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Multiscaling and skew separation in regional floods

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Abstract. Matalas et al. (1975) (MSW) found that the simulated values of flood peaks in a region using various common forms of flood frequency distributions did not reproduce the empirical skew statistics for 14 different regions covering the conterminous United States. Specifically, the field data always demonstrated a higher value of standard deviation of skew for a given sample value of mean skew than did the simulations. MSW termed this difference "separation" and further showed that it could not be explained either by autocorrelation of flood peaks or as a small sample property. In this paper, we discuss an explanation of this property using the recently developing scaling theories of regional floods. It is shown that in a homogeneous region, recently defined by us, separation would result from the multiscaling structure of flood peaks. Separation would not occur if floods obey simple scaling, nor would separation necessarily occur with heterogeneity or mixing among different homogeneous regions. Mixing must be of a particular kind in order to cause separation. The use of normalized flood frequencies having mean of zero and variance of 1 in the simulations carried out by MSW is shown to be consistent with the assumption of index flood or simple scaling but not multiscaling. In the 14 "megaregions" analyzed by MSW, mixing among subregions within each megaregion may add to the magnitude of separation. The separation in 14 regions is physically interpreted based on different physical mechanisms that have been recently hypothesized by us to be responsible for the presence of simple scaling or multiscaling in floods.

1. Introduction

There has been a continuing interest in regional flood frequency analysis for over 30 years [*Dalrymple*, 1960; *Dawdy*, 1961; *Benson*, 1962]. Two broad lines of research are responsible for this interest. The first has been devoted to an understanding of quantitative relationships between landforms and peak flows or floods, because floods are viewed as one of the main geomorphic agents that shape drainage networks and landscapes [*Gupta and Waymire*, 1989; *Leopold et al.*, 1964]. The second broad line of research has been motivated by the flood insurance program, its need for consistency in the methods of analysis, and its establishment of standards for flood frequency analysis [*Hydrology Subcommittee*, 1982]. Another source of interest has been in the optimization of information content concerning regional hydrology derived from the samples of flood peaks collected at surface water streamgaging stations. Some of these issues are discussed in a report of the *National Research Council (NRC)* [1988].

Within the last few years, a new theoretical framework is beginning to be developed which is aimed at understanding the statistical structure of regional floods in terms of their physical generating mechanisms. This framework for regional flood frequency is based on the contemporary ideas of scaling invariance [*Gupta et al.*, 1994; *Smith*, 1992]. Specifically, one of the key issues is to understand how the scaling invariance in floods is related to that in precipitation (both rainfall and snowmelt) and to the three-dimensional (3-D) geometry of river networks [*Gupta and Waymire*, 1995]. For example, *Gupta and Dawdy* [1994, 1995] have published preliminary findings with respect to the physical basis of scaling invariance in floods in terms of physical processes responsible for generating floods. Since this new theory is in its infancy, it is very important to understand the extent to which it can unify flood data, as well as the physics and the statistical structure of runoff and floods. This premise would serve as a basis for the further development of this theory.

The past approaches to flood frequency can be grouped into three broad categories. The first is the "quantile regression approach" of the U.S. Geological Survey (USGS) [*Benson*, 1962]. It has been used extensively by the USGS since the mid-1960s. In this approach, each flood quantile (i.e., flows with specified probability of exceedance) is regressed against multiple basin characteristics, such as drainage area, mean basin slope, etc., using multiple regression analysis. Although various basin characteristics are used in the regional relations, there is no physical foundation for their inclusion or exclusion, and they are treated purely as statistical variables. Consequently, this line of investigation has remained data intensive and essentially statistical in nature.

Development of the second set of approaches to regional flood frequency has been based mostly on the index flood assumption. The main emphasis has been on the development of statistically robust regional estimators of flood distributions [*Vogel and Fennessey*, 1993] and the delineation of homoge-

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neous regions, or those regions in which the index flood assumption holds. It implies that the coefficient of variation (CV) of annual floods is a constant. It is also widely recognized that these homogeneous regions are not simply geographical areas and that it may be difficult to portray them effectively on maps [*NRC*, 1988, p. 38]. Extensive data provided by the USGS exhibit that the index flood assumption does not hold widely; see the discussion by *Gupta et al.* [1994] and the data analysis by *Gupta and Dawdy* [1995]. Moreover, this approach does not address the basic issue of how to delineate homogeneous regions physically and understand the statistics of regional floods in terms of flood hydrology.

The third set of approaches to flood frequency combine a basin response function with the rainfall input of a given frequency. In this respect, they can be called "physically based" [*Eagleson*, 1972; *Sivapalan et al.*, 1990]. The existing body of work on this topic has focused mostly on deriving flood frequency distribution at a fixed site rather than on their regional behavior. Therefore connections and consistency between physically based approaches and regional approaches, either empirical or statistical, for the most part have remained unexplored; see *Gupta and Dawdy* [1994] for a further discussion of this issue. This brief overview illustrates a real need to develop a unified theoretical framework that includes data, the hydrology of the system, and the statistics of the system. This issue furnishes the context within which the physical basis of simple scaling and multiscaling in floods, and its implications for the regional structure of skewness separation are examined in this paper.

Three papers in the early discussions of the statistical basis for regional flood frequency analysis raised several important points. The first of these was a study of the moments of sample statistics for several standard flood frequency distributions that might be used for the analysis of flood data [*Wallis et al.*, 1974]. Wallis et al. showed that sample estimates of skewness approached an upper bound as a function of the sample size. This means that no matter how large the population skew, the sample values of skew are bounded, and the bound is determined by the sample size. The second of the three papers gave a mathematical explanation for the upper limit on sample skew estimates [*Kirby*, 1974]. The third paper [*Matalas et al.*, 1975] (hereinafter referred to as MSW), the significance of which appears to have been lost in the literature, applied the findings of the first two papers to the analysis of field data. That paper drew two major conclusions. The first was that the estimates of mean and standard deviation of regional skew depend on the sample size. The second was that simulated values of peaks using various common forms of flood frequency distribution could not reproduce the empirical skew statistics for 14 different "mega" regions covering the conterminous United States. Specifically, the field data always demonstrated a higher value of standard deviation of skew for a given sample value of mean skew than did the simulations. MSW termed this difference "separation" and further showed that it could not be explained by autocorrelation of flood peaks and was not a small sample property. They mentioned but did not explore two other possible causes of separation, namely, spatial mixing of values of skewness g among subregions within each megaregion, and mixing of values of g in time [MSW, 1975, p. 818].

MSW chose as regions for analysis the 14 parts into which the USGS divides the United States for publication of data. Each of these regions shown in Figure 1 comprises a major basin, such as the Missouri or Colombia. In its regional flood frequency analysis, the USGS analyzes data on a state by state basis. It divides states into several regions, each of which is considered "homogeneous." The issue of what is meant by homogeneity is controversial [*Gupta et al.*, 1994]. However, the definition of homogeneity given by *Gupta et al.* [1994] refers to physical geographic regions, and for many regions it agrees with the delineation of homogeneous regions by the USGS. Therefore, each of the 14 regions analyzed by MSW combines many of the homogeneous regions into a megaregion. This combination may cause separation by spatial mixing of values of skewness g, as was mentioned by MSW. We demonstrate that not all mixing causes separation but rather that the mixing must be of a particular kind. Moreover, there is an underlying physical cause that creates skewness separation in otherwise homogeneous regions defined by *Gupta et al.* [1994]. Therefore by looking at the structure of flood peaks in terms of the scaling statistics and their physical generating mechanisms, the separation observed by MSW can be interpreted physically in a qualitative manner. Our main focus in this paper is to explain this issue and discuss its implications for the further development of a physically based theory of regional flood frequency.

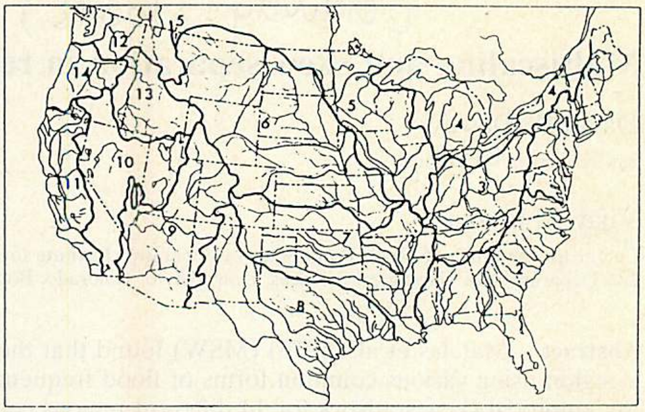

Figure 1. Regional hydrologic division of the United States [from *Matalas et al.*, 1975].

2. Brief Review of MSW's Findings Regarding Skew Separation

MSW showed the results of 100,000 simulations of various lengths assuming several common statistical distributions: normal, Gumbel, three-parameter lognormal (3PLN), Pearson III (not log Pearson III), Weibull, Pareto, and uniform. The sample values of skew and their standard deviations were compared with field data. These results are shown in their Figures 2–4, of which Figure 4 for record length of 30 years is reproduced here as Figure 2. The skewness separation can be clearly seen in Figure 2, since the simulated values of the average skew versus its standard deviation fall below the sample statistics from field data.

Several of the candidate flood frequency distributions could be rejected a priori from consideration for use in flood frequency analysis. The normal and the uniform distributions have an expected skew of zero. The Gumbel has a fixed population skew. Therefore they should not be expected to reproduce the variation of the field data. In addition, the simulated Pareto distribution had a small range of skew values, the sizes of which were beyond the range of the field data for most of

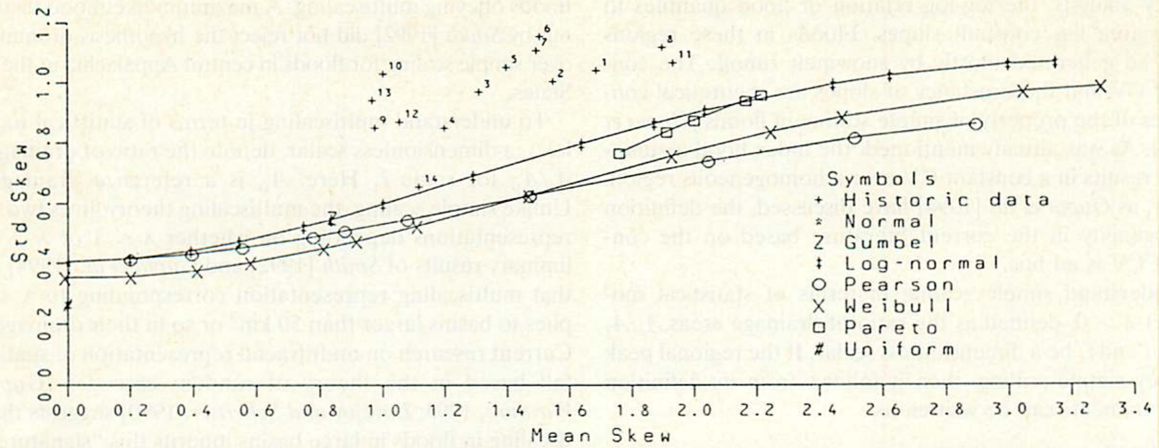

Figure 2. Mean skew versus standard deviation of skew for $n = 30$, historic and simulated data. [from *Matalas et al.*, 1975]. The numbers indicate the regions.

the 14 regions. This left three candidate distributions, namely, Pearson III, Weibull, and 3PLN. Of those three potential candidate distributions, 3PLN comes closest to the data, insofar as it gives a greater skew standard deviation for a fixed mean compared with the other two. Therefore it is instructive to investigate the assumptions of MSW in their use of the 3PLN.

Let X be lognormal with mean $\mu_x = 0$, variance $\sigma_x^2 = 1$, and skewness g; let Y be normal with mean μ_y and variance σ_y^2; and let a be the third parameter added as a lower bound to the values of X, i.e.,

$$X = a + \exp(Y) \tag{1}$$

The three equations relating the three unknowns, μ_y, σ_y^2, and g, are given by equations (8)–(10) of MSW. It can be seen from their equation (10) that once a population skew g is chosen, it determines σ_y^2. Equation (9) of MSW shows that σ_y^2 determines the mean μ_y, and these two together determine the additive term a. Thus for any population skew chosen, the distribution of the normalized random variable X with mean of zero and variance of 1 can be determined from (1). MSW dealt only with normalized random variables in their simulations of regional floods. They found, as is shown in Figure 2, that for a given mean skew the empirical skew standard deviation was always systematically larger than the simulated standard deviation. As we have already discussed, this separation could not be explained either by autocorrelation of flood peaks or from the small sample property, although spatial mixing of values of skewness g may partially explain their results. Their basic assumption that a normalized random variable can describe floods at different gauges within a region implies that these variates do not depend on their spatial location within a region, however defined. This issue is very important to keep in mind in order to understand the consequences of two different scaling assumptions on skew separation in floods, which will now be discussed.

3. Simple and Multiscaling Theories and Skew Separation

Gupta et al. [1994] have argued that a key step in the formulation of a physically based spatial statistical theory of annual floods is to view a channel network as a natural indexing parameter set for the random field of flows. Invariance of the probability distributions of peak flows under translation on this indexing set defines statistical homogeneity. It suggests that floods in a homogeneous region can be indexed by network magnitude, or equivalently the drainage area, and nothing else. *Gupta and Dawdy* [1995] have illustrated that the widely used quantile regression method of the USGS provides one simple criterion to approximately designate geographic regions which are hydrologically homogeneous. Those empirically derived USGS homogeneous regions that do not meet the criterion of homogeneity defined by *Gupta et al.* [1994] because of the use of other variables in addition to drainage area will require a general look at the issue of flood heterogeneity.

Let the random variables representing the peak floods from k basins of drainage areas A_i, $i = 1, 2, \cdots, k$ within a homogeneous region be denoted by $Q(A_i)$, $i = 1, 2, \cdots, k$. Let $E[Q^r(A_i)]$, $r = 1, 2, \cdots$ denote the rth expectation or the statistical moment. Then the mean μ_i, the standard deviation σ_i, and the coefficient of skew, g_i, $i = 1, 2, \cdots, k$, can be expressed as

$$\mu_i = E[Q(A_i)] \tag{2}$$

$$\sigma_i = \{E[Q^2(A_i)] - E^2[Q(A_i)]\}^{1/2} \tag{3}$$

$$g_i = \{E[Q^3(A_i)] - 3\mu_i E[Q^2(A_i)] + 2\mu_i^3\}/\sigma_i^3 \tag{4}$$

Given the k values of skew within a homogeneous region, one can define the regional mean skew $m(g)$ and the regional standard deviation of skew, $s(g)$, as

$$m(g) = \frac{1}{k} \sum_{i=1}^{k} g_i \tag{5}$$

$$s(g) = \frac{1}{k} \sum_{i=1}^{k} g_i^2 - m^2(g) \tag{6}$$

We will now examine the implications of the assumptions of simple sealing and multiscaling of peak floods on the statistics of regional skew defined in (5) and (6). It has been shown that simple scaling leads to a constant coefficient of variation of floods over a homogeneous region [*Smith*, 1992; *Gupta et al.*, 1994]. Further, *Gupta and Dawdy* [1995] have found that for some of the regions used by the USGS in their regional flood frequency analysis, the log-log relation of flood quantiles to drainage area has constant slopes. Floods in these regions seem to be generated mostly by snowmelt runoff. The constancy of CV and the constancy of slopes are theoretical consequences of the property of simple scaling in floods [*Gupta et al.*, 1994]. As was already mentioned, the index flood assumption also results in a constant CV over a homogeneous region. However, as *Gupta et al.* [1994] have discussed, the definition of homogeneity in the current literature based on the constancy of CV is ad hoc.

To understand simple scaling in terms of statistical moments, let $\lambda > 0$, defined as the ratio of drainage areas A_i/A_j for some i and j, be a dimensionless scalar. If the regional peak flows obey simple scaling, then it follows from its definition that the moments can be written as

$$E[Q^r(A_i)] = (A_i/A_j)^{r\theta} E[Q^r(A_j)] \qquad r = 1, 2, \cdots \tag{7}$$

where the parameter θ can be either positive or negative and is known as the scaling exponent [*Gupta and Waymire*, 1990, p. 2001]. Equation (7) says that given the moments of floods with respect to a reference basin with area A_j, the moments for any other basin of area A_i can be computed from (7) simply by knowing the exponent θ. In further discussion we take the reference basin to be A_0, and let $E[Q^r(A_0)] = m_r$ in (7) to write it as

$$E[Q^r(A_i)] = m_r(A_i/A_0)^{r\theta} \qquad r = 1, 2, \cdots \tag{8}$$

The reader can check by substituting (8) into (4) that

$$g_i = g \qquad i = 1, 2, \cdots \tag{9}$$

because the dependence of $Q(A_i)$ on the scale parameter A_i drops out. Another way to view this result is that if $Q(A_i)$ obeying simple scaling are "normalized" by subtracting the mean μ_i and dividing by σ_i, then the normalized variables Z_i have a common probability distribution, Z, i.e.,

$$\frac{Q(A_i) - \mu_i}{\sigma_i} \overset{d}{=} Z_i \overset{d}{=} Z \qquad i = 1, 2, \cdots \tag{10}$$

where $\overset{d}{=}$ denotes the equality in probability distributions. It is easy to see from (10) that normalized floods in a homogeneous region obeying simple scaling exhibit a common skewness. This property is similar, but not identical to the index flood assumption widely used in the flood frequency literature [*NRC*, 1988; *Gupta et al.*, 1994]. In view of this property, it follows simply from (5) and (6) that

$$m(g) = g \qquad s(g) = 0 \tag{11}$$

Equation (10) shows that the regional skewness is the same as the station skewness and therefore does not exhibit skewness separation. MSW considered only normalized floods and assumed that (10) holds. The reader should note that the expressions regarding skew statistics in (11) follow from simple scaling and do not depend on any specific probability distributional assumptions about the flood peaks.

Gupta and Dawdy [1995] have observed that the log-log relations of flood quantiles to drainage areas do not have constant slopes for many regions used in the USGS regional flood frequency analyses. In fact, in most cases the slopes decrease as the return period increases. Floods in these regions seem to be generated mostly by rainfall. *Gupta et al.* [1994] have shown that this property of quantiles is exhibited by floods obeying multiscaling. A maximum likelihood test carried out by *Smith* [1992] did not reject the hypothesis of multiscaling over simple scaling for floods in central Appalachia in the United States.

To understand multiscaling in terms of statistical moments, let λ, a dimensionless scalar, denote the ratio of drainage areas A_i/A_0 for some i. Here, A_0 is a reference drainage area. Unlike simple scaling, the multiscaling theory gives two distinct representations depending on whether $\lambda < 1$ or $\lambda > 1$. Preliminary results of *Smith* [1992] and *Gupta et al.* [1994] suggest that multiscaling representation corresponding to $\lambda < 1$ applies to basins larger than 50 km^2 or so in their drainage areas. Current research on multifractal representation of spatial rainfall based in the theory of random cascades [*Gupta and Waymire*, 1993; *Lovejoy and Schertzer*, 1990] suggests that multiscaling in floods in large basins inherits this "signature" from the spatial variability of rainfall. *Gupta et al.* [1994] have suggested that the scaling structure of floods in small basins is determined by basin response rather than precipitation input. Their results from a simple rainfall-runoff simulation experiment show that the scaling in floods in small basins may be exhibited by the representation corresponding to $\lambda > 1$, but this issue needs to be investigated further.

In order to fix ideas, we will take $\lambda < 1$, even though this constraint does not affect the results given below. This condition requires that the reference area A_0 is larger than all drainage areas in a homogeneous region. Under the assumption of multiscaling of floods within a homogeneous region, it follows from its definition that the moments can be written as

$$E[Q^r(A_i)] = (A_i/A_0)^{\theta(r)} E[Q^r(A_0)] \tag{12}$$

$$r = 1, 2, \cdots \qquad i = 1, 2, \cdots, k$$

Here, the scaling exponent function is non-linear and concave in r, i.e.,

$$\theta(r) \neq r\theta, \frac{d^2\theta(r)}{dr^2} < 0 \qquad r = 1, 2, \cdots \tag{13}$$

For example, $\theta(r) = a - br^2$, if $Q(A)$ is lognormal [*Gupta et al.*, 1994; *Smith*, 1992]. This feature of multiscaling for the broad class of log-Levy models of which the lognormal is a special case is explained by *Gupta and Waymire* [1990]. This nonlinearity in the exponents of the drainage area distinguishes it from simple scaling. As we will now explain, this property leads to skew separation in a homogeneous hydrologic region.

First note that if we apply (10) to normalize the flood peaks $Q(A_i)$, $i = 1, 2, \cdots, k$, in a region obeying multiscaling given by (12), the variables Z_i do not become independent of their scale parameters A_i. In other words, the probability distributions of the normalized peak flows still depend on their drainage areas and therefore are not identically distributed. Consequently, any two basins with different drainage areas in a homogeneous region have different skews, i.e.,

$$g_i \neq g_j \qquad A_i \neq A_j \tag{14}$$

Typically, in a homogeneous region, such as those identified in the USGS regional flood frequency reports, the drainage areas of gaged streams cover a broad range of variation. Therefore, skew separation directly follows from (12), because for a given mean skew $m(g)$ one obtains

$$s(g) > 0 \tag{15}$$

This result in (15) depends only on the multiscaling assumption and is independent of the specific distributional assumptions on flood peaks. The reader should note that (15) will still be true even when the population of basins in a region consists of both small and large basins, as typically is the case.

We have already discussed in section 2 that in the theoretical simulations carried out by MSW, different normalized flood frequency models were assumed to describe regional floods. It was assumed that each of these models applies to all the gaging stations within a region after normalization. This is tantamount to assuming either simple scaling or the index flood method. This assumption implies that theoretically, $s(g) = 0$, regardless of what specific flood frequency model is chosen. In their simulations, MSW fixed the theoretical skew to match the empirical mean skew $m(g)$ in a region and noted that the simulated skew standard deviation could not reproduce the empirically computed standard deviation of skew values for any of the 14 regions. The latter were found to be systematically higher than the simulated values and could not be explained by any of the seven models simulated by MSW, which included the 3PLN. Clearly, if floods obey multiscaling in a region, it then follows from (13) that the skew standard deviation will always be higher than if they obey simple scaling, thereby exhibiting separation. This result is independent of specific model assumptions about flood frequency distributions. Since each of the 14 regions analyzed by MSW combines many of the homogeneous regions into a megaregion, this combination may cause separation by spatial mixing of values of skewness g, as was mentioned by MSW. Let us now examine this issue of mixing among regions.

4. Mixing, Heterogeneity and Separation

Mixing implies heterogeneity. Combining two homogeneous regions creates mixing through their difference. Therefore what constitutes a homogeneous region? We will now explain homogeneity [*Gupta et al.*, 1994, p. 3409] in terms of the scaling assumptions discussed above. A homogeneous region has, for all gaging locations within the region, flood peaks that follow probability distributions which are rescaled versions of one another, based only on drainage area and nothing else. Rescaling can obey either simple scaling or multiscaling in a homogeneous region.

Consider two homogeneous regions, 1 and 2, for which flood peaks in each obey simple scaling given by (8). The exponents are θ_1 and θ_2, and the moments of floods with respect to the reference drainage basin A_0 are m_r^1 and m_r^2, $r = 1, 2, \cdots$, respectively. Now assume that the moments are related by

$$m_r^2 = c^r m_r^1 \tag{16}$$

where $c > 0$ is some constant. It now follows from (10) that these two regions have the same CV and skewness g. Therefore mixing of these two regions would not create separation. It may also be noted that regions 1 and 2 have the same CV, but the flood peak probability distributions are not the rescaled versions of each other with respect to drainage area, and therefore are not homogeneous. This point illustrates that the constancy of CV as a definition of homogeneity is ad hoc and not useful, as was emphasized by *Gupta et al.* [1994]. A similar argument can be made for multiscaling, where mixing of homogeneous regions will not cause separation but multiscaling will. The condition given in (16) implies that the quantiles of the floods for the reference drainage area are constant multiples of each other by a factor c. We discuss below that four USGS regions in western Washington State exhibit this condition: they are simple scaling, nonhomogeneous, or a mixture, but they do not exhibit separation.

5. Physical Interpretations of Skew Separation Observed by MSW

The regions analyzed by the USGS in its statewide regional flood frequency reports are much smaller than the 14 regions of the United States analyzed by MSW. In the USGS reports, each state is typically divided into 6–10 regions. On the basis of our theoretical discussion given above, one expects that the regions in a state in which floods exhibit simple scaling would more closely approach the relation of mean simulated skew to standard deviation of simulated skew developed by MSW. By contrast, regions exhibiting multiscaling would exhibit a higher skew standard deviation for the same mean skew than that given by regions with simple scaling. In addition, the distribution of flood peaks exhibiting either simple scaling or multiscaling need not be the same from one region to another. When several such regions are combined, as by MSW, the pooled data would tend to have an even greater skew standard deviation than data from individual regions because of spatial mixing.

Gupta and Dawdy [1995] hypothesized that simple scaling as opposed to multiscaling results from differences in the physical mechanism for generating floods. Regions controlled primarily by snowmelt peaks in the spring might tend to exhibit simple scaling. Flood peaks in such a region are a function of average basin snowpack for the year and maximum incoming radiation, and they tend to be similar over the region. Similarity in statistical variability over a region is what simple scaling exhibits. On the other hand, in regions where most peaks are caused by convection-dominated rainfall, such as summer thunderstorms and other frontal storms, floods tend to exhibit multiscaling.

It is instructive to analyze the separation of the 14 parts from the general tendency of the 3PLN line (double daggers) in Figure 2 in terms of the physical generating mechanisms of floods discussed above. Region 14 contains the Pacific slope basins in Oregon and the lower Columbia River Basin. The desert regions of eastern Oregon flow into the Snake River or the Great Basin and are not included in region 14. Region 14 has a Mediterranean climate with high rainfall during some 6–8 months of the year. Almost all of the high peaks are caused by frontal systems from off the Pacific Coast, and all basins are subject to similar systems. Therefore there is some degree of uniformity of flood hydrology. The hydrologic consequence of that seems to be exhibited in Figure 2, as the data most nearly approach the 3PLN line. The plot of these data is supported by the results of the USGS regional flood frequency report for Washington State [*Jennings et al.*, 1994]. Regions I to IV, which comprise the western third of the state, all have drainage area exponents for all quantiles of either 0.85 or 0.86, which shows a simple scaling structure for all the regions [*Gupta and Dawdy*, 1995]. Similarly, western Oregon has uniform exponents of 0.88 in the Willamette and 0.86 to 0.90 in the Rogue-Umpqua basins. The coast region has exponents varying from 0.92 to 0.96. The almost constant exponents throughout part 14 would lead to the lack of separation shown by the MSW analysis. Thus uniform frontal systems appear to produce simple scaling. The physical basis for this remains unclear and should be investigated further.

Even more interesting, MSW region 14 exhibits the fact that mixing of regions need not necessarily produce separation. USGS regions I to IV in western Washington have flood quantile discharges which differ from region to region by a constant factor of up to 3 times. Their mixing is of the kind described in section 4. Thus the combined USGS regions are not homogeneous in that they produce floods of considerably different magnitudes for the same drainage area. Yet when combined with other similar homogeneous regions, they do not produce separation. Thus mixing may be a cause of some of the separation in MSW, but region 14 demonstrates that mixing need not cause separation. The mixing must be of a particular kind in order to cause separation.

The regions with greatest deviation from 3PLN in Figure 2 are 7, 8, and 10. Region 7 is the lower Mississippi River Basin, which includes the Red River and the Canadian River. Thus the western drainage of region 7 has less than 2.5 cm of runoff per year and is combined with parts of Louisiana and Arkansas with much greater annual runoff. Thus region 7 should be expected to have variability among basins in their flood distributions, and this is exhibited in Figure 2. Region 8 drains most of Texas, including the Pecos River and most of west Texas plus the Rio Grande. The upper Rio Grande is snowmelt driven, but most of region 8 drains semi-arid and arid regions. That variability also is exhibited in Figure 2.

Region 10 drains the Great Basin. Some parts of Utah in region 10 are snowmelt driven. Those basins are combined with most of Nevada and the southern California desert. The overall variability of part 10 is exhibited by the large separation in Figure 2.

Much of the eastern United States is characterized by a humid climate, with rainfall distributed rather evenly throughout the year. Occasional hurricanes pass through and cause major floods locally. In the northern parts, spring snowmelt may cause the annual flood event in many years. However, there is not the extreme variability of rainfall and floods experienced in the arid west. Thus parts 1, 2, 3, and 4, the part of the country east of the Mississippi but including the Ohio River, exhibit similar separation in Figure 2. Their separation is much less than that of 7, 8, or 10, but more than that of 14. They have similar climates and they exhibit similar separation.

If regions for analysis in the MSW simulations coincided with the USGS statewide flood frequency report regions, the amount of separation would decrease, but many of those regions would exhibit separation because they exhibit multiscaling. On the other hand, those regions that exhibit simple scaling in their quantile flood prediction equations would exhibit little or no separation, other than as the result of random scatter of estimation or due to small sample size. Therefore multiscaling provides a physical explanation for separation in a hydrologically homogeneous region defined by *Gupta et al.* [1994] and is independent of any effects of mixing among regions.

6. Conclusions and Recommendations

The first conclusion of MSW is that regional estimates of skew should be conditioned on the length of record. This is an important result, and it follows directly from *Kirby* [1974]. The second conclusion of MSW is that skew separation is "essentially" independent of flood frequency model assumptions. Our results support this conclusion. As we have shown here, the magnitude of skew separation depends on (1) the scaling structure or the generating mechanism within a homogeneous region and (2) heterogeneity (mixing) among regions, if several regions are pooled to form a "megaregion" as was done by MSW and if the mixing is of a particular type. In addition, simple scaling seems to be related to snowmelt regions and to some regions with floods caused by rainfall. This finding augments the results of *Gupta and Dawdy* [1995]. The physical driving mechanism for simple scaling and multiscaling in floods needs further investigation.

It is our view that the 14 regions defined by the USGS are too large to enable one to test the implications of two different scaling assumptions on skew separation. USGS statewide regions defined as homogenous on the basis of field data would be a more valid means for comparison [see *Jennings et al.*, 1994]. Perhaps a study similar to that of MSW could be undertaken to determine how the sample skew varies with length of record, and that variation could be related to the characteristics of simulated records to try to recover the lost information as a result of limited sampling. However, any such simulations must be based upon a proper underlying generating mechanism for flood flows. The choice of generating mechanism can be determined by inspection of the field data, which in turn would suggest whether simulations should be based upon simple scaling or multiscaling. As has already been discussed by *Gupta and Dawdy* [1995], information concerning simple scaling and multiscaling is embedded in the USGS quantile regression equations for many regions used by the USGS. However, because of the use of other variables in addition to drainage area in the quantile analysis, not all the empirically defined USGS homogeneous regions meet the criterion of homogeneity defined by *Gupta et al.* [1994]. A proper resolution of this issue requires a general look at the issue of regional heterogeneity within the scaling framework.

In view of basin-response-dominated scaling structure of floods being suggested for small basins [*Gupta and Dawdy*, 1995], the multiscaling theory in small basins differs from that in large basins and should be developed separately. Similarly, further research is needed on the important topic of computation of scaling exponents of floods in terms of precipitation, and other parameters governing basin response, for instance, e.g., channel network geomorphology, so that small sample information in the estimation of flood statistics can be augmented on the basis of a physical understanding of regional flood frequencies.

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