arXiv:cond-mat/0403299 v1 11 Mar 2004

Discrete Hierarchical Organization of Social Group Sizes

Wei-Xing Zhou,[1] Didier Sornette,[1,2,3*] Russell A. Hill,[4] Robin I.M. Dunbar[5*]

[1]Institute of Geophysics and Planetary Physics,
University of California, Los Angeles, CA 90095, USA
[2]Department of Earth and Space Sciences,
University of California, Los Angeles, CA 90095, USA
[3]Laboratoire de Physique de la Matière Condensée,
CNRS UMR 6622 and Université de Nice-Sophia Antipolis,
06108 Nice Cedex 2, France
[4]Evolutionary Anthropology Research Group,
Department of Anthropology University of Durham, 43 Old Elvet,
Durham DH1 3HN, UK
[5]British Academy Centenary Project, School of Biological Sciences,
University of Liverpool, Crown St., Liverpool L69 7ZB, England

*To whom correspondence should be addressed.
E-mail: sornette@moho.ess.ucla.edu (D.S.) and rimd@liverpool.ac.uk (R.I.M.D.).

The "social brain hypothesis" for the evolution of large brains in primates has led to evidence for the coevolution of neocortical size and social group sizes. Extrapolation of these findings to modern humans indicated that the equivalent group size for our species should be approximately 150 (essentially the number of people known personally as individuals). Here, we combine data on human grouping in a comprehensive and systematic study. Using fractal analysis, we identify with high statistical confidence a discrete hierarchy of group

sizes with a preferred scaling ratio close to 3: rather than a single or a continuous spectrum of group sizes, humans spontaneously form groups of preferred sizes organized in a geometrical series approximating $3, 9, 27, ...$ Such discrete scale invariance (DSI) could be related to that identified in signatures of herding behavior in financial markets and might reflect a hierarchical processing of social nearness by human brains.

Attempts to understand the grouping patterns of humans have a long history in both sociology (*1*) and social anthropology (*2,3*). However, these approaches have been largely ecological in focus. In contrast, recent attempts to understand the evolution of sociality in primates have focussed in part on the cognitive constraints that may limit the ecological flexibility of group size (*5,4*). The social brain hypothesis, as it has come to be known, argues that the evolution of primate brains (and in particular, the neocortex) was driven by the need to coordinate and manage increasingly large social groups. Since the stability of these groupings is based on intimate knowledge of other individuals and the ability to use this knowledge to manage social relationships effectively, the volume of neural matter available for cognitive processing inevitably imposes a species-specific limit on group size. Attempts to increase group size beyond this threshold result in reduced social stability and, ultimately, group fission. Extrapolating these findings to humans led to the prediction that humans had a cognitive limit of about 150 on the number of individuals with whom coherent personal relationships could be maintained (*6*). Evidence to support this prediction has come from a number of ethnographic and sociological sources. It has, however, always been recognised that both human and nonhuman primate groups are internally highly structured. Further analyses (*7*) have indicated that at least one level of structuring (the grooming clique) also correlates with neocortex size. While it is not always clear what the significance of these tiered groupings is, it is clear that human social groups (like those of other primates) consist of a series of hierarchically organised sub-groupings. We first

review previous quantifications of group sizes and then provide a systematic analysis. There is no universally accepted procedure and all methods attempting of identify group sizes suffer from several sources of bias (small sample size, large inter-individual variability, and the criteria used to include individuals). Our strategy is to include all the reasonable data and attempt to extract useful signals above the noise level by a careful analysis of the global data set.

The core social grouping is called the support clique, defined as the set of individuals from whom the respondent would seek personal advice or help in times of severe emotional and financial distress, whose mean size is typically 3-5 individuals (*8, 9*). Above this may be discerned a grouping of 12-20 individuals (often referred to as a sympathy group) that characteristically consists of all the individuals with whom one has special ties; these individuals are typically contacted at least once a month (*8, 9*). The ethnographic data on hunter-gatherer societies (*6*) point to a grouping of 30-50 individuals as the size of overnight camps (sometimes referred to as bands). These groupings are often unstable, but their membership is always drawn from the same set of individuals, who typically number in the order of 150 individuals. This last grouping is often identified in small scale traditional societies as the clan or regional group. Beyond these, at least two larger scale groupings have been identified in the ethnographic literature: the megaband of about 500 individuals and the tribe (a linguistic unit, commonly of 1000-2000 individuals) (*6*).

We complement the data used in Refs. (*6,8,9*), and sources therein with new data as follows. The USA 1998 General Social Survey reports a mean size of 3.3 for support clique (*10*). The sizes of sympathy groups are reported to be 14.0 in Egypt, 15.1 in Malaysia, 13.5 in Mexico, 13.8 in South Africa (*11*), 10.2 in USA (*12*), 15 in The Netherlands (1995) (*13, 14*), 15.0 in The Netherlands (1992), 14.3 in The Netherlands (1992-1993), 14.8 in The Netherlands (1995-1996), 14.2 in The Netherlands (1998-1999) (*15, 16*), and 14.4 in Mali of West Africa (*17*). See Figure 1.

Method 1: Average sizes of different network layers. To summarize the previously cited data, we denote S_1 as the mean support clique size, S_2 the mean sympathy group, S_3 the mean band size, S_4 the mean cognitive group size, and S_5 and S_6 the size of small and large tribes. Here, we do not address the relevance of this classification (which will be done below) but only characterize it quantitatively. The previously cited data gives $S_0 = 1$ (individual or ego), $S_1 = 4.6$, $S_2 = 14.3$, $S_3 = 42.6$, $S_4 = 132.5$, $S_5 = 566.6$, and $S_6 = 1728$. In order to determine the possible existence of a discrete hierarchy, we construct the series of ratios S_i/S_{i-1} of successive mean sizes:

$$S_i/S_{i-1} = 4.58, 3.12, 2.98, 3.11, 4.28, 3.05\ , \quad \text{for } \ i = 1, \cdots, 6\ . \tag{1}$$

This result suggests that humans form groups according to a discrete hierarchy with a prefered scaling ratio between 3 and 4: the mean of S_i/S_{i-1} is 3.50.

Method 2: Probability density function and generalized q-analysis of the complete data set. In order to avoid any potential biases in the published group classifications defined, we employ a more systematic method of analysis that uses all the available data and not just the mean group sizes.

The sample has 61 grouping clusters (including the ego) with size s_i available for $i = 1, 2, \cdots, 61$. Figure 1 presents the data in a form attributing group sizes to their relevant studies in an arbitrary order. We consider this sample to be a realization of a distribution whose sample estimation can be written as

$$f(s) = \sum_{i=1}^{61} \delta(s - s_i)\ , \tag{2}$$

where δ is Dirac's delta function. Figure 2 shows the probability density function $f(s)$ obtained by applying a Gaussian kernel estimation approach (*18*).

Our challenge is to extract a possible periodicity in this function in the $\ln s$ variable, if any. For instance, if the ratios given in (1) are genuine, one would expect a periodic oscillation

of $f(s)$ expressed in the variable $\ln s$ with mean period $\ln(3.5) = 1.24$. This is called "log-periodicity" (*19*).

Standard spectral analysis applied to $f(s)$ is dominated by the trend seen in Figure 2 giving a peak at a very low log-frequency corresponding to the whole range of the group sizes. We thus turn to generalized q-analysis, or (H, q)-analysis (*20*), which has been shown to be very sensitive and efficient for such tasks. The q-analysis is a natural tool to describe DSI in fractals and multifractals (*21, 22*). The (H, q)-analysis consists in constructing the (H, q)-derivative

$$D_q^H f(s) \stackrel{\triangle}{=} \frac{f(s) - f(qs)}{[(1-q)s]^H} \ . \tag{3}$$

Introducing an exponent H different from 1 allows us to detrend $f(s)$ in an adaptive way. Note that the limit $H = 1$ and $q \to 1$ retrieves the standard definition of the derivative of f. A value of q strictly less than 1 allows one to enhance possible discrete scale structures in the data. To keep a good resolution, we work with $0.65 \leq q \leq 0.95$, because smaller q's require more data for small s's. To put more weight on the small group sizes (which are probably more reliable since they are obtained by conducting general surveys in larger representative populations), we use $0.5 \leq H \leq 0.9$. A typical (H, q)-derivative with $H = 0.5$ and $q = 0.8$ is illustrated in a semi-log plot in Figure 3.

We then use a Lomb periodogram analysis (*23*) to extract the log-periodicity in $f(s)$. Figure 4 presents the normalized Lomb periodograms of $D_q^H f(s)$ for different pairs of (H, q) with $0.5 \leq H \leq 0.9$ and $0.65 \leq q \leq 0.95$. This figure illustrates the robustness of our result. For the specific values $H = 0.5$ and $q = 0.8$ shown in Figure 3, the highest peak is at $\omega_1 = 5.40$ with height $P_N = 8.67$. The preferred scaling ratio is thus $\lambda = e^{2\pi/\omega_1} = 3.20$, which is consistent with the previous result using the "grouping analysis" (1). The confidence level is 0.993 under the null hypothesis of white noise (*23*). If the underlying noise decorating the log-periodic structure is correlated with a Hurst index of 0.6, the confidence level decreases to 0.99; if the

Hurst index is 0.7, the confidence level falls to 0.85 (*24*).

The Lomb periodograms also exhibit a second peak at $\omega_2 = 9.80$ with height $P_N = 5.48$. This can be interpreted as the second harmonic component $\omega_2 \approx 2\omega_1$ of the fundamental component at $\omega_1 = 5.40$. The amplitude ratio of the fundamental and the harmonic is 1.26. The co-existence of the two peaks at ω_1 and $\omega_2 \approx 2\omega_1$ strengthens the statistical significance of a log-periodic structure. To see this, we constructed 10000 synthetic sets of 61 values uniformly distributed in the variable $\ln s$ within the interval $[0, \ln(2000)]$. By construction, these 10000 sets, which are exactly of the same size as our data and span the same interval, do not have log-periodicity and thus have no characteristic sizes. We then applied the same procedure as for the real data set to these synthetic data sets and obtain 10000 Lomb periodograms. We then performed the following tests on their Lomb periodograms. Find the highest Lomb peak (ω, P_N). If $P_N > 8.5$, check if there is at least another peak at $2\omega \pm 1$ with its P_N larger than 5.5. 238 sets among the 10000 passed the test, suggesting a probability that our signal results from chance equal to 0.024. The probability that there are at least two peaks (one in $4.9 < \omega < 5.9$ with $P_N > 8.5$ and the other in $9.5 < \omega < 11.5$ with $P_N > 5.5$) is found equal to 77/10000, giving another estimation of 0.993 for the statistical confidence of our results. Another metric consists in quantifying the area below the significant peaks found in the Lomb periodogram of our data and comparing them with those in the synthetic sets. We count the area of the main peak of the Lomb periodogram at ω and add to it the areas of its harmonics whose local maxima fall in the intervals $[(k - (1/5))\omega, (k + (1/5))\omega]$ for $k = 2, 3, ...$ around all its harmonics. The area associated with a peak is defined as the region around a local maximum delimited by the two closest local minima bracketing it. The fraction of synthetic sets which give an area thus defined larger than the value found for the real data is 6-7%, depending on the specific values H and q used in the analysis. Summarising, all these tests suggest that the evidence in support of our hypothesis data is significantly unlikely to result from chance, but rather reflects the fact

that human group sizes are naturally structured into a discrete hierarchy.

Method 3: Probability density function and generalised q-analysis of individual networks. We apply the same analysis to individual social networks based upon the exchange of Christmas cards in contemporary Western Society (*9*). This study indicated that contemporary social networks may be differentiated on the basis of frequency of contact between individuals, but that both 'passive' and 'active' factors may determine contact frequency. Controlling for the passive factors allowed the hierarchical network structure to be examined on the basis of residual (active) contact frequency. Starting from the residual contact frequencies, we constructed their (H, q)-derivative with respect to the number of people contacted for each individual, obtained the Lomb spectrum of the (H, q)-derivative and then averaged them over the 42 individuals in the study (Figure 5). The very strong peak at $\omega = 5.2$ is consistent with the previous results with a preferred scaling ratio from the expression $\lambda = e^{2\pi/\omega} \approx 3.3$ (*19*) for the smaller grouping levels in this study (group sizes below 150).

Discussion. Putting together a variety of measures collected under a wide range of experimental conditions and in different countries, we have documented a coherent set of characteristic group sizes organized according to a geometric series with a preferred scaling ratio close to 3. Similar hierarchies can be found in other types of human organizations, of which the military probably provides the best examples. In the land armies of many countries, one typically finds sections (or squads) of about 10-12 soldiers, platoons (of 3 sections, ≈ 35), companies (3-4 platoons, $\approx 120 - 150$), battalions (usually 3-4 companies plus support units, $\propto 550 - 800$), regiments (or brigades) (usually three battalions, plus support; $2500+$), divisions (usually 3 regiments), and corps (2-3 divisions). This gives a series with a multiplying factor from one level to the next close to three. Could it be that the army's structures have evolved so as to mimic the natural hierarchical groupings of everyday social structures, thereby optimising the cognitive

processing of within-group interactions?

The existence of a discrete hierarchy of group sizes may provide a key ingredient in rationalizing the reported existence of discrete scale invariance (DSI) in financial time series in so-called "bubble" regimes characterized by strong herding behaviors between investors (*25*). Johansen et al. (*26, 27*) have proposed a model to explain the observed DSI in stock market prices as resulting from a discrete hierarchy in the interactions between investors. Recent analysis of DSI in regimes with strong herding component have also identified the presence of a strong harmonic at 2ω, similar to the findings reported here (*28, 29*). The fact that DSI is found only during stock market regimes associated with a strong herding behavior suggests that it may reflect the fact that a discrete hierarchy of naturally occurring group sizes characterizes human interactions whether they be hunter-gatherers or traders. The present work suggests that this discrete hierarchy may have its origins in the fundamental organization of any social structure and be deeply rooted within the cognitive processing abilities of human brains.

When dealing with discrete hierarchies, it may be important to distinguish between the specific group sizes on the one hand and their successive ratios on the other. It may be that the absolute values of the group sizes are less important than the ratios between successive group sizes. If the ratio of group sizes is interpreted as a fractal dimension (specifically, the ratio is related to the imaginary part of a fractal dimension: see (*19*) and references therein), this would imply that, depending on the social context, the minimum "nucleation" size may vary, but the ratio (close to 3) might be universal. The fundamental question, then, is to determine the origin of this discrete hierarchy. At present, there is no obvious reason why a ratio of 3 should be important. Equally, however, we have little real understanding of what cognitive mechanisms might limit the nucleation point to a particular value. Considerable additional work will need to be done on both these components if we are to understand why these constraints on human grouping patterns exist and exactly what their significance might be.

References and Notes

1. J.S. Coleman, *An Introduction to Mathematical Sociology* (Collier-Macmillan, London, 1964).
2. C.P. Kottak, *Cultural anthropology* (McGraw Hill, New York, 1991).
3. R. Scupin, *Cultural anthropology – a global perspective* (Prentice Hall, Englewood Cliffs, 1992).
4. R.I.M. Dunbar, *Evol. Anthropology* **6**, 178 (1998).
5. R.I.M. Dunbar, *J. Hum. Evol.* **22**, 469 (1992).
6. R.I.M. Dunbar, *Behav. Brain Sci.* **16**, 681 (1993).
7. H. Kudo, R.I.M. Dunbar, *Anim. Behav.***62**, 711 (2001).
8. R.I.M. Dunbar, M. Spoors, *Hum. Nature* **6**, 273 (1995).
9. R.A. Hill, R.I.M. Dunbar, *Hum. Nature* **14**, 53 (2003).
10. P.V. Marsden, *Social Networks* **25**, 1 (2003).
11. C.J. Buys, *Psychol. Rep.* **45**, 789 (1992).
12. C. Latkin *et al.*, *Social Networks* **17**, 219 (1995).
13. S. Kef, *Journal of Visual Impairment and Blindness* **91**, 236 (1997).
14. S. Kef, J.J. Hox, H.T. Habekothé, *Social Networks* **22**, 73 (2000).
15. T.G. van Tilburg, M.I.B. van Groenou, *J. Soc. Issues* **58**, 697 (2002).
16. M.I.B. van Groenou, T.G. van Tilburg, *Ageing and Society* **23**, 625 (2003).

17. A.M. Adams, S. Madhavan, D. Simon, *Soc. Sci. Med.* **54**, 165 (2002).

18. B.W. Silverman, *Density Estimation for Statistics and Data Analysis* (Chapman and Hall, London, 1986).

19. D. Sornette, *Phys. Rep.* **297**, 239 (1998).

20. W.-X. Zhou, D. Sornette, *Phys. Rev. E* **66**, 046111 (2002).

21. A. Erzan, *Phys. Lett. A* **225**, 235 (1997).

22. A. Erzan, J.P. Eckmann, *Phys. Rev. Lett.* **87**, 3245 (1997).

23. W. Press, S. Teukolsky, W. Vetterling, B. Flannery, *Numerical Recipes in FORTRAN: The Art of Scientific Computing* (Cambridge University, Cambridge, 1996).

24. W.-X. Zhou, D. Sornette, *Int. J. Mod. Phys. C* **13**, 137 (2002).

25. D. Sornette, *Why Stock Markets Crash – Critical Events in Complex Financial Systems* (Princeton University Press, Princeton, NJ, 2003).

26. A. Johansen, D. Sornette, O. Ledoit, *J. Risk* **1**, 5 (1999).

27. A. Johansen, O. Ledoit, D. Sornette, *Int. J. Theor. Appl. Finance* **3**, 219 (2000).

28. A. Johansen, D. Sornette, *Int. J. Mod. Phys. C* **10**, 563 (1999).

29. D. Sornette, W.-X. Zhou, *Quant. Finance* **2**, 468 (2002).

30. Research by WXZ and DS was partially supported by the James S. Mc Donnell Foundation 21st century scientist award/studying complex system. Research by RH and RD was funded by the ESRC's Research Centre in Economic Learning and Social Evolution (ELSE). RD's

research is supported by the British Academy Centenary Project and by a British Academy Research Professorship.

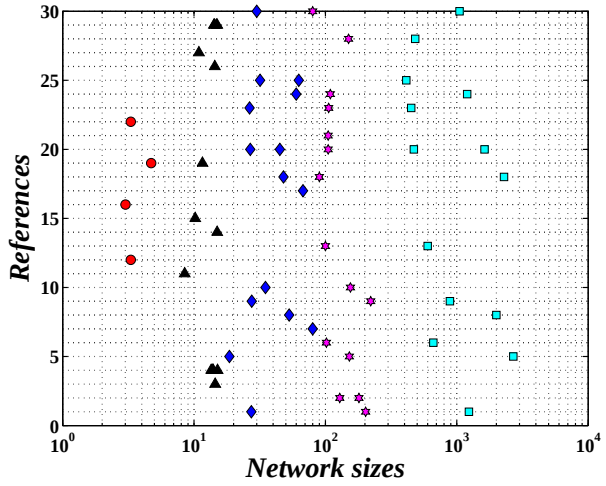


Figure 1: Presentation of our data set of 61 group sizes. The ordinate is an arbitrary ordering of data sources and the abscissa gives the group sizes reported in each sources. The symbols refer to the classification used in each of the studies: circle (support clique), triangle (sympathy group), diamond (bands), stars (cognitive groups), squares (small and large tribes). This classification is not used in our systematic analysis summarized in the other figures, to avoid any bias.

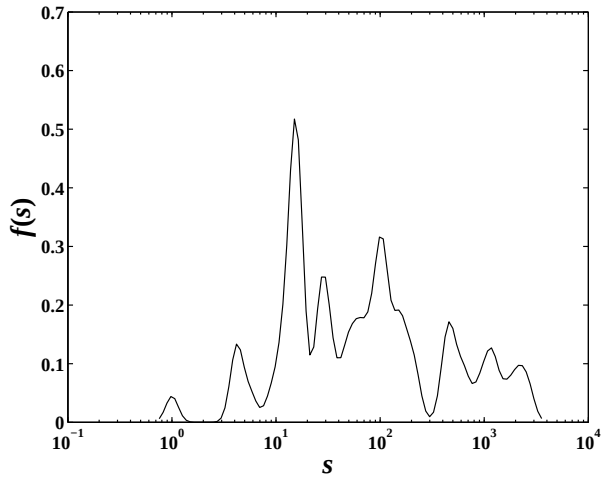


Figure 2: Probability density function $f(s)$ of size s estimated with a Gaussian kernel estimator in the variable $\ln s$ with a bandwidth $h = 0.14$. Varying h by 100% does change $f(s)$ much and gives similar results.

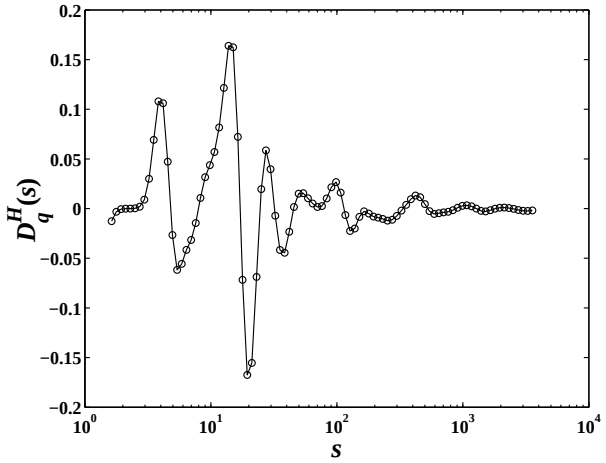


Figure 3: Typical (H, q)-derivative $D_q^H(s)$ of the probability density $f(s)$ as a function of size s with $H = 0.5$ and $q = 0.8$.

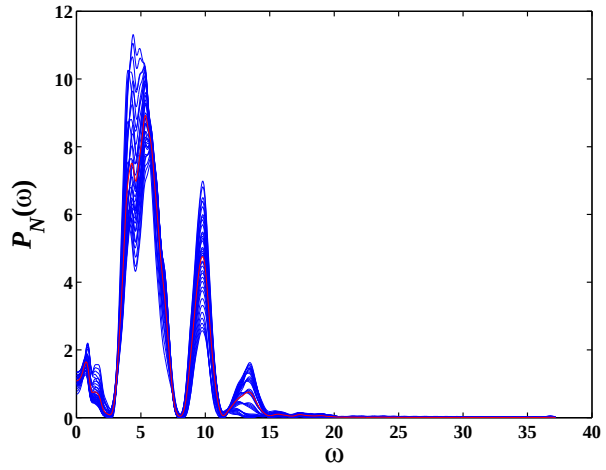


Figure 4: Normalized Lomb periodograms $P_N(\omega)$ as a function of angular log-frequency ω of the (H, q)-derivative $D_q^H(s)$ for different pairs of (H, q) with $0.5 \leq H \leq 0.9$ and $0.65 \leq q \leq 0.95$. The red line gives the averaged Lomb power.

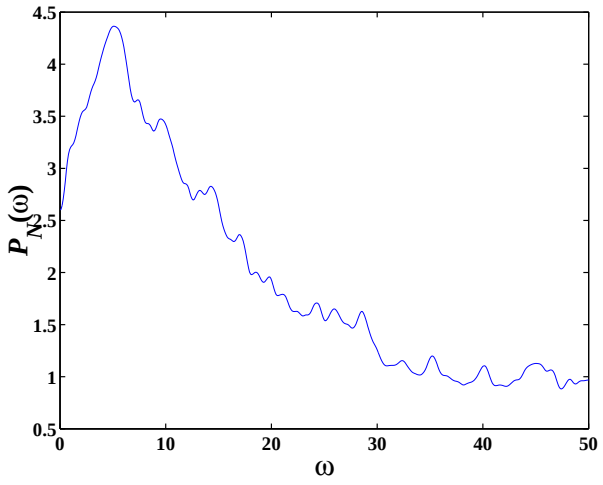


Figure 5: Average Lomb periodogram $P_N(\omega)$ of the (H, q)-derivative $D_q^H(s)$ with respect to the number of receivers of the residual contact frequency for each individual in the Christmas card experiment, as a function of the angular log-frequency ω of the (H, q)-derivative, over the 42 individuals and different pairs of (H, q) with $-1 \leq H \leq 1$ and $0.80 \leq q \leq 0.95$.