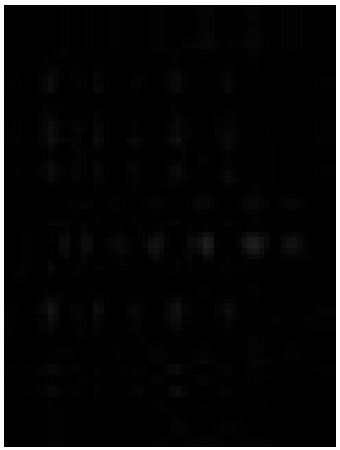


Image approximation (80x60)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



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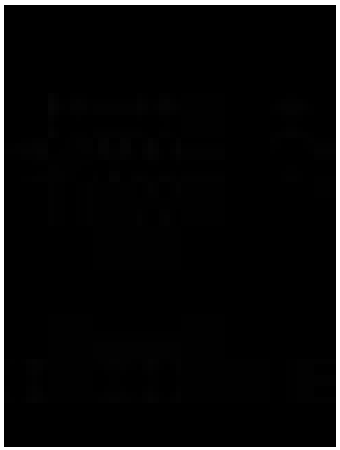


Image approximation (80x60)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

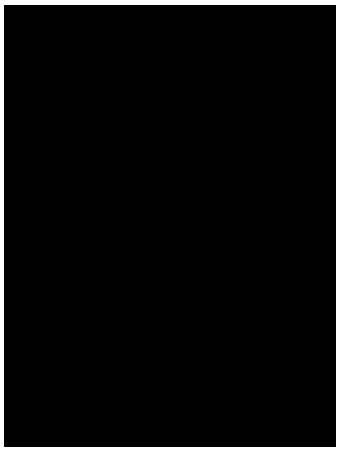


Image approximation (80x60)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

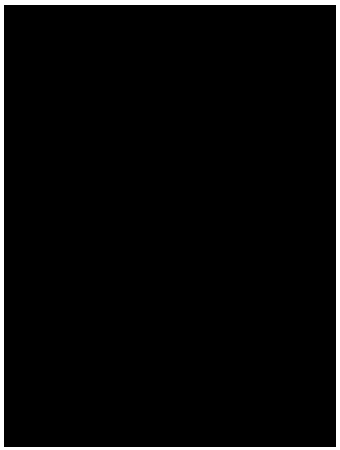
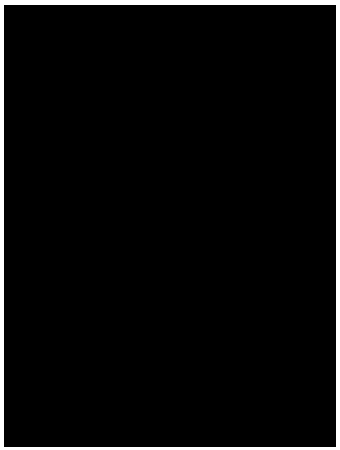


Image approximation (80x60)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



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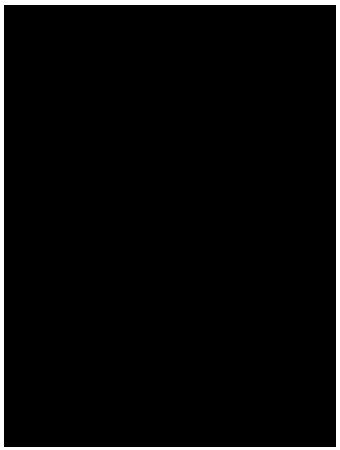

Decay of sigma values: Einstein

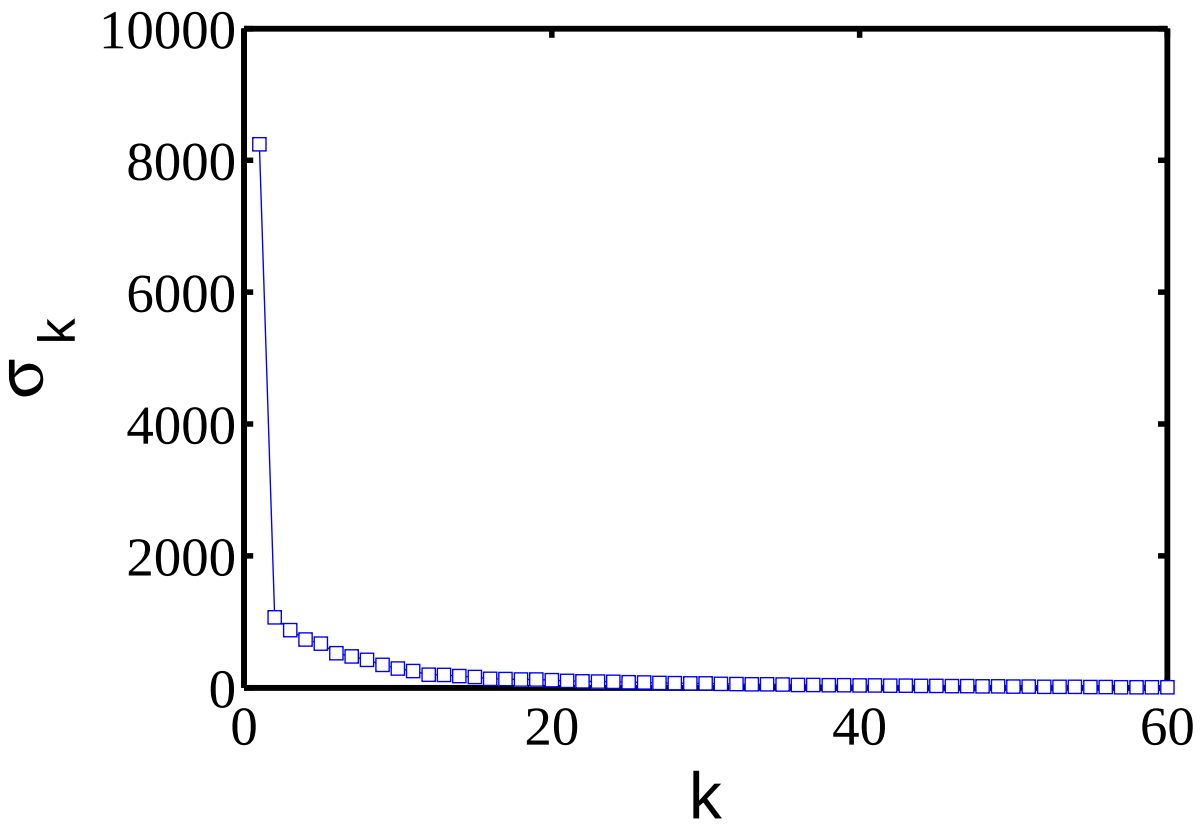

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Image approximation (480x640)

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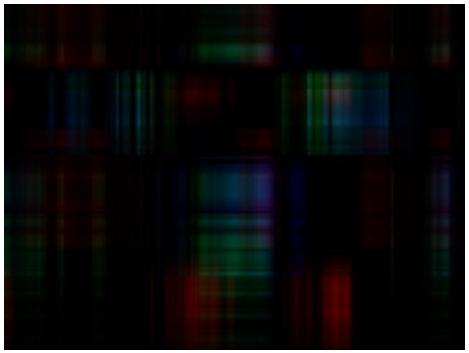

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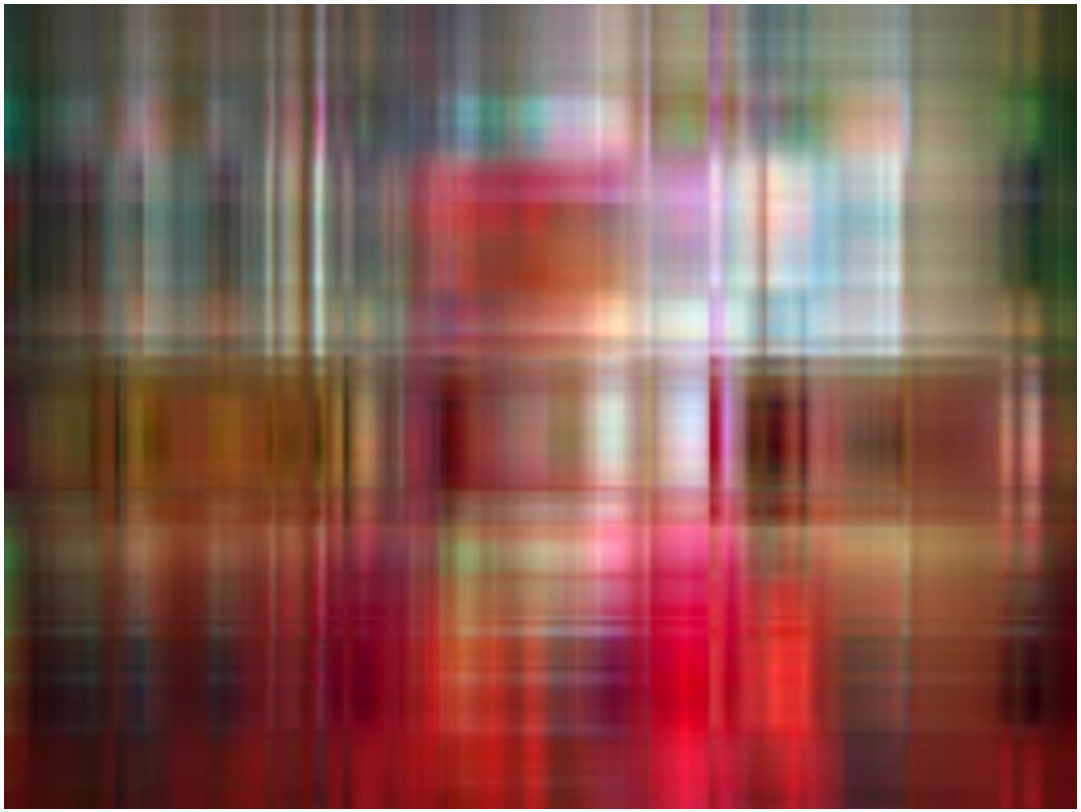

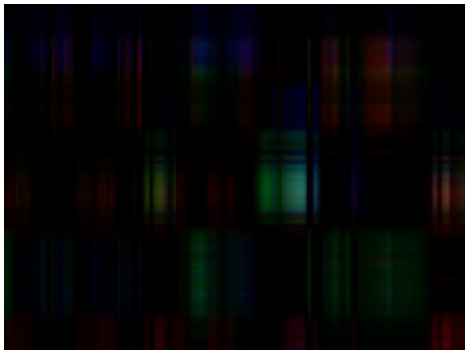

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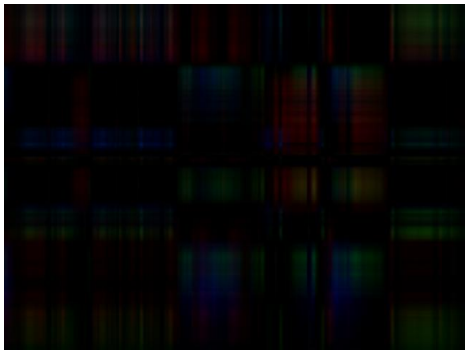

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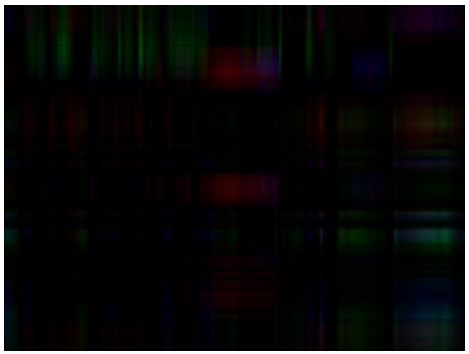

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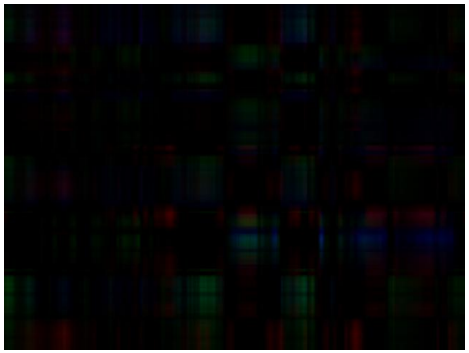

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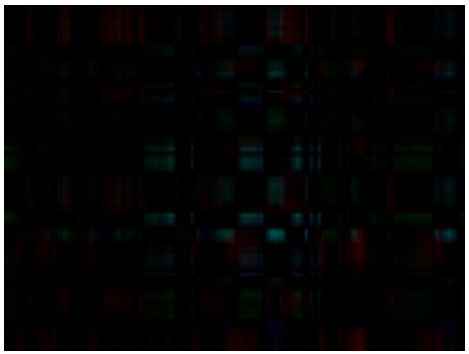

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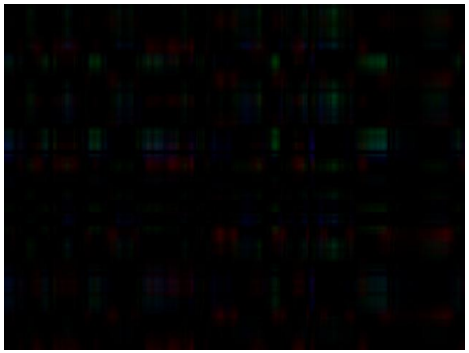

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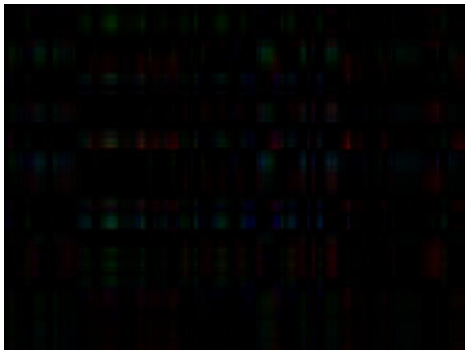

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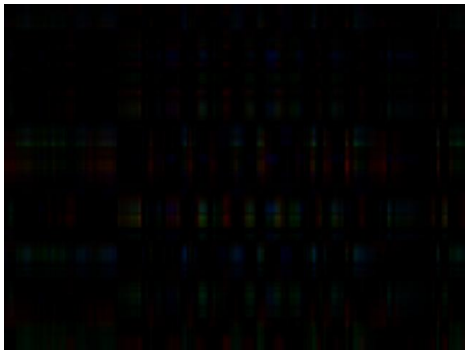

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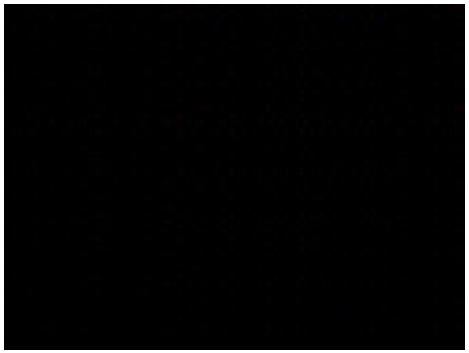

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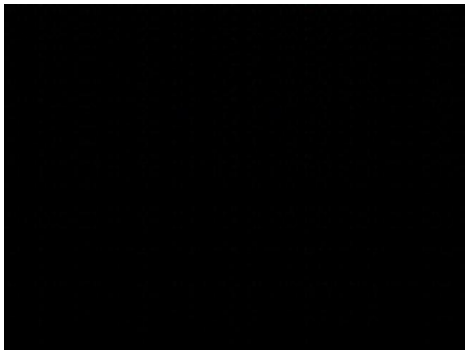

Image approximation (480x640)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

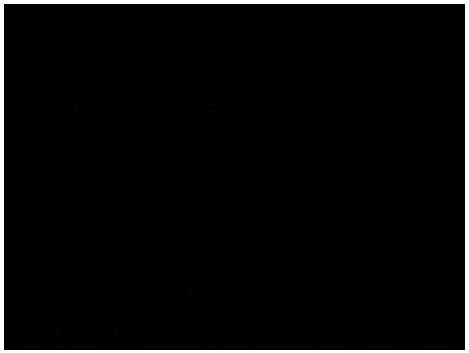

Image approximation (480x640)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Outline

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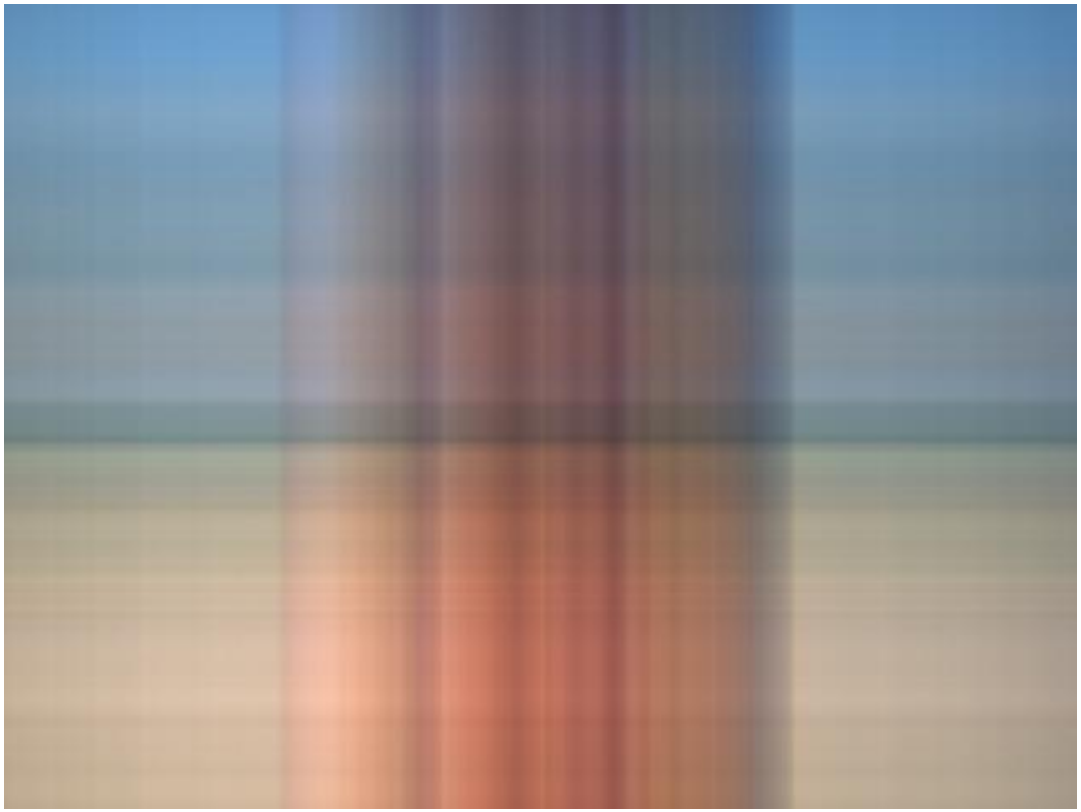

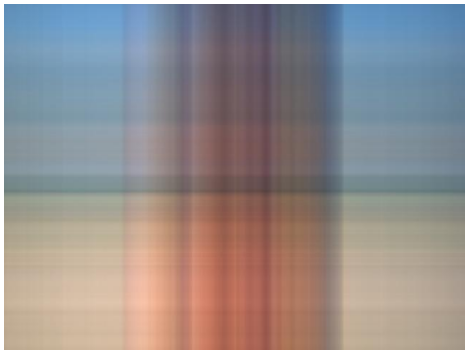

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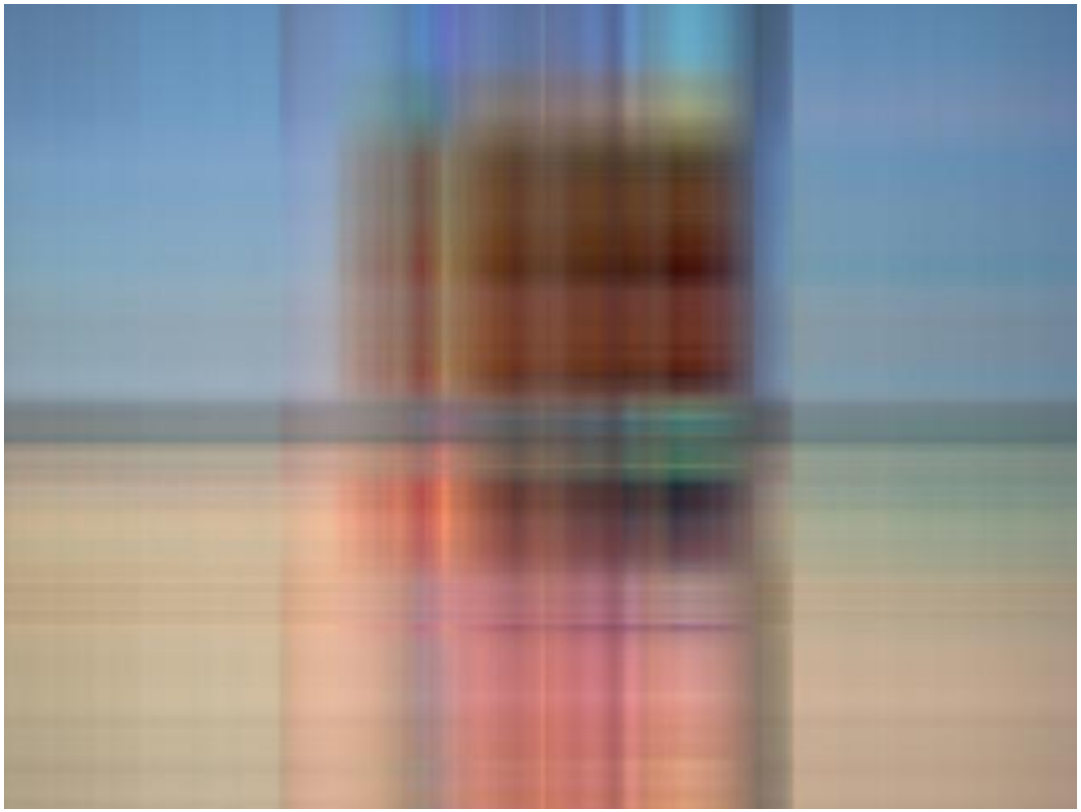

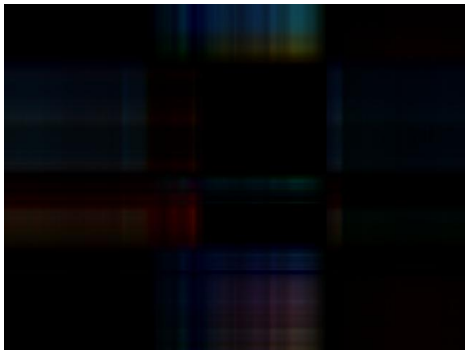

Image approximation (480x640)

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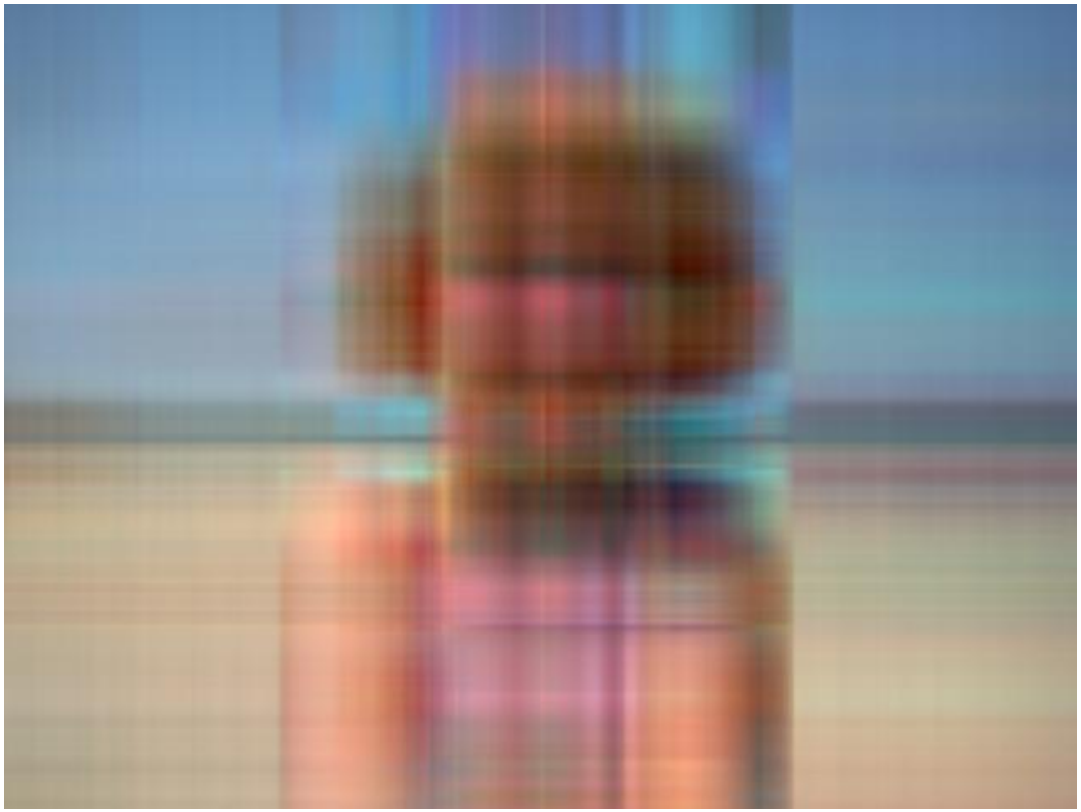

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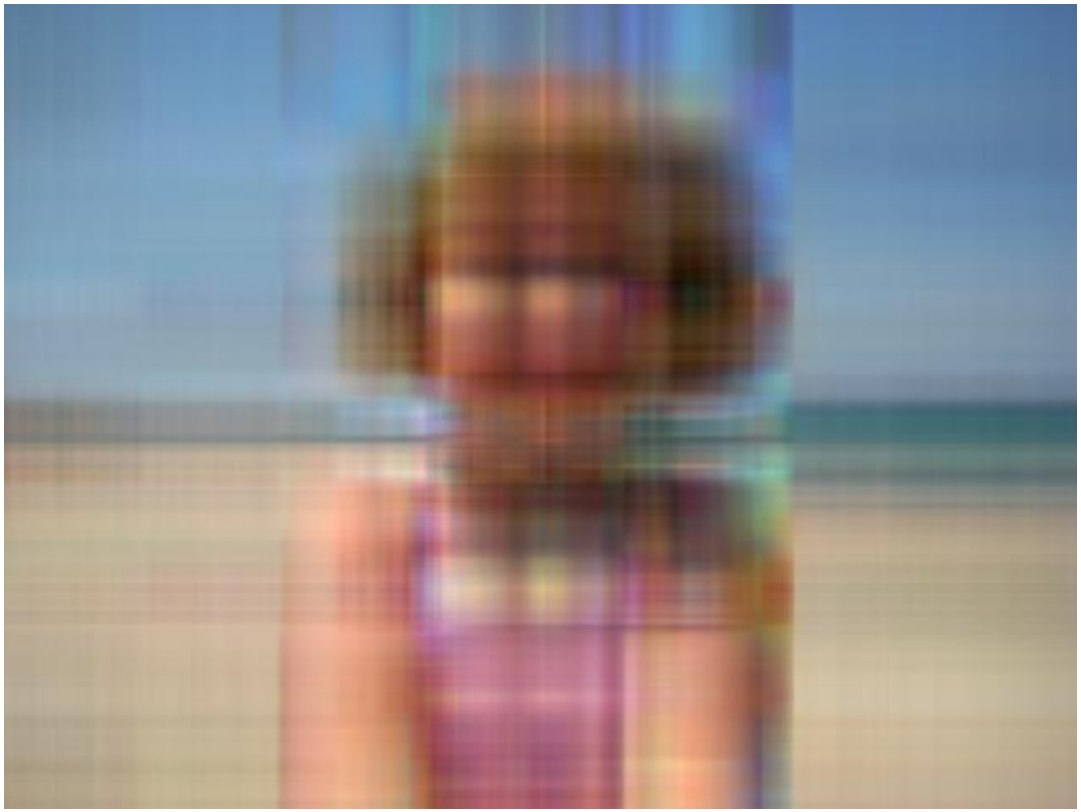

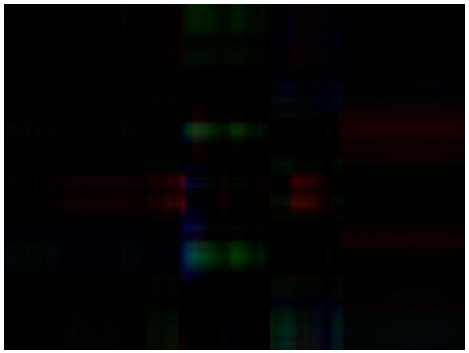

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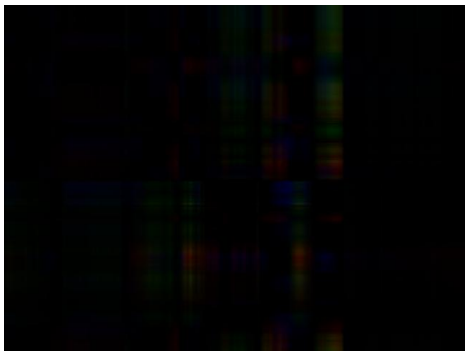

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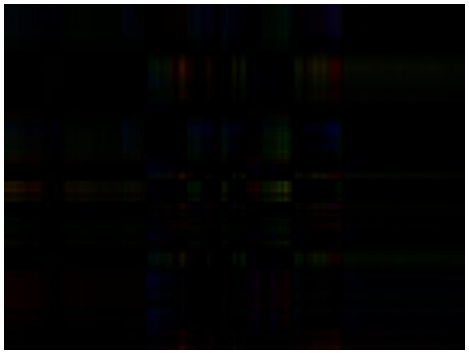


Image approximation (480x640)

$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

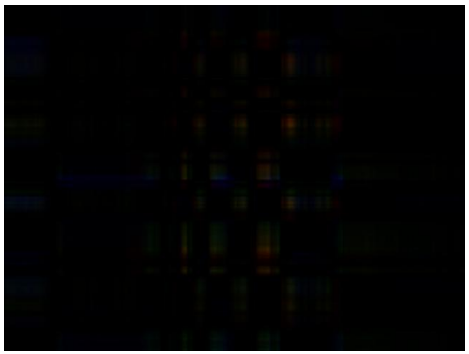


Image approximation (480x640)

$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

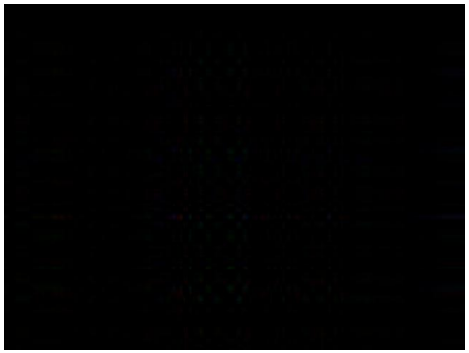


Image approximation (480x640)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

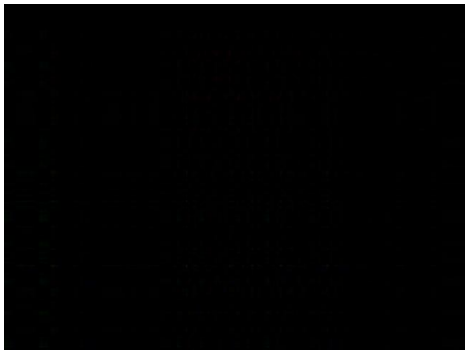


Image approximation (480x640)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

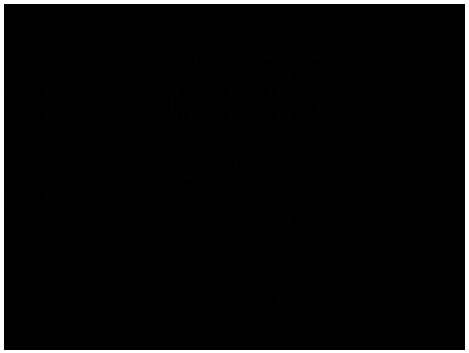


Image approximation (480x640)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

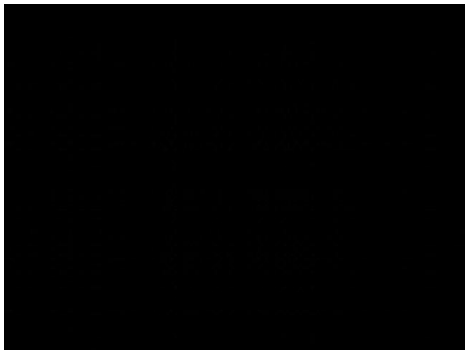


Image approximation (480x640)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

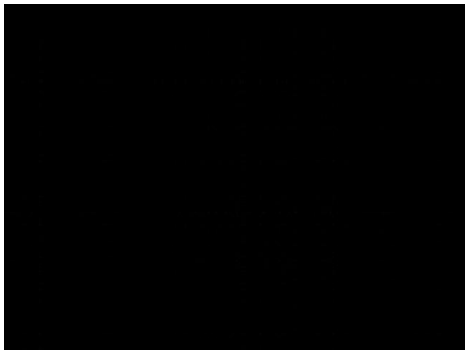


Image approximation (480x640)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Outline

Image approximation (480x640)

$$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

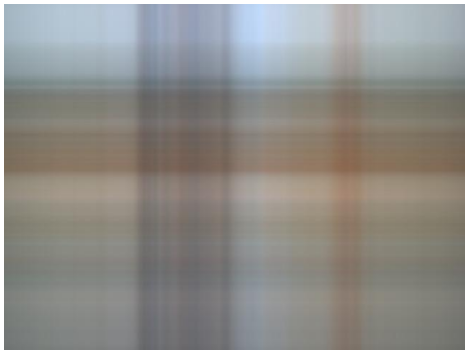
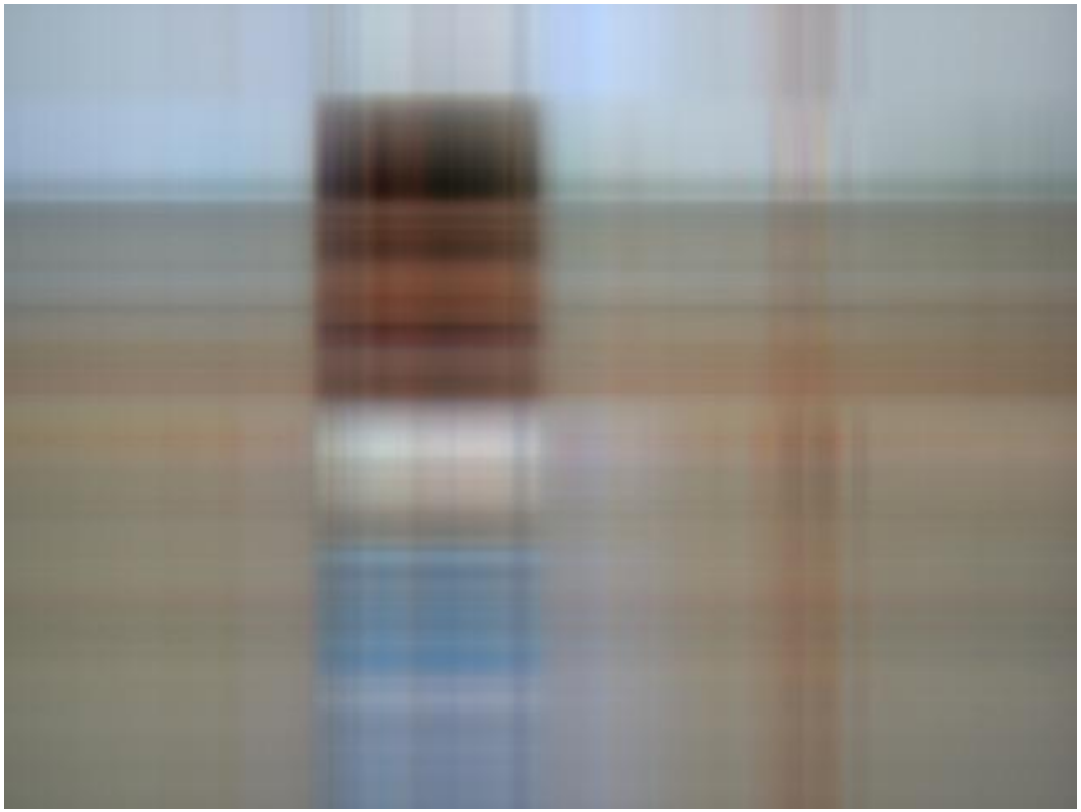


Image approximation (480x640)

$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



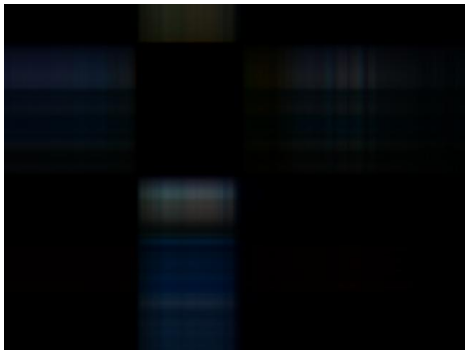


Image approximation (480x640)

$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

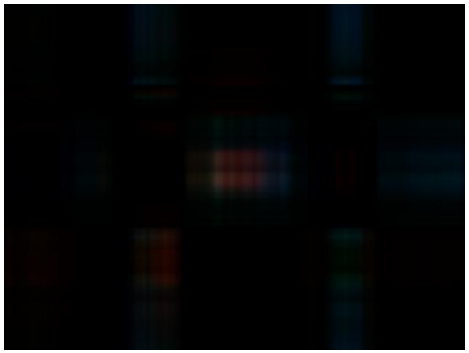
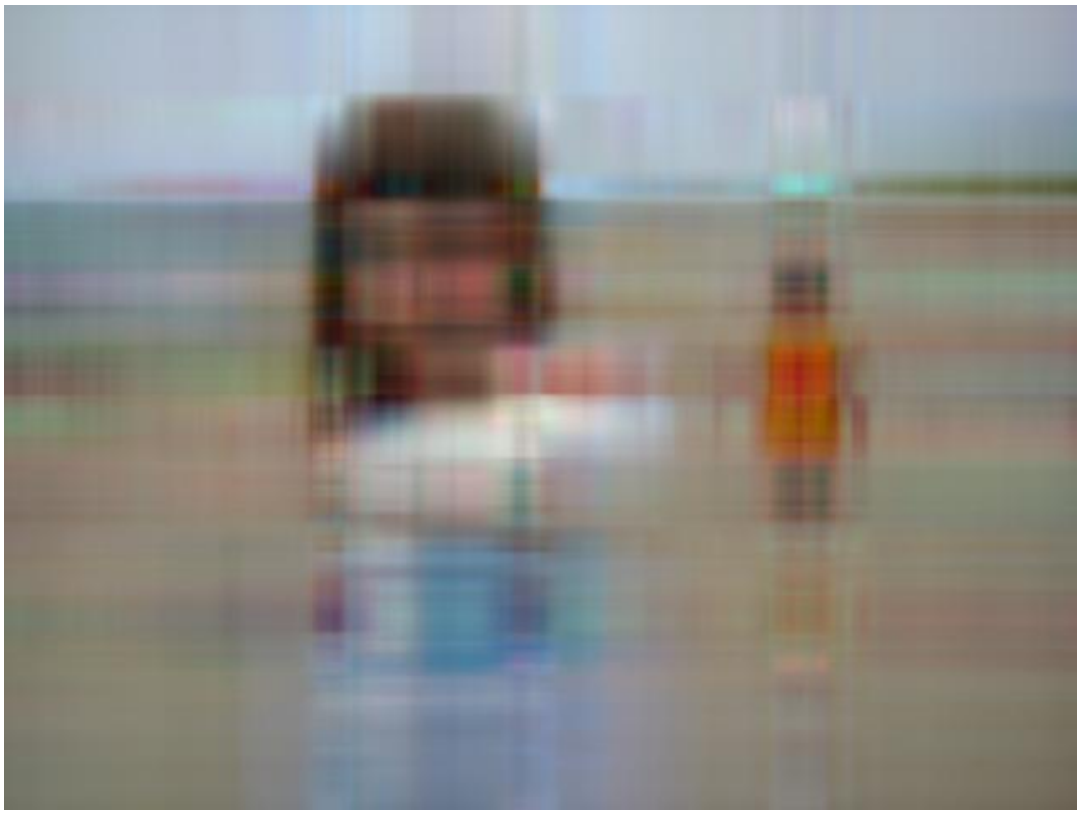


Image approximation (480x640)

$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



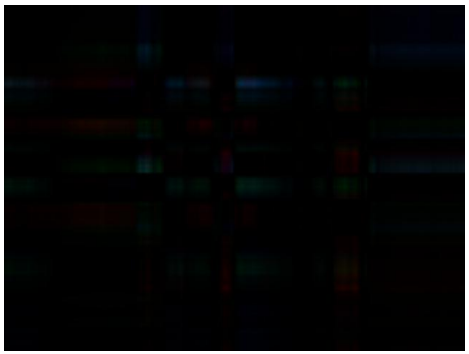


Image approximation (480x640)

$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

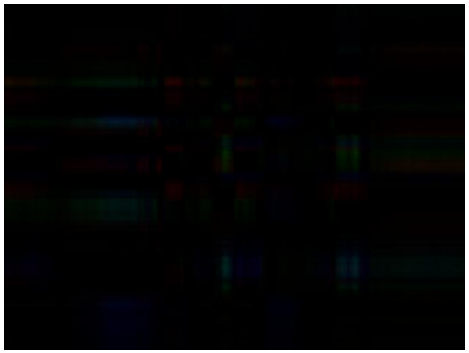
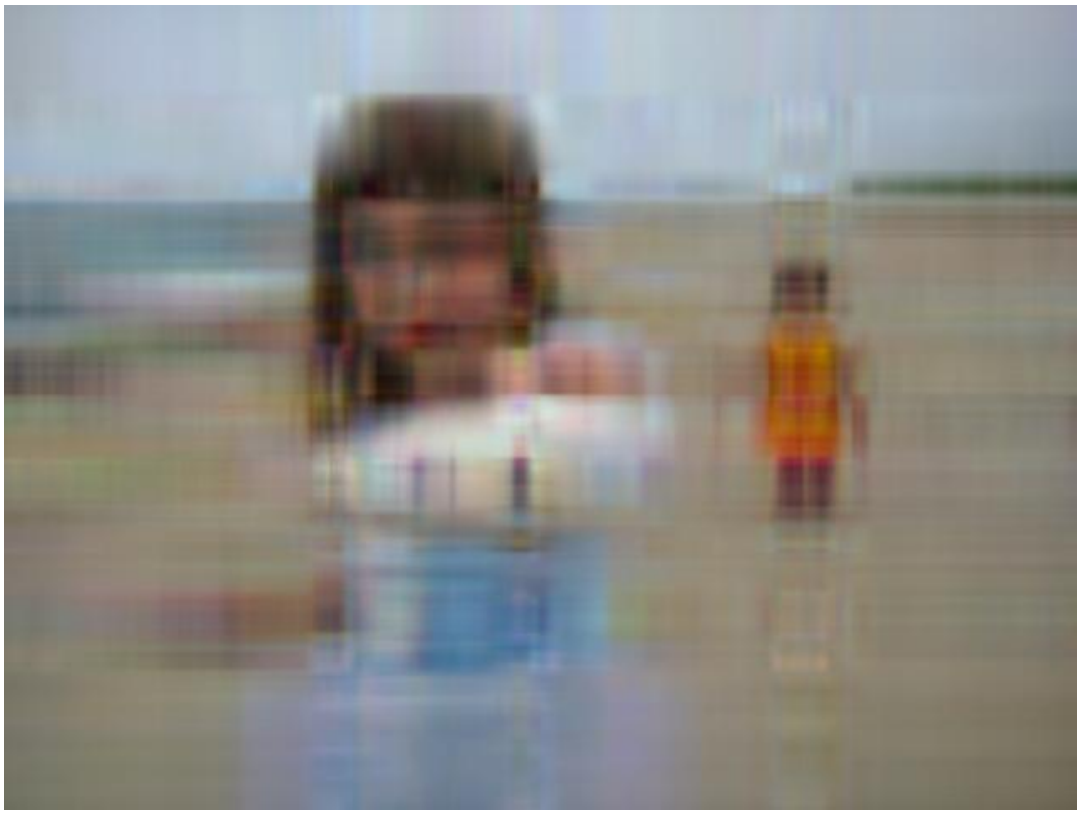


Image approximation (480x640)

$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



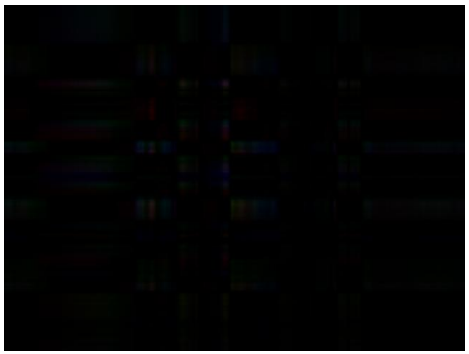


Image approximation (480x640)

$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

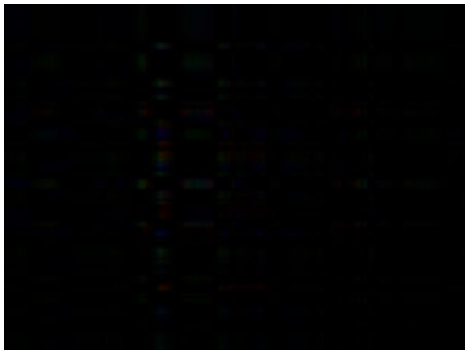


Image approximation (480x640)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

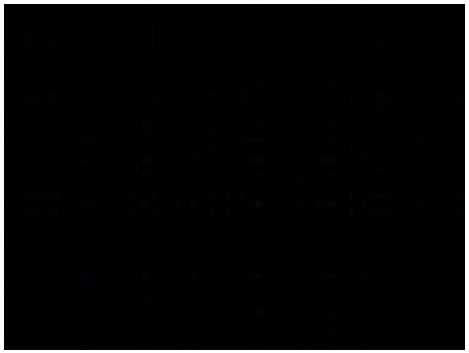


Image approximation (480x640)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

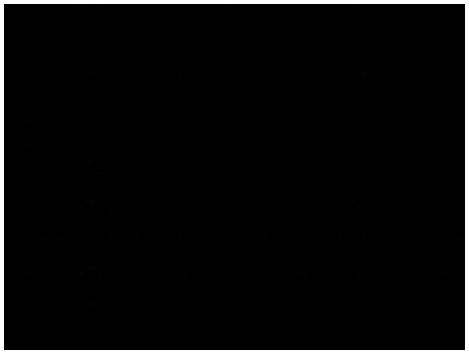


Image approximation (480x640)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

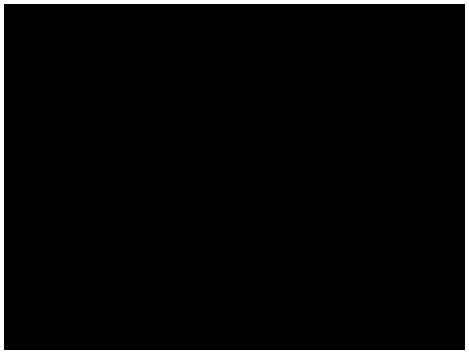


Image approximation (480x640)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

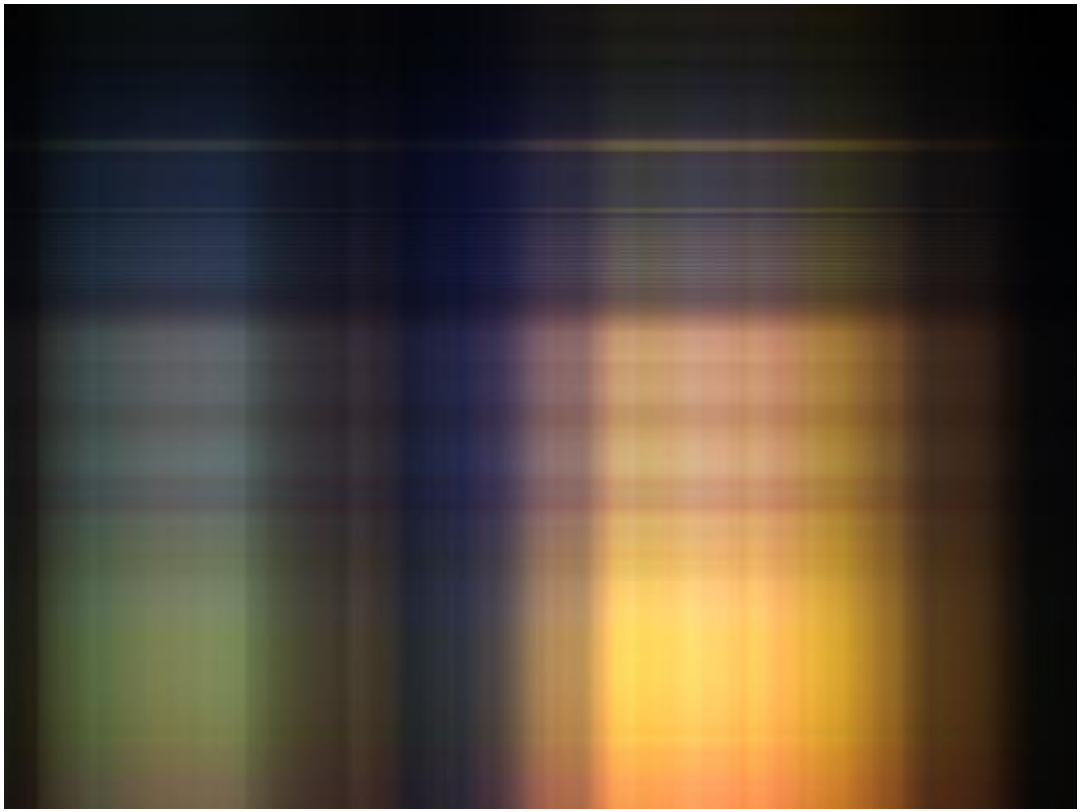
Image approximation (480x640)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Outline

Image approximation (480x640)

$$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



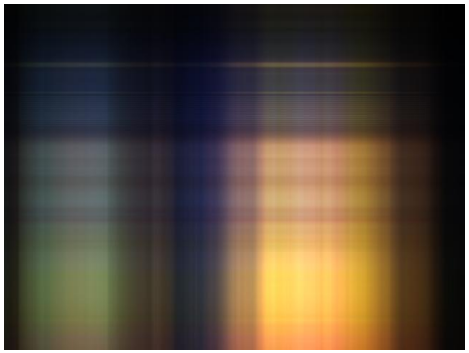


Image approximation (480x640)

$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

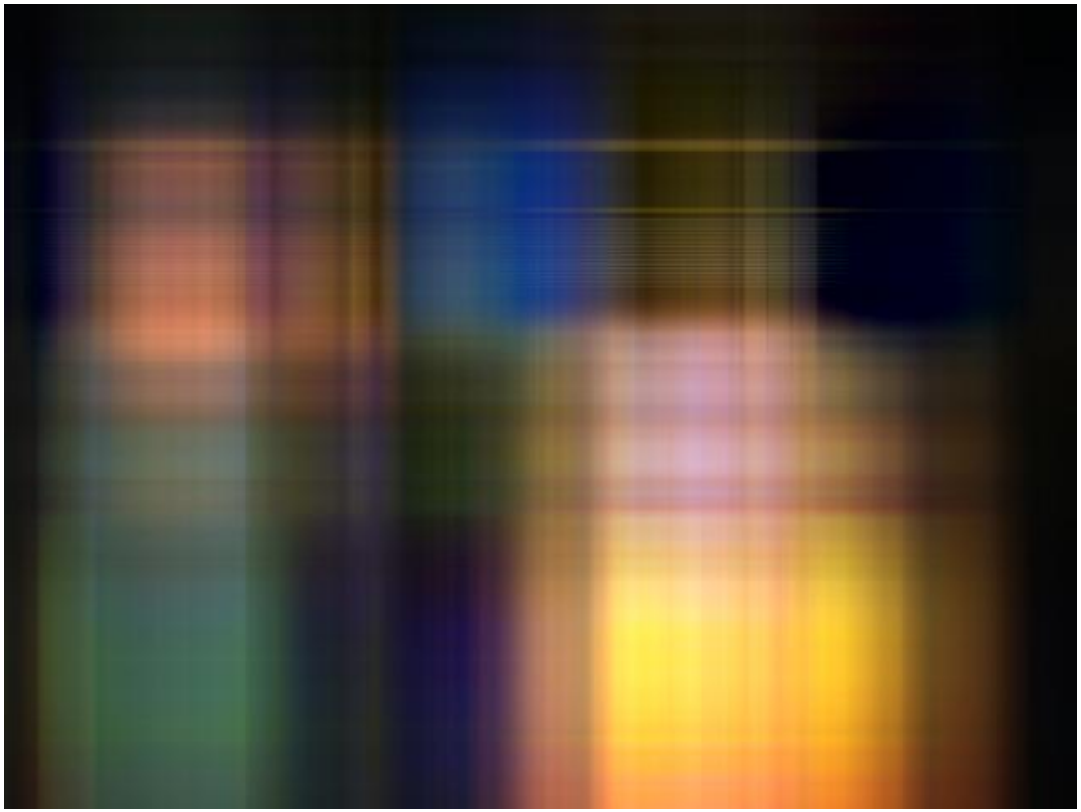


Image approximation (480x640)

$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

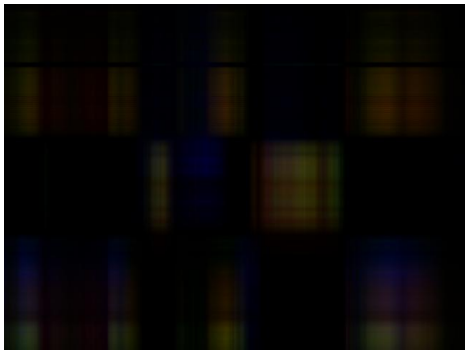


Image approximation (480x640)

$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

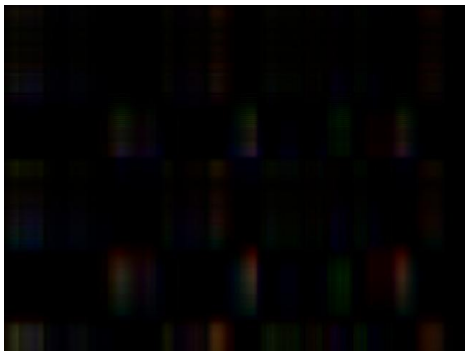


Image approximation (480x640)

$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

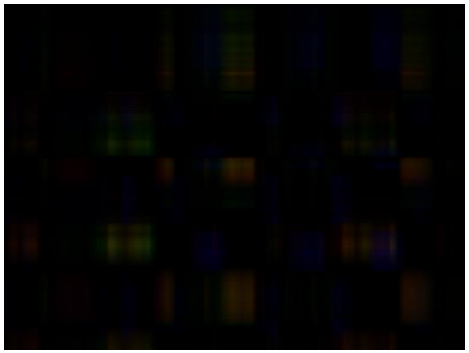


Image approximation (480x640)

$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

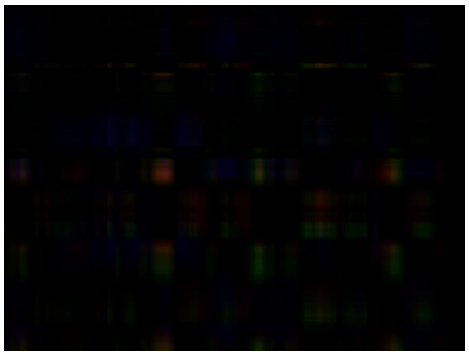


Image approximation (480x640)

$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

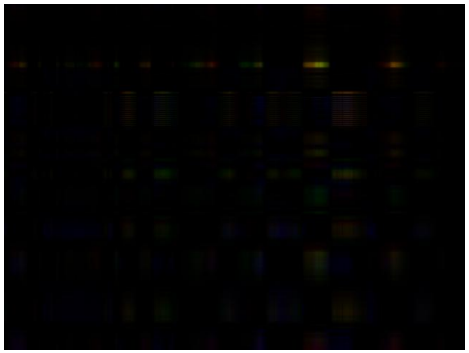
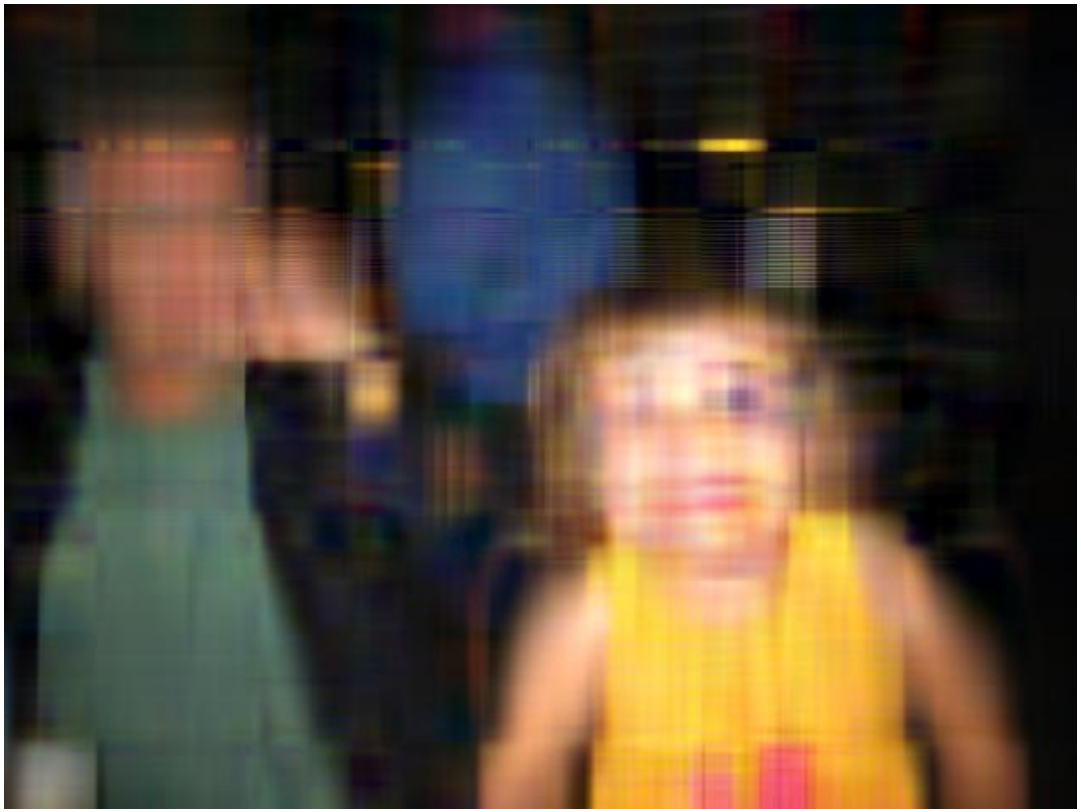


Image approximation (480x640)

$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



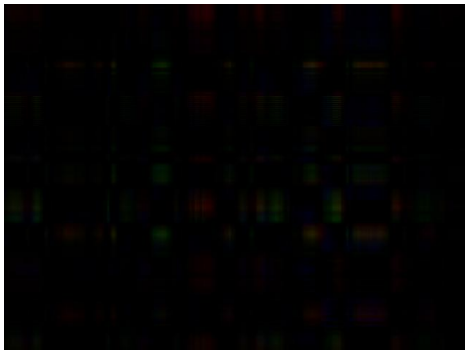


Image approximation (480x640)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

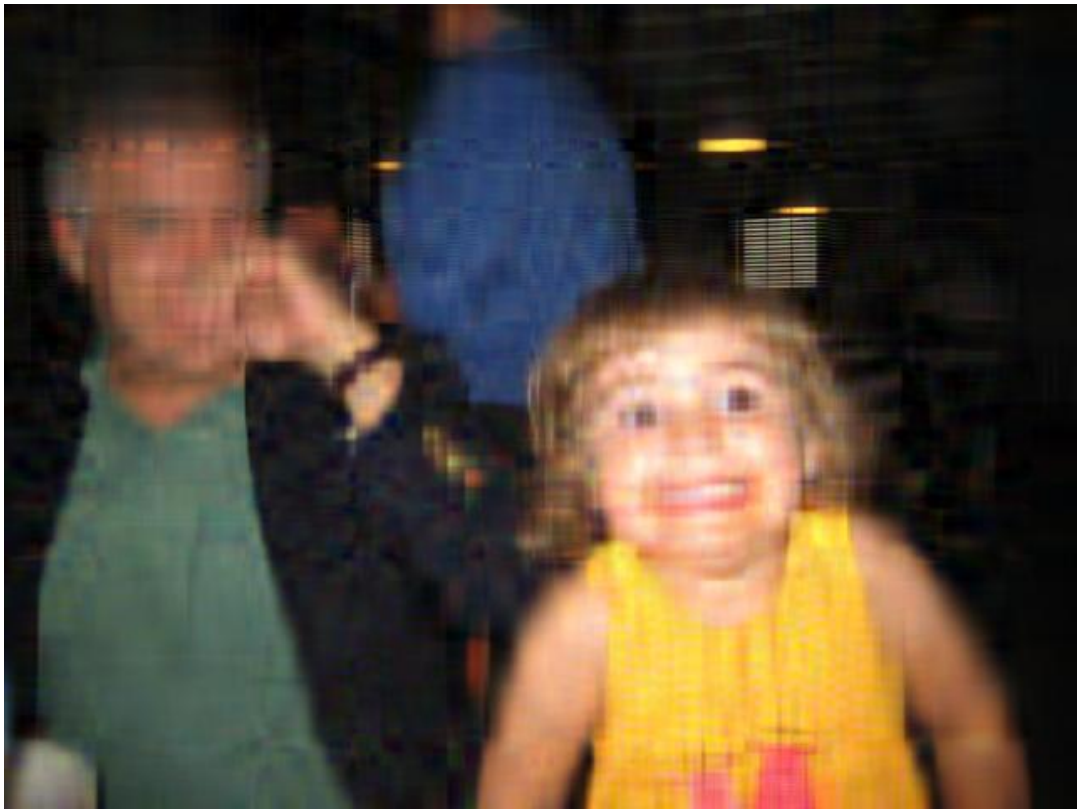


Image approximation (480x640)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

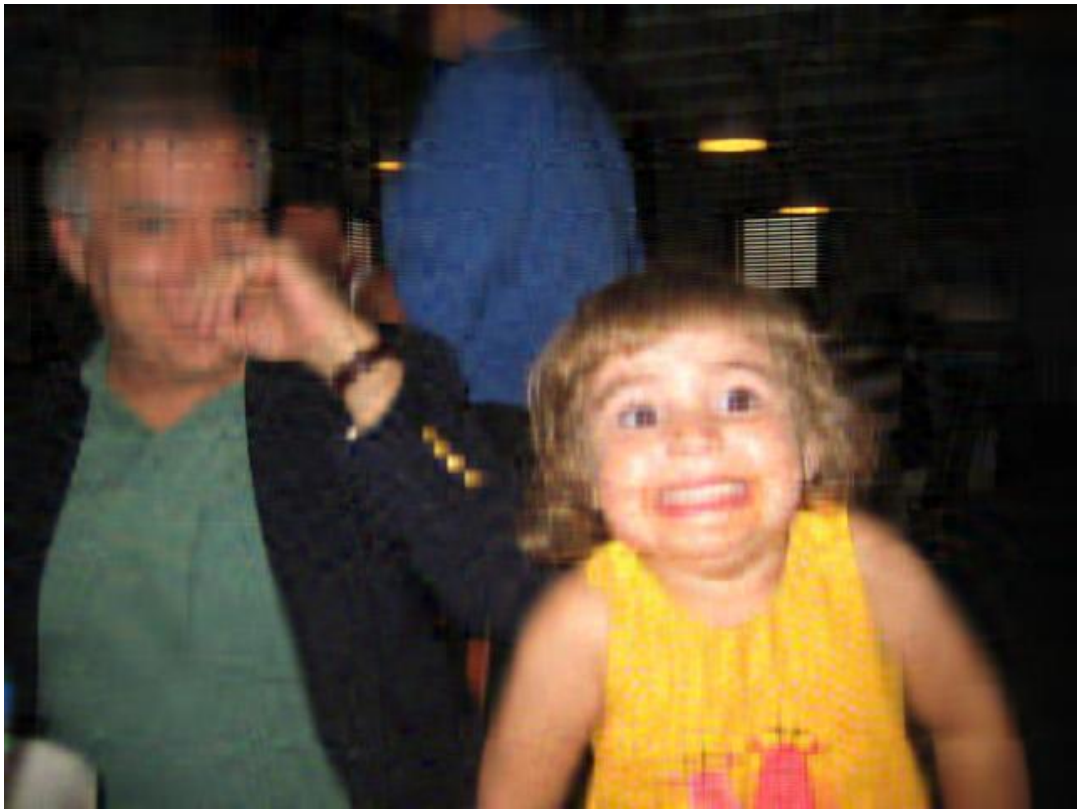


Image approximation (480x640)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

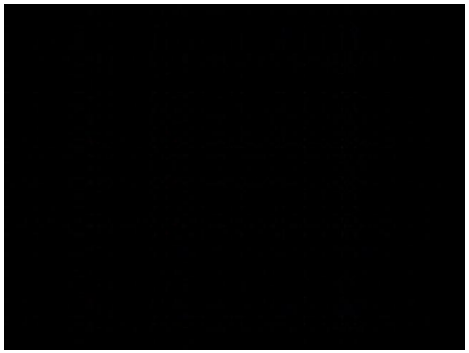


Image approximation (480x640)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

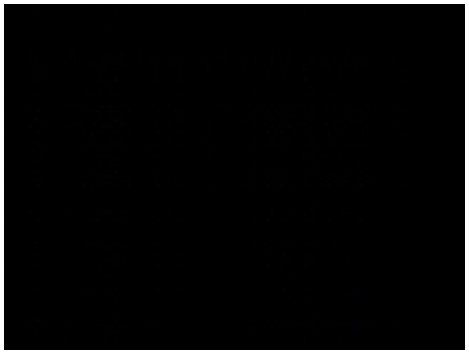


Image approximation (480x640)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

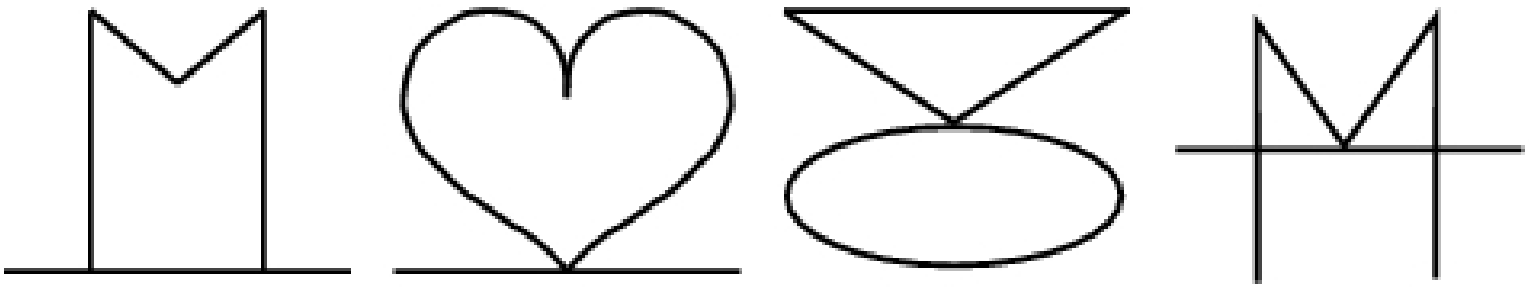
$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

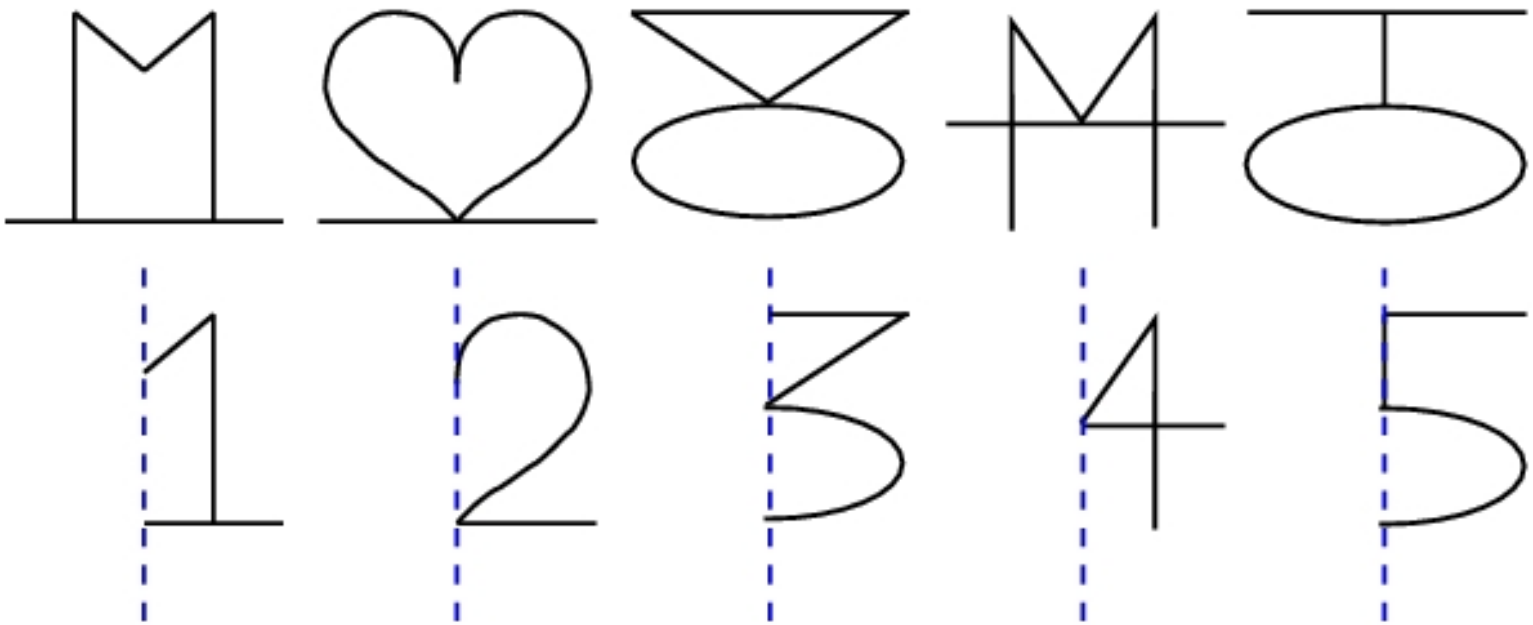
$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Outline

The bonus puzzle



The bonus puzzle



Outline

The truth about mathematics

The Colbert Report on Math (Feburary 2, 2007)