Modeling the Self-Organized Criticality of War Size

From Billiard Balls to Sandpiles*

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May 22, 2001

Abstract:

Richardson's finding that the severity of interstate wars is power-law distributed belongs to the most striking empirical regularities in world politics. As with earthquakes, there are many small events, fewer large ones, and a very small number of huge disasters. Yet, this is a regularity in search for a theory. Drawing on the principles of self-organized criticality, I propose a computational model of war and state-formation that exhibits power-law behavior in terms of war magnitudes. As a step toward developing systematic non-equilibrium theory of world politics, the model illuminates the underlying mechanisms responsible for generating the regularities. In the current framework, wars spread even in the absence of alliances due to exogenous shifts in the offense-defense balance, which force states to reassess their balance-of-power relationships.

*) Earlier drafts of this paper were prepared for presentation at the University of Michigan, the University of Chicago, and Ohio State University. I am grateful to the participants of those meetings and to Robert Axelrod and Claudio Cioffi-Revilla for excellent comments. Needless to say, I bear the full responsibility for any inaccuracies and omissions.

**) The Department of Government, Harvard University, from July 1, 2001.

1. Introduction

Since Richardson's (1948; 1960) pioneering statistical work, we know that casualty levels of wars are power-law distributed. As with earthquakes, there are many events with few casualties, fewer large ones, and a very small number of huge disasters. More precisely, power laws tell us that the size of an event is inversely proportional to its frequency. In other words, doubling the severity of wars leads to a decrease in frequency by a constant factor regardless of the size in question. This remarkable finding belongs to the most accurate and robust ones that one is likely to find in world politics.

Despite the strength of Richardson's law, IR scholars have paid little attention to it. While some recent confirmatory studies exist, to my knowledge, there are few, if any, attempts to uncover the mechanisms generating it. Drawing on recent advances in non-equilibrium physics, I suggest that the theory of

?self-organized criticality? (SOC) goes a long way toward providing an explanation. Relying on the explanatory strategy utilized in non-equilibrium physics, I regenerate the regularity with the help of a computational model that traces disruptions between meta-stable balance-of-power equilibria. It is shown that small changes in the aggressor states' offensive inclinations trigger geopolitical ?avalanches? of the type anticipated by SOC theory. Once power laws have been generated artificially, the conditions under which they appear can be investigated. Yet, rather than probing extensively into the robustness of this relationship, the main purpose of this paper is to generate it at all.

The paper is organized as follows. The next section describes the empirical puzzle. Section 3 introduces the literature on self-organized criticality. Following up the general notions of this concept, Section 4 highlights some candidate mechanisms that account for war expansion. Equipped with those conceptual tools, section 5 presents the modeling framework in which they will be implemented. Section 6 specifies the experimental research design used to generate geopolitical avalanches, followed by a section presenting the computational findings. Section 8 discusses the robustness and limitations of these results. Finally, the concluding section presents a number of consequences for IR theory in general.

2. Richardson's puzzle

As early as 1948, the English physicist and meteorologist Lewis F. Richardson published a landmark paper entitled ?Variation of the Frequency of Fatal Quarrels with Magnitude.? The essay divides domestic and international cases of violence between 1820 and 1945 into logarithmic categories *?* = 3, 4, 5, 6, and 7 corresponding to powers of ten (i.e. about 1,000, 10,000, 100,000, 1,000,000 and 10,000,000 respectively). Based on his own updated compilation of conflict statistics, Richardson (1960) recorded 188, 63, 24, 5, and 2 events that matched each category respectively, the latter two of course being the two world wars. His calculations revealed that the frequency of each size category follows a simple multiplicative law: for each ten-fold increase in severity, the frequency decreased by somewhat less than a factor three.

To investigate if these findings hold up in the light of more recent quantitative evidence, I use data from the Correlates of War Project (Geller and Singer 1998) while restricting the focus to interstate wars. Instead of relying on direct frequency counts through binning as did Richardson, which leads to information loss, my calculations center on the cumulative relative frequencies of war sizes n(S > s) = N(S > s)/N where S is the random variable of war sizes, and N the total number of wars. Assuming we have enough events, this quantity can be used as an estimate of the probability P(S > s) that there are wars of greater severity than s. Thus, whereas for small wars the likelihood of larger conflicts occurring has to be close to one, this probability approaches zero for very large events since it is unlikely that there will be any larger calamities.

In formal terms, it can be postulated that the cumulative probability scales as a power law:

$$P(S>s) = C\, S^{D}$$

where C is a positive constant and D is a negative number. Using double logarithmic scales, Figure 1 plots the *cumulative* frequency P(S > s) as a function of the severity S of interstate wars between 1820 and 1997. If there is a power law, the fit should be linear:

$$\log P(S > s) = \log C + D \log S$$

with the intercept log C and the slope D.

As can be readily seen, the linear fit is strikingly good confirming that the distribution follows a power law. While the data points in the lower right-hand corner correspond to the world wars, the vast majority of all other wars reach considerably lower levels of severity, though without straying very far from the estimated line. The slope estimate ?0.41 implies that a ten-fold increase of war severity decreases the probability of war by a factor of 26.

This regularity appears to be robust. It can be shown that these findings generalize beyond the two last centuries covered by the COW data. Similar calculations applied to Jack Levy's (1983) compilation of European great power wars from 1495 to 1965 yields a similarly straight line in a log-log diagram with an R^2 of 0.99, though with a steeper slope (the coefficient ?0.57 instead of ?0.41). A more steeply descending line corresponds to a less conflictual process, because by reaching smaller probabilities along the y-axis for lower severity levels, such a line makes really catastrophic events less likely.

Given these strong results it may seem surprising that so few scholars have attempted to account for what seems to be an important empirical law. In fact, the situation is not very different from the economists' failure to explain the power law governing the distribution of city sizes, also known as Zipf's law. Using urban population data from many countries, researchers have established that the rank of city size typically correlates astonishingly well with city size. In an innovative book on geography and economics, Paul Krugman (1995, p. 44) admits that ?at this point we have to say that the rank-size rule is a major embarrassment for economic theory: one of the strongest statistical relationships we know, lacking any clear basis in theory.?

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Figure 1. Cumulative frequency distribution of severity of interstate wars, 1820-1997
Source: COW data

By the same token, Richardson's law remains an equally acute embarrassment to IR theory. For sure, the law has been known for a long time, but the vast majority of researchers have paid scant attention to it. For example, there's no mention of it in Geller and Singer's (1998) comprehensive survey of quantitative peace research dating back several decades (see also Midlarsky 1989; Vasquez 1993). To my knowledge there are extremely few studies that attack this problem head-on. The only recent exception in the social sciences that I am aware of is a paper by Claudio Cioffi-Revilla (2000), who shows that the power-law regularity applies not only to interstate warfare but also to civil wars (see also Cioffi-Revilla and Midlarsky forthcoming; Wesley 1962; Weiss 1963). In addition, similar linear patterns can be found in log-log plots of war duration and time until the next onset. Although these are very important results, in this study I focus entirely on the sizes of interstate wars.

Given the discrepancy between the empirical findings and the almost complete absence of theoretical foundations on which to rely to account for them, we are confronted with a classical puzzle. The puzzle becomes all the more puzzling because of the notorious scarcity of robust empirical laws in IR. Despite decades of concerted efforts to find regularities in IR, why haven't scholars followed in the footsteps of Richardson, who, after all, is considered to be one of the pioneers of quantitative analysis of conflict? Postponing consideration of this question to the concluding section, I instead turn to a literature that appears to have more promise in accounting for the regularity.

3. Self-organized criticality

If IR theory, and the social sciences more generally, offer few hints about how to resolve the Richardsonian puzzle, where would one look for a solution? Natural scientists have been studying power laws in various settings for more than a decade. Usually organized under the notion of self-organized criticality (SOC), the pioneering contributions by Per Bak and others have evolved into a burgeoning literature that covers topics as diverse as earthquakes, biological extinction events, epidemics, forest fires, traffic jams, city growth, market fluctuations, firm sizes, and indeed, wars (for popular introductions see Bak 1996; Buchanan 2000; technical references are given below).

Self-organized criticality is the umbrella term that connotes slowly driven threshold systems that exhibit a series of meta-stable equilibria interrupted by disturbances with sizes scaling as power laws (Jensen 1998, p. 126; Turcotte 1999, p. 1380). In this context, thresholds stand for non-linearities in the system's operation that allow tension to build up inside. As the name of the phenomenon indicates, there has to be both an element of self-organization and criticality. To start with the latter, physicists have known for a long time that, if constantly fine-tuned, complex systems, such as magnets, sometimes reach a critical state between order and chaos (Jensen 1998, pp. 2-3; Buchanan 2000, Chap. 6). What is unique about SOC systems, however, is that they do not have to be carefully tuned to stay in the critical point where they generate the scale-free output responsible for the power laws.

Using a sandpile as a master metaphor, Per Bak (1996, Chap. 3) constructed a simple computer model that produces this type of regularity (see Bak, Tang, and Wiesenfeld 1987; Bak and Chen 1991). If grains of sand trickle down slowly on the pile, avalanches will be triggered from time to time. For most parts, the ?sand slides? will be very small, but once in a while, huge events are unleashed. This example illustrates the abstract idea of SOC: a steady, linear input generates tensions inside a system that in turn leads to non-linear and delayed output ranging from small events to huge ones. Without some type of internal thresholds, the system would settle instantly as do liquids. On the other hand, it is less stable than a solid rock.

Several important things can be said about SOC: First, such systems feature scale-invariant behavior in that standard sample statistics fail to summarize the distribution (Liebovitch and Scheurle 2000). In fact, the statistical mean is not even defined for power law distributions. Unlike normally distributed random variables, then, the system output of SOC systems has no typical scale of events, and there is in principle no upper bound on their size. Scale-invariant distributions are characterized by ?fat tails? that taper off less quickly for extreme values than do those of normal distributions. Since they are typically generated by self-similar processes characterized by recursive ?Chinese boxes?, power laws correspond closely to fractal geometry (Mandelbrot 1981). In fact, the slope coefficient for regression lines in log-log plots of the type shown in Figure 1 serves as a measure of the process' fractal dimension (Schroeder 1991; Jensen 1998, p. 5).

Moreover, whereas the macro-level distribution emerges as a stable feature of SOC systems, at the micro-level, such systems exhibit a strong degree of path-dependence. To use the sandpile as an illustration, it matters exactly where and when the grains land. Of course, the extreme sensitivity to small changes is what defines path-dependence (Arthur 1994). This means that point predictions often turn out to be futile in these cases, as exemplified by earthquakes. This does not mean, however, that no regularities exist. In particular, it is important to distinguish complex self-organized systems of the SOC kind from mere chaos, which also generates unpredictable behavior. By contrast, the former remain at the ?edge of chaos?, to resort to a popular buzz word (e.g. Kaufmann and Johnsen 1991). While excluding micro-level predictions, then, SOC is perfectly compatible with macro-level regularities (Bak 1996, pp. 29-31; Axelrod and Cohen 1999, p. xv; Buchanan 2000, pp. 14-15).

The radical difference from conventional equilibrium-based systems should also be noted. Though at rest most of the time, configurations in a state of SOC never fully settle down. In fact, the interesting behavior happens when the system moves from one meta-stable equilibrium to another. This property explains why SOC can be seen as a branch of non-equilibrium or historical physics (Buchanan 2000, pp. 210-212). In other words, power-law distributions can be seen as static footprints of dynamic processes. SOC helps us grasp the connection between the macro-level regularities and the underlying causal mechanisms generating the emergent patterns.

Finally, SOC has methodological consequences. Because of the inherent complexity of such systems, researchers have had little choice but to rely extensively on computational experiments to find the underlying mechanisms responsible for real-world equivalents. It is therefore no coincidence that the study of SOC has grown exponentially in the last decade as computing power has become increasingly cheap and accessible (see the extensive references in Turcotte 1999).

All this is interesting, the student of international politics may say, but do these insights really generalize to interstate warfare? It is important not to be put off by some of the exaggerated universality claims advanced by some of the proponents of SOC. In view of the strong findings in a wide variety of areas, complexity researchers can be forgiven some of their enthusiasm. Indeed, one is inclined to agree with Mark Buchanan (2000, p. 17) that ?the ubiquity of the critical state may well be considered the first really solid discovery of complexity theory?. On the other hand, sweeping speculations suggesting that this is ?how nature works? (Bak 1996) have to be taken with a liberal pinch of salt (or perhaps sand!). While useful as a diagnostic, the mere presence of power laws does not guarantee that the underlying process is characterized by SOC. As any other class of explanations, SOC-based accounts ultimately hinge on the theoretical and empirical plausibility of their causal mechanisms.

Precisely this is the weakness afflicting the few attempts that have so far been made to explain why wars are power-law distributed. Recently, the geologist Donald Turcotte (1999, pp. 1418-1420) has observed that Richardson's result resembles a model of forest fires (see also Roberts and Turcotte 1998). Computational models of such phenomena are known to produce slope coefficients not unlike the one observed in Figure 1. If forest fires start by lightnings igniting sparks that spread from tree to tree, Turcotte (1999, p. 1419) suggests, ?a war must begin in a manner similar to the ignition of a forest. One country may invade another country, or a prominent politician may be assassinated. The war will then spread over the contiguous region of metastable countries.? While suggestive, it goes without saying that this analogy cannot serve as an explanation in its own right, because at the level of mechanisms, there are simply too many important differences between forests and state systems. In particular, the scenario has rather little to do with main explanations for how wars spread.

Nevertheless, Turcotte's conjecture points in the right direction. The key to any explanation of war sizes resides in the way that wars spread. Moreover, the linking of the phenomenon to SOC makes a very promising theoretical contribution. Similarly referring to ?tensions? in history and referring to Paul Kennedy's theory of uneven growth rates, Mark Buchanan (2000, pp. 189) recognizes that such ideas remain too general as explanations to be fully convincing:

> Neither the ?web of international relations? nor the social fabric of any society is a thing easily grasped. Even so, it is probably beyond argument that some form of stress must build up in both. It seems likely as well that as this stress builds, it is not always (or even often) immediately released by ?adjustments?. It has to build beyond some threshold before anything changes.

What is needed is a more realistic model of warfare that renders the ?web of international relations? explicit and in which the conditions and mechanisms generating self-organized criticality can be systematically explored. It is exactly this that is the main purpose of current paper. Thus my research strategy differs very little from the way that natural scientists have relied on computers to study SOC in complex settings. If, for instance, a simple sliding-block model exhibits power-law regularities, we can be more confident that continental plates behave in a similar way. Similarly, the discovery of power laws in a computational model featuring reasonably realistic representations of war and state-formation would strengthen the case for SOC in world politics. The immediate task at hand, then, consists in identifying a set of mechanisms that expand wars.

4. Explaining the scope of warfare

To account for the size of wars is equivalent to explaining how conflict spreads. Rather than treating large wars, such as the world wars, as qualitatively distinct events that require separate explanations (e.g. Midlarsky 1990), it is preferable to advance a unified theory that explains all wars regardless of their size (e.g. Kim and Morrow 1992). Apart from the inherent desirability of more general explanations, the focus on SOC encourages us to search for scale-invariant explanations.

Although the overwhelming majority of the literature on warfare focuses on explaining the outbreak of war, several attempts have been made to account for how wars expand in time and space. The theoretical literature points to a number of plausible culprits beyond the domestic conditions that Turcotte referred to in his forest-fire analogy (see Vasquez 1993, pp. 244-246). The perhaps most important mechanism relates to alliances. The consensus among empirical researchers confirms that alignment can serve as a conduit of conflict by entangling more states than without alliances (for references, see Vasquez 1993, pp. 234-237). Apart from this, the basic fact of territorial contiguity serves as an obvious factor enabling conflictual behavior to spread (Ibid., pp. 237-240). Geopolitical adjacency in itself says little about how warfare expands, however. An explanation needs to account for the active motivation driving the process. The main logic pertains to how geopolitical instability changes the strategic calculations in the context: ?An ongoing war, no matter what its initial cause, is likely to change the existing political world of those contiguous to the belligerents, creating new opportunities, as well as threats? (Vasquez 1993, p. 239; see also Wesley 1962).

Despite their inherent theoretical and actual importance, alliances will be put aside (although I will return to this topic below). Instead, the focus will be entirely on territorial processes of geopolitical expansion through conquest. Because of its simplicity, the alliance-free scenario serves as a useful baseline for further investigations featuring alignment and other complications. How could wars spread without alliances? Drawing on Vasquez' reference to strategic context, it can be suggested that military victory resulting in conquest changes the balance-of-power calculations of the affected states. The conqueror typically grows stronger while the weaker side loses power. This opens up new opportunities for conquest, sometimes prompting a chain reaction that will only stop until deterrence or infrastructural constrains dampen the process (e.g. Liberman 1996).

What could turn the balance of power into such a period of instability? Clearly, the list of sources of change is long, but here I will highlight one important class of mechanisms, namely technological or doctrinal innovations. As Robert Gilpin (1981, p. 60) points out, technological changes often give a particular state an advantage that can translate into territorial expansion. Yet, it needs to be remembered that ?international political history reveals that in many instances a relative advantage in military technique has been short-lived. The permanence of military advantage is a function both of the scale and complexity of the innovation on which it is based and of the prerequisites for its adoption by other societies.? Under the pressure of geopolitical competition, new military or logistical techniques typically travel quickly from country to country until the entire system has adopted the more effective solution. It is especially in the window that such transitions open up that war and conquest take place.

Thus innovations are a potentially disruptive source of change. Gilpin (1981) explains that there are at least two ways in which they can influence geopolitical conditions: the transformation may affect the effectiveness of resource extraction: ?Military innovation gives a particular society a monopoly of superior armament or technique and dramatically decreases the cost of extending the area of domination, thus providing a society with a considerable advantage over its neighbors and an incentive to expand and to change the international system? (Ibid., p. 60). Second, there could be an effect on the relationship between offense and defense: ?Military innovations that tend to favor the offense over the defense stimulate territorial expansion and the political consolidation of international systems by empires or great powers? (p. 61).

Without prejudging the substantive importance of the first of these mechanisms, the remainder of this paper concentrates entirely on the second one. While ?offense-defense theory? (e.g. van Evera 1999) has come in for a lot of trenchant criticism as an explanation of specific wars (e.g. Sagan 1984), there can be no doubt that sudden shifts favoring offensive strategies are bound to have momentous repercussions for warfare: ?From the early modern era to the Napoleonic period the balance between defense and offense oscillated. However, the Napoleonic revolution in military affairs led to a significant predominance of the offense that resulted in a continuous political consolidation of Western and Eastern Europe? (Gilpin 1981, p. 62; for more historical examples, see McNeill 1982; though see Thompson 2000, Chap. 5).

The theory of SOC presupposes a process of reasonably slow change driving the system together with some type of internal thresholds, which allow the tension to build up. Once released, the geopolitical avalanches are then expected to be scale-free. To return to familiar metaphor of sandpiles, what corresponds to the stream of sand in the case of geopolitics? To fix our thought, it can be suggested that the gradual diffusion of an offense-promoting innovation exposes a balance-of-power system to similar stress as the falling grains of sand. In fact, the model I propose traces balance-of-power systems undergoing transitional periods that introduce a more offensive orientation. But innovations take some time to spread and even more time to have an actual impact on decision making. It is not farfetched to imagine that strategic caution introduces a set of thresholds that adds some inertia to not unlike the friction holding together a sandpile.

5. Modeling geopolitics

Now it is time to confront the challenge of model-building. How could we move beyond models of sandpiles and forest fires? Because the power law of Figure 1 stretch over centuries, it is clearly necessary to focus on Braudel's *longue durée* of history. But this raises the explanatory bar considerably, because rather than treating states as billiard balls, it is necessary to view them as territorial entities with dynamically growing borders. Levy's data, focusing on great power wars in Europe, for example, coincide with massive rewriting of the geopolitical map of Europe. At the dawn of early modern Europe, there were half a thousand independent geopolitical units in Europe, a number that decreased to some twenty states by the end of Levy's sample period (Tilly 1975, p. 24; cf. also Cusack and Stoll 1990, Chap. 1).

It therefore seems hopeless to trace macro patters of warfare without endogenizing the very boundaries of states. In my previous work, I have constructed an explicitly spatio-dynamic model of power politics that lends itself ideally to this type of exploration. Though constructed primarily to study state-formation and polarity structures, it relies heavily on warfare as the key to geopolitical consolidation. In doing so, it formalizes Charles Tilly's famous aphorism: ?War made the state and states made war?.

The current model represents a slight elaboration of the previous framework. At the methodological level, the new modeling effort required a complete re-coding of the earlier Pascal-based specification. Here I rely on a Java-based simulation package called RePast which is developed and supported by the University of Chicago's social-science computing center.

As opposed to my earlier work, which relied on rather small grid sizes, the standard initial configuration consists of a 30 x 30 square lattice populated by state-like agents interacting locally. Initially, each actor has up to four neighbors with which it can cooperate or fight. Soon, however, some of them will grow through conquest which means that the number of neighbor has to be variable. In fact, each sovereign state possess a

dynamic list of all its neighbors.
In each time period, the state-like actors allocate resources to each of their fronts and then choose whether or not to fight with their territorial neighbors. While a small share of each state's resources are allocated evenly to its fronts, the bulk thereof goes to a pool of mobile resources that are distributed in proportion to the power and activities of each neighbor. This scheme assures that military action on one front dilutes the remaining resources available for mobilization, which in turn creates a strong strategic interdependence that ultimately affects other states' decision-making. Because the main rationale of the paper is to study geopolitical consolidation processes, the current model excludes the possibility of secession. The appendix describes this and all the other computational rules in detail.
For the time being, let us assume that all states use exactly the same ?grim-trigger? strategy in their relations. Normally, they reciprocate their neighbors' actions. Should one of the adjacent actors attack them, however, they respond in kind without relenting until the battle has been won by either side or ends with a draw. Unprovoked attacks happen as soon as a state finds itself in a sufficiently superior situation vis-à-vis a neighbor. Set at a ratio of 2.2 of the locally allocated resources, the stochastic threshold defines the defense-offense balance of each state (see Figure 2). As described in the following section, this is the crucial dimension that will be manipulated in the computational experiments (i.e. the shift to the new thresholds). For the moment we focus on the bold curves. The figure also displays the less precise criterion determining if the aggressor can claim to have won the battle. As with the decision to attack, the threshold is located slightly above the double force ratio, but because this threshold is flatter, and thus more stochastic, than the one governing decisions, the victory often becomes possible for much lower levels of superiority than 2.2.

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Figure 2. The criteria of attack and battle victory as functions of the force ratio.

When the local capability balance tips decisively in favor of the stronger party, conquest results, implying that the victor absorbs the targeted unit. This is how hierarchical actors form. If the target was already a part of another multi-province state, this state loses its province. Successful campaigns against the capital of corporate actors lead to their complete collapse. The appendix provides further details showing exactly how structural change unfolds.
Territorial expansion has important consequences for the states' overall resource levels. After conquest, the capitals of conquered territories are able to ?tax? the incorporated provinces including the capital province. The extraction rate depends on a distance function that approaches one for the capital province but that falls quickly as the distance form the center increases. Far away, the rate flattens out around 20% (again see the appendix for details). This property introduces considerable infrastructural inertia that captures the effect of the loss-of-strength gradient (Boulding 1963; Gilpin 1981, p. 115). The other constraint on expansion relates to the competitive pressure by other powers (Gilpin 1981, p. 147).
These rules entail four things: First, the number of states will decrease as the power-seeking states absorb their victims. Second, as a consequence of conquest, the predatory actors increase in size, their territory expanding into the conquered areas. Third, decentralized competition creates emergent boundaries around the composite actors. Fourth, once both sides of a border reach a point at which no one is ready to launch an attack, a local balance-of-power equilibrium obtains. If all borders are characterized by such balances, a global, meta-stable equilibrium emerges. Such an equilibrium is meta-stable rather than perfectly stable because, as suggested by Figure 2, decision making always involves an element of chance. As an illustration, Figure 3 shows one such system after 500 iterations. Of all the 900 states in initial system, only 66 considerably larger units managed to survive in this case. The lines indicate the states' territorial borders and the dots their capitals.
Of course, the situation represents a drastically simplified picture of real interstate systems, especially since there is no room for norms. In this sense, the geopolitical model renders a situation that is Hobbesiean rather than Lockean (Wendt 1999). As neorealists assume, the only sources of stability are logistical constraints and opportunistically enforced balances.

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Figure 3. The world with 66 surviving states at time 500.

6. Experimental design

Now that we have bred a rudimentary geopolitical balance-of-power system, it is time to explore conflict patterns systematically. In order to measure the latter, this section introduces a two-stage experimental design. In the first phase, a massively multipolar system of the type displayed in Figure 3 is created. Once such a meta-stable, non-equilibrium setting has been generated, it can be exposed to the experimental treatment, which introduces a technological change that increases the states' offensive orientation.

The first phase requires precise tuning in order to generate multi-polar systems. In the absence of secession, insufficient polarity levels would result in too little geopolitical variation. After all, the regularities to be measured can only be observed at the aggregate level. Furthermore, it is important that the first stage generate reasonably stable outcomes or the treatment could not be separated from the background ?noise? of geopolitical change. For the time being, I will use the 66-polar world represented by Figure 3.

In the second stage, which starts after 500 time periods, the main issue concerns technological diffusion and its behavioral consequences. To start with the latter, technological change can be simulated by altering the threshold function shown in Figure 2. While there are many ways of altering the curve, here I merely consider a general lowering of the threshold from 2.2 by a given amount. The shift from the bold curves to the non-bold ones, labeled ?new?, illustrates a reduction by 10% to a threshold at about two. The increased offensive ?appetite? will make the state in question less cautious in its strategic calculations. In addition, the offensive change also shifts the victory criterion in exactly the same way. This means that the new offense-defense balance not only affects the calculations but also the combat effectiveness of the forces benefiting from the innovation.

How does this type of innovation spread through the system? Rather than letting all the states benefit from the new technology at once, I assume that it is going to be adopted with a fixed probability set at 0.001 per state and time period. In the current configuration, this probability is not affected by neighboring states' threshold functions. This means that the new offensive orientation is going to appear gradually throughout the entire system until all states have adopted the new technique.

Let us now study an example of such an innovation cycle and its geopolitical impact. Figure 4 shows the system in Figure 4 at time period 760. At this later point, a number of states have already adopted the new technology (as indicated by the shading). While no conquest has taken place, it is only a few steps away. Note the non-reformed (white) state that is completely surrounded by offensive states somewhat above the center of the grid. It will soon become clear that its days are numbered.

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Figure 4. Technological diffusion at time period 760.

Figure 5 illustrates the situation at time period 809 after the first wave of conquest has taken place. In period 770, hostilities broke out that wiped out not only the surrounded state, but also one of the shaded actors that previously surrounded it. As result, some of the larger unreformed states joined the fray and enlarged their territory. Yet, the overall impact on the grid was local. This time no major chain reaction started.

If this process is allowed to continue, a drastically different picture emerges (see Figure 6). At time period 10,000, after a long series of wars, only six states coexist in a meta-stable balance-of-power equilibrium. As would be expected, they all possess the new technology (thus the entire system is now shaded).

Before turning to systematic replications, it is necessary to operationalize war size more precisely. This task involves two problems. First, spatio-temporal conflict clusters have to be identified as wars. Once identified, their size needs to be measured. To solve the measurement problem, I use a proxy for severity that is usually referred to as ?magnitude? in the literature (Levy 1983; Vasquez 1993; Geller and Singer 1998). In such empirical studies, magnitude measures the number of unit-years. It can be safely assumed that this indicator is strongly correlated with severity. In the simulations, I use a similar indicator of war size defined as the total number of dyad-periods per conflict cluster.

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Figure 5. The situation after conquest at time period 809.

It should be clear that the first question pertaining to case identification presents a more difficult challenge. In real historical cases, empirical experts bring their intuition to bear in determining the boundaries of conflict clusters. While it is not always a straight-forward problem to tell ?what is a case? (Ragin and Becker 1992), wars tend to be reasonably well delimited (though see Levy 1983, Chap. 3). In the computation model, by contrast, this task poses considerable problems because of the lack of historical intuition. The modeling framework therefore includes a spatio-temporal cluster-finding mechanism that draws a distinction between active and inactive states. Active states are those that are currently fighting, or that fought in the last five time periods. The latter provision eliminates noise due to briefs lulls in the combat. To keep things simple, a cluster in a specific period is defined as any group of adjacent fighting states whether they fight each other or not. The temporal connection is made by letting the states that are still fighting in subsequent periods retain their cluster identification. Once there is no more active state belonging to the conflict cluster, it is defined as a war and the accumulated conflict count belonging to it is reported as its magnitude. This counter makes it possible to separate smaller conflicts that happen concurrently in different parts of the system as long as they do not merge into the same war.

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Figure 6. The situation after conquest at time period 10,000.

7. Computational findings

This section begins by presenting the key findings from the sample system traced in the two previous sections, before widening the scope to cover other settings. The logical place to start is to use the output of our sample system to generate a chart similar to the empirical log-log diagrams displayed in Figure 1. Starting in period 500 and stopping at period 10,000, the magnitude counter records the following series of 20 war magnitudes in chronological order: 1, 33, 15, 3, 82, 5, 9, 1, 1, 307, 881, 1, 10, 208, 6, 6, 16, 1, 1, 849, and 28. Whereas the second recording amounting to 33 dyad-periods corresponds to the war that happened between time periods 770 and 809, the first one records a single conflict dyad without territorial consequences in the lower right of the system (see Figures 4 and 5).

Based on the same type of calculations as in the previous figures, Figure 9 plots the cumulative frequency against the corresponding magnitudes. Already in this small example, it is possible to discern a negative linear trend with a slope of about ?0.36. Interestingly enough, the estimated slope resulting from the simulated run falls not very far from the empirical result from the last two centuries reported in Figure 1.

While it cannot be excluded that these findings describe a power law, not too much should be concluded from the fact that our first computational results happen to fall close to the empirical regressions. So far, I have only considered a single run with one size of the offensive shocks, i.e. 10%. It could be expected that the regression line would become steeper were the shocks smaller, because for each war, the likelihood that a larger one could occur should fall more quickly.

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Figure 7. Cumulative frequency distribution of war events in the computational model (single run in 66-state sample system).

To test this intuition, I ran similar experiments with offensive increases at 5% and 15% in addition to 10%. Here the second stage was repeated with separate streams of random numbers starting at time period 500. All in all, a batch of 30 runs was run for each of the three shock sizes. Thus, in each of these series, the 66-state system shown in Figure 4 was first regenerated identically 30 times in the first stage. From time period 500, however, each reenactment of the second stage differed stochastically in that path-dependence generated outcomes at time period 10,000 quite different from the final outcome shown in Figure 8, though the shape is still linear. As sandpiles erode in path-dependent ways, geopolitical systems take different ways depending on the order of the geopolitical events.

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Figure 8. Cumulative frequency distribution of sample system comparing three levels of offensive shocks (30 replications of the 66-state system)

Figure 8 shows that the system behaves as expected. First of all, the changes do not alter significantly the linear shape of the estimated curves. For large war magnitudes, however, there is a precipitous fall in cumulative frequency that reflects the limited run times and equally limited size of the grid. Similar effects have been discovered in simulations with limited grid sizes (see e.g. Jensen 1998; Turcotte 1999). This boundary effect prevents very large events from occurring. Below, I explore a larger grid to verify whether this is indeed a boundary phenomenon. Again, the lower numbers also have to be taken with a grain of salt since the counting mechanism fails to account properly for smaller events.
As regards the slopes, the findings also support our intuition. With small disruptions at 5% the line falls quickly with a slope of ?0.75. The descending trend is much slower for revolutionary worlds in which the innovation implies a 15% change reduction of the threshold. Here the line becomes much flatter with a coefficient of ?0.41. In between these two extremes, shocks at the 10% level produce an intermediate slope of ?0.53.

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Figure 9. Cumulative frequency distribution of a 45 x 45 system (10 replications)

Since the main theoretical claim of SOC refers to scale-invariance, one obvious question pertains to whether these results can be replicated in an even larger system. Experiments with a 45 x 45 system with 131 states a time 500 were run until time period 15,000 and yielded encouraging results, as indicated by Figure 9.
Most importantly, the linear shape holds up very convincingly. In this case, the slope is somewhat steeper than in the 66-state system (but not very different from the other runs reported on in footnote 15). Thus, the power-law regularity does not seem to be an artifact of the system's size. Again, a boundary effect appears for high magnitudes, but the cutoff has shifted almost all they way up to log s = 4, which is to be expected since there are roughly twice as many states at the beginning of the second phase.

8. Discussion: how robust are the results?
After having looked at many straight lines, it is easy to forget that the previous section has shown findings based on only one specific set of parameters. Table A1 in the appendix reveals that there are many knobs to turn and, as a matter of fact, the calibration process turned out to be rather difficult. Since the main purpose of this paper is to generate power laws at all, something that has so far not been done in any model, extensive sensitivity analysis lies beyond the scope of this paper. Yet a few remarks on the robustness of these results are in order.
The trick is to construct a system that ?simmers? without ?boiling over?. We want to see some big conflicts but if the system were to explode in perennial warfare, there would be too few observations. Since the intermediate range of geopolitical consolidation actually does require a lot of parameter adjustment in the current model, the pure case of parameter-insensitivity characterizing SOC cannot be said to have been fully satisfied, at least to the extent that parameter selection isn't guided by detailed knowledge about the micro-level mechanisms. Generally, the current model is too ?nervous? and warfare too frequent for the system to be fully realistic, at

least within a European setting (though see the ancient Chinese case, Hui 2000). Ultimately, there have to be decentralized, adaptive mechanisms that somehow deliver the desired power laws within a large domain of parameter settings, for otherwise the correspondence with the empirical regularities would be a mere coincidence. Currently this domain is probably too narrow, at least if we want to generate power laws ranging over four orders of magnitude as that reported in Figure 1.

It is true that some of the tuning is of less concern since it is linked to the adjustment of the first-stage systems rather than to the generation of power laws. If there are going to be any interesting power-law phenomena at all, it is necessary to generate a substantial number of states to begin with. The specific tuning of the superiority threshold reflects this priority. With too high a superiority threshold, the polarity would sink due to reduced geopolitical competition. It was found that a ratio of 2.2 generated about the right number of states in the first stage, but other values also generated regimes of power laws (such as 2.0, 2.5 and 3.0). As would be expected from the threshold logic of SOC, it became harder to generate power laws once the superiority ratio was reduced to lower levels. Higher thresholds generally called for larger shocks, or the larger events would not materialize. My preliminary experiments also suggest that the logistical loss-of-gradient function plays a very important role in slowing down conquest processes (thus keeping the ?pot simmering!?). Yet, the constraints should not be too overwhelming, or large-scale wars would never occur. Indeed, the more one plays with the parameter tuning of the model, the more remarkable the empirical law appears.

What self-correcting mechanisms could render the scale-free behavior more insensitive to parameter variations? Along the long road of model construction, I made a number of simplifying assumptions that might have undermined the system's robustness. First of all, the attention must turn to alliances since, as we have seen above, they have been singled out in the theoretical and empirical literature on war diffusion. Some preliminary experimentation with strictly defensive mechanisms managed to calm the system down but with the (theoretically) undesirable side-effect of thwarting large wars. This might change if a less defensively oriented alignment process is introduced.

Another important source of self-adjustment that could increase the competitiveness is the adoption of defensive technologies in response to offensive innovations. The current model lowers the bar for all states where it then stays in all perpetuity, but in the real world, the offense-defense threshold is likely to rebound after an offensive dip. Such ?action-reaction? phenomena have been extensively described in the literature and could also provide the key to a more supple, self-organizing system.

At the structural level, it would be necessary to consider the possibility of secession and civil wars. Secession could interrupt positive-feedback cycles of imperial expansion through implosion. In addition, by depicting mos wars as being decisive triggers of conquest, the model probably ties conflictual behavior too closely to state-formation. Obviously not all wars start with territorial conquest as the main goal and they certainly do not always end with such outcomes. Moreover, there is also nationalism, which affects not only the extractive capacity but also the boundaries of states through national unification and secession. In a separate model, I have been able to generate power-law distributed wars as a consequence of nationalist mobilization.

As already stated, much remains to be done in terms of calibrating the temporal distribution of wars. The issue of calibration also shows up in the case the specific innovation mechanism on which the model has been relying. Here there is obviously room for the introduction of more realistic processes, at least to the extent that data can be gathered on the offense-defense ratios and the extractive capacity of states. For example, it cannot be excluded that the empirical power laws derive at least in part from the exponential growth in weapons' own destructiveness.

9. Conclusion: consequences for IR theory

Despite the complications introduced by parameter sensitivity, the fact remains that the model in its present ?stripped? form has fulfilled its primary purpose, namely that of generating power laws similar to those observed in empirical data. The current framework may well be the first model of international politics that does precisely that. In doing so, it takes us one step closer to resolving Richardson's original puzzle first stated more than half a century ago. The computational reconstruction of this regularity strengthens our confidence in the conjecture that interstate warfare does follow the principles of self-organized criticality. However, stronger confidence does not equal conclusive corroboration, which requires considerably more accurate portrayal of the causal mechanisms generating the phenomenon in the first place.

Assuming that the SOC conjecture holds after all, important consequences for theory-building in IR flow from these principles. By using the method of exclusion, we have to ask what theories are capable of generating regularities of this type. Most obviously, the logic of SOC casts doubt on static equilibrium theories as blue-prints for systemic theorizing. Whereas equilibria clearly exist at the micro-level, it cannot be imputed that they exist at the macro level. If wars emanate from disequilibrium processes, then these theories' narrow focus on equilibrium is misguided. It is not hard to find the main reason for this ontological closure: micro-economic theory has served as a very important source of inspiration for theory-builders in IR, and, if there has been any change in recent years, this influence has grown stronger with the surge of rational-choice research (Thompson 2000, p. 26).

At the level of general theorizing, it is Waltz (1979) who represents this analogical borrowing by stressing the prevalence of negative feedback and rationality in history. Yet, if SOC is a correct guide to interstate phenomena such as war, it seems less likely that static frameworks such as that suggested by Waltz are the right place to start in future attempts to build systems theory. In fact, his sweeping anarchy thesis remains too vague to be particularly helpful in explaining particular wars or any aggregate pattern of warfare for that matter (Vasquez 1993).

This reasoning does not render realist analysis of warfare is obsolete, however. What the SOC analysis does tell us, however, is that realist theorizing needs to be put on explicitly spatio-temporal foundations. With respect to the former, the findings of this paper suggest that Robert Gilpin's (1981) analytical sketch of war and change offers a much more fruitful point of departure than does Waltz. Partly anticipating the SOC perspective, Gilpin (1981) advances an explicitly dialectical theory that interprets wars as releases of built-up tensions in the international system: ?As a consequence of the changing interest of individual states, and especially because of the differential growth in power among states, the international system moves from a condition of equilibrium to one of disequilibrium? (p. 14). Once the tension has been accumulated, it will sooner or later be released, usually in a violent way: ?Although resolution of a crisis through peaceful adjustment of the systemic disequilibrium is possible, the principal mechanism of change throughout history has been war, or what we shall call hegemonic war? (p. 15). Yet, rather than adopting an exclusively revolutionary approach to warfare, Gilpin realizes that most adjustments produce much smaller conflicts. By adopting an explicit non-equilibrium focus, Gilpin is clearly on the right track. Yet, despite his search for invariant ?law of change?, Gilpin admits that his theoretical framework remains an untested set of statements rather than a firmly integrated substantive theory.

The current paper has attempted to bring fresh theoretical thinking and new methods to what has so far been a rather amorphous set of propositions on ?historical institutionalism? (cf. March and Olsen 1998). This is particularly important since constructivist research has tended to degenerate into unsystematic ?narration.? By over-emphasizing contingency, post-modernists and many historians risk getting sucked into a black hole of relativism from which very little light escapes. It is against such ?chaos? thinking that complexity theory and computational modeling provide a useful antidote.

Viewed as a source of theoretical inspiration, then, the sandpiles of non-equilibrium physics may prove more useful as master analogies than both the billiard balls of classical physics and than the ?butterfly effect? of chaos theory. Earthquakes, forest fires, biological evolution, and other historically formed complex systems, serve as better metaphors for the broad picture of world history than ?ahistorical? pool tables or intractable turbulence. It may not be a coincidence that scholars trying to make sense of historical disruptions have been prone to use seismic analogies. According to John Lewis Gaddis' (19992, p. 22) analysis of the end of the Cold War, ?[w]e know that a series of geopolitical earthquakes have taken place, but it is not yet clear how these upheavals have rearranged the landscape that lies before us.?

Following in the footsteps of modern natural science, computational modeling enables IR theory to move from such intriguing, yet very loose, analogies to detailed investigations of how causal mechanisms interact in time and space. While ?system effects? are well understood by qualitative theorists (e.g. Jervis 1997), they have not been integrated into coherent theorizing. Today's static ?variable-based thinking? and piling of *ceteris-paribus* ?hypotheses? miss the point by failing to appreciate the contextual features of world history (Abbott 1997).

Most importantly, careful modeling may help us avoid the pitfalls of the simplistic analogizing that has so often haunted IR theory. For example, a seismic analogy supported by statistical parallels between wars and earthquake magnitudes could tempt realist ?pessimists? to conclude that wars are as unavoidable as geological events. Such a conclusion does not follow from SOC theory at all, for unlike continental plates, democratic security communities can emerge on a comparatively quick historical scale. As in the geological case, we know that while some areas of the world are prone to frequent outbreaks of interstate violence, in others, catastrophic events are virtually unthinkable. But unlike continental plates, the pacific regions are socially constructed features of the international system. As conjectured by Immanuel Kant, the emergence of democratic security communities over the last two centuries shows that the ?laws? of geopolitics can be transcended.

If SOC provides an accurate guide to world politics, it can be concluded that disaster-avoidance through the ?taming? of *Realpolitik* by boosting the defense (e.g. through a missile defense system) or by avoiding ?bandwagoning behavior? may be as futile as hoping that the ?new economy? will prevent stock-market crashes in all perpetuity. In the long run, we may be willing to pay the price of market upheavals to benefit from the wealth-generating effect of decentralized markets. By contrast, it is less obvious that the world can afford to run the risk of catastrophic geopolitical events, such as nuclear wars. The only safe way of managing security affairs is to transcend the balance of power into a situation of trust, which is exactly what happened between France and Germany in the last half century. While nuclear calamities would fit nicely into the Richardsonian law, there may be few people around who could enjoy the advances of social science should the ostensibly ?impossible? turn out to be just another huge low-probability event.

Appendix: Detailed model specification

As specified in the text, a variable-size grid constitutes the core of the model. Primitive actors live in each square and can be thought of as the basic units of the framework. Although they can never be destroyed, many of them do expand territorially. This also implies that some them can lose their sovereignty as other actors come to dominate them hierarchically.

All actors, whether primitive or compound, keep track of their geopolitical context with the help of a portfolio holding all of their current relationships. These can be of three types:

0 *territorial relations* point to the four territorial neighbors of each primitive actor (north, south, west, and east).

1 *interstate relations* refer to all sovereign neighbors with which an actor interacts.

2 *hierarchical relations* specify the two-way link between provinces and capitals.

The behavior of the geopolitical base model depends crucially on these three types of relationship. Whereas all strategic interaction is located at the political level, territorial relations become important as soon as structural change happens. Combat takes place locally and results in hierarchical changes that will be described below.

As in my previous models, the logic of execution is quasi-synchronous with double buffering. As opposed to picking one actors at a time and giving it all the ?limelight?, the process follows stages in which each actor executes code (cf. Cusack and Stoll 1990). To achieve quasi-parallel execution the list of actors is scrambled each time structural change occurs. The actors keep a memory of one step and thus make up an advanced Markov process (though with an almost infinite number of states and without fixed transition probabilities).

In addition to a model-setup stage, the main simulation loop contains five phases that will be presented in the following. In the first phase, the actors determine their resource levels. Then they allocate resources to their fronts followed by a decision procedure, during which they decide on whether to cooperate or defect in their neighbor relations. The interaction phase determines the winner of each battle, if any. Finally, the structural change procedure carries out conquest and other border-changing transformations.

The reader is referred to Table A1 at the end of this appendix for a summary of all parameters and their values.

Table A1. Parameters and variables in the geopolitical model.

		Description
System Parameters		
nx, ny	dimensions of the grid	30 x 30 or 45 x 45
propHegemon	share of units with 10 resource units as opposed to 0 in initial grid	0.05
propMobile	share of mobile resources to be allocated as opposed to fixed ones	0.85
sup.t,sup.c	superiority criterion (logistical parameters t and c)	2.2, 50
vic.t,vic.c	victory criterion (logistical parameters t, c)	2.2, 10
pStalemate	probability of undecided battle per round	0.2
offset	flat ?tax rate? for long distances	0.2
d.t,d.c	location of threshold for distance function	3.0, 10

	Actor
res	resource level
	Relation
res(i,j)	relational resources of actor i with actor j
act(i,j)	action {C, D}
agent(i,j)	agent province in state i carrying out attack against j
target(i,j)	target province in state j absorbing attack by state i
claim(i,j)	conquest claim made by state i against state j

Model Setup

At the very beginning of each simulation, the square grid is created and populated with the primitive units. In order to trigger geopolitical competition, a random selection amounting to propHegemon = 5 per cent of the states are then given a 10 resource units, whereas all others receive nothing. This creates an uneven world in which a small minority of the states are going to be aspiring hegemons in the initial stage.

Resource updating

As the first step in the actual simulation loop, the resource levels are updated. The simple ?metabolism? of the system depends directly on the size of the territory controlled by each capital. It is assumed that all sites in the grid are worth ten resource units. A sovereign actor i begins the simulation loop by extracting resources from all of its provinces. It accumulates a share of these resources determined by a distance-dependent logistical function f:

```
f(d) = offset + (1−offset)/{1+(d/d.t)^d.c}
```

where 0? offset ? 1.0 sets the flat extraction rate for long distances. In all runs reported on in this paper, it is fixed at 0.2. The location of the threshold is determined by d.t = 3, and the slope by d.c = 10. Thus, the extraction rate is 100% for small distances, but because of technological constraints, it falls rapidly to a the flat part of the function beyond the threshold. The rationale for the low value of d.t relates to the desire to generate reasonably stable, multipolar systems for the first stage. Moreover, if we are going to be able to detect any conflict clusters at all, the offset rate cannot be too large or the system gets engulfed immediately in huge conflict because conquest becomes too easy. Furthermore, the tribute gained from each province is discounted for in proportion to the province's distance dist(i,j) from the capital. Finally, the resources of the actor res(i) can be computed by adding the non-discounted resources of the capital together with the sum of all tribute:

```
tribute = 0
for all provinces j of state i do
  tribute = tribute + 10 * f(dist(i,j)).

res(i) = 10 + tribute
```

Resource allocation

Before the states can make any behavioral decisions, resources must be allocated to each front. For unitary states, there are up to four fronts, each one corresponding to a territorial relation (those located in the corners of the grid have a few as two neighbors). Resource allocation proceeds according to a hybrid logic. A preset share of each actor's resources are considered to be fixed and have to be evenly spread to all external fronts. Yet, this scheme lacks realism because it underestimates the strength of large actors, at least to the extent that they are capable of shifting their resources around to wherever they are needed. The remaining part of the resources, here `propMobile` = 0.85, are therefore mobilized in proportion to the opponent's strength and the previous activity on respective front. The current model features a simplified version of Proportional Resource Allocation. Mobile resources are proportionally allocated to fronts that are active (i.e. where combat occurs will occur), but also for deterrent purposes in anticipation of a new attack. Resources are always allocated under the assumption that no more than one new attack might happen.

For example, a state with 50 mobile units could use them in the following way assuming that the five neighboring states control 10, 15, 20, 25, and 30 respectively. If the previous period featured warfare with the second and fourth of these neighbors, these two fronts would be allocated 15/(15+25) ? 50 = 18.75 and 25 / (15+25) ? 50 = 31.25. Under the assumption that one more war could start, the first, third, and fifth states would be allocated respectively: 10/(15+25+10) ? 50 = 10, 20/(15+25+10) ? 50 = 20, and 30/(15+25+10) ? 50 = 30.

Formally, resource allocation for state `i` starts with the computation of the fixed resources for each relationship `j`. A preset proportion of the total resources `res` are evenly spread out across the `n` fronts:

```
fixedRes(i,j) = (1-propMobile) * res / n
```

The remaining part `mobileRes = propMobile * res` is allocated in proportion to the activity and the strength of the opponents. To do this, it is necessary to calculate all resources that were targeted at actor i:

```
enemyRes(i) = ?{j}{res(j,i)}
```

The algorithm of actor `i`?s allocation can thus be summarized:

```
for all relations j do
  in case enemyRes(i) = 0 then [actor not under attack]
    res(i,j) = fixedRes(i,j)+mobileRes
  in case i and j were fighting in the last period then
res(i,j) = fixedRes(i,j) +
           r(j,i)/enemyRes(i)*mobileRes
  in case i and j were not fighting the last period then
    res(i,j) = fixedRes(i,j)+
               r(j,i)/(enemyRes(i)+r(j,i))*mobileRes
```

Decisions

Once each sovereign actor has allocated resources to its external fronts, it is ready to make decisions about future actions. This is done by recording the front-dependent decisions in the corresponding relational list. As with resource allocation, this happens in parallel (or to be more precise in randomized order) without any reference to other actors' decisions during the current time step.

All actors behave very much like the ?predatorial? states in my previous modeling framework. This means that they all start by playing unforgiving ?tit-for-tat? with all their neighbors, a strategy that resembles the game-theoretic strategy ?grim trigger? in conventional iterated games. The predatory aspect of the strategy relates to the second step featuring an unprovoked attack provided that no combat is already under way. First a potential victim `j'` is selected. Ultimately, the decision to attack depends on a probabilistic criterion to which we will return below. If an attack is approved, the aggressor chooses a ?battle path? consisting of an agent and a target province. The target province is any primitive actor inside `j'` (including the capital) that borders on `i`. The agent province is a province inside state `i` (including the capital) that borders on the target.

In summary, the decision algorithm of a power-seeking state `i` can be expressed in pseudo-code:

Decision rule of state i:

```
for all external fronts j do
  if i or j  played D in the previous period then
    act(i,j) = D
  else
    act(i,j) = C  [Grim Trigger]

if there is no action on any front then
  randomly select a neighbor state j'
  with  p(i,j') do
    change to act(i,j') = D [launch attack against j']
    randomly select target(i,j') and agent(i,j')
```

The precise criterion for attacks `p(i,j')` remains to be specified. The current version of model relies on a stochastic function of logistic type. The power balance plays the main role in this calculation:

`bal(i,j') = res(i,j')/res(j',i).`

Hence, the probability can be computed as:

`p(i,j') = 1/{1+(bal(i,j')/sup.t)^sup.c}`

where `sup.t` is a system parameter specifying the threshold that has to be transgressed for the probability of an attack to reach 0.5, and `sup.c` a tunable parameter that determines that slope of the logistic curve. A zero `sup.c` makes the curve completely flat and the decision insensitive to the power balance. High values, by contrast, turn the curve into a sharp step at `sup.t`. The sharper step-shaped curve (`sup.t` = 2.2, `sup.c` = 50) corresponds to the geopolitical decision criterion used in all simulations reported in this paper. Intuitively speaking, the likelihood of an attack rises steeply as soon as state becomes twice as powerful as its potential victim on any front. The second, somewhat less slanted curve determines battle performance, which is the topic of the next sub-section (see Figure 3 above).

Interaction

After the decision phase, the system executes all actions and determines the consequences in terms of the local power balance. The current version of the model introduces a simplified combat mechanism that assumes away damage for a more elaborate version that includes battle costs. The only thing that matters is how powerful both sides are in any particular two-way battle. Combat has consequences only when both sides are playing `D`.

As shown in Figure 3, the outcome of combat is determined probabilistically. If the updated local resource balance `bal(i,j')` (including damage) tips far enough in favor of either side, victory becomes more likely for that party. In the initial phase, the logistical probability function `q(i,j')` has the same shape as the decision criterion with the same threshold set at `vic.t` = 2.2 but with a less steep slope: `vic.c` = 10:

`q(i,j') = 1/{1+(bal(i,j')/vic.t)^vic.c}`

These parameter values were chosen such that combat is less than fully predictable and not always immediate. Because the ?victory? curve is still no more than 0.5 at the decision threshold, more than one time step might be needed before victory is attained. Yet, provided that the attacking actor doesn't absorb hits on other fronts, the aggressor is prone to win the battle in due course.

Theoretically, each time-step of a battle can generate one of three outcomes: it may remain undecided, one or both sides could claim victory. In the first case, combat is discontinued with a stalemate with a probability `pStalemate` = 0.2. Otherwise it continues in the next round because of the grim-trigger strategy of the decision phase. If there is a claim, however, it is recorded and processed in the structural change phase, to which we now turn.

Structural change

The module carrying out structural change contains the most formidable algorithmic challenge. Structural change is defined as any change of the actors boundaries. This version of the framework features conquest as the only structural transformation, but other extensions of the modeling framework include secession and voluntary unification. Combat happens locally rather than at the country level, as in Cusack and Stoll (1990).