

Small-world networks

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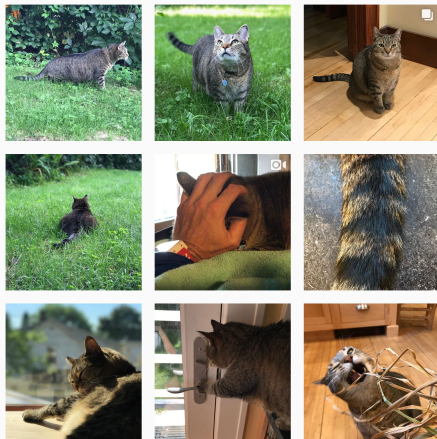
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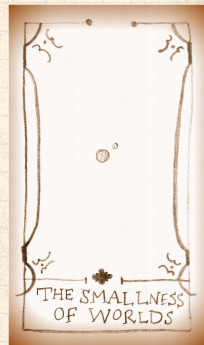
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People thinking about people:

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How are social networks structured?

- How do we define and measure connections?
- Methods/issues of self-report and remote sensing.

What about the dynamics of social networks?

- How do social networks/movements begin & evolve?
- How does collective problem solving work?
- How does information move through social networks?
- Which rules give the best 'game of society?'

Sociotechnical phenomena and algorithms:

- What can people and computers do together? (google)
- Use **Play + Crunch** to solve problems. Which problems?



A small slice of the pie:



Q. Can people pass messages between distant individuals using only their existing social connections?



A. Apparently yes ...



Milgram's social search experiment (1960s)

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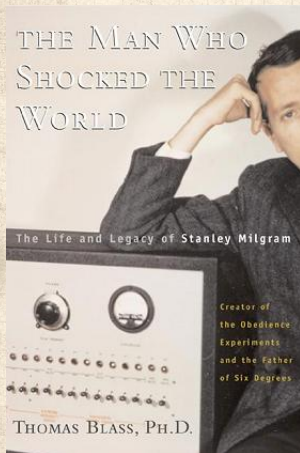
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<http://www.stanleymilgram.com>

- Target person = Boston stockbroker.
- 296 senders from Boston and Omaha.
- 20% of senders reached target.
- chain length $\simeq 6.5$.

Popular terms:

- The Small World Phenomenon;
- “Six Degrees of Separation.”




From Frigyes Karinthy's "Chain-links"  in "Everything is Different", 1929:

'A fascinating game grew out of this discussion. One of us suggested performing the following experiment to prove that the population of the Earth is closer together now than they have ever been before. We should select any person from the 1.5 billion inhabitants of the Earth—anyone, anywhere at all. He bet us that, using no more than five individuals, one of whom is a personal acquaintance, he could contact the selected individual using nothing except the network of personal acquaintances. For example, "Look, you know Mr. X.Y., please ask him to contact his friend Mr. Q.Z., whom he knows, and so forth."




Six Degrees of Kevin Bacon:

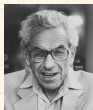


It's a game : "Kevin Bacon is the Center of the Universe"




The Oracle of Bacon 

Six Degrees of Paul Erdős:




Academic papers.




Erdős Number 



Erdős Number Project 



So naturally we must have the Erdős-Bacon Number 




One Story Lab alum has $EB\# < \infty$.



Natalie Hershlag's (Portman's) $EB\# = 5 + 2 = 7$.



The EBS# is also a thing: erdosbaconsabbath.com 

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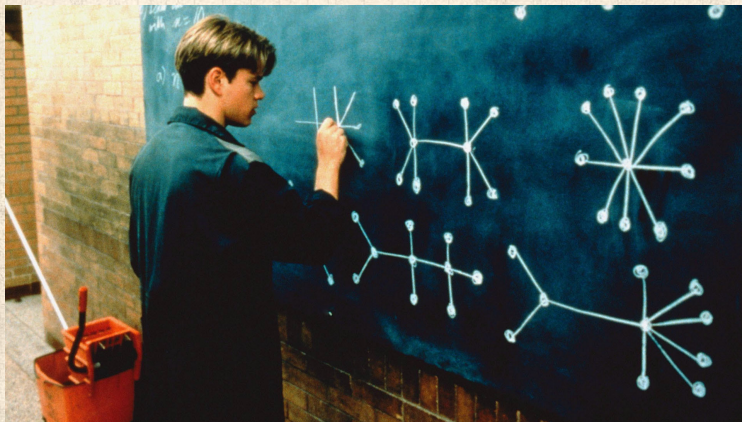
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Good Will Hunting:



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Boardwork by Dan Kleitman , $EB\# = 1 + 2 = 3$.



See Kleitman's sidebar in
Mark Saul's Movie Review 
(Notices of the AMS, Vol. 45, 1998.)



You may already be a winner in NSA's "three-degrees" surveillance sweepstakes!

NSA's probes could cover hundreds of millions of Americans. Thanks, Kevin Bacon.


by Sean Gallagher - July 18 2013, 4:00pm EDT

BIG DATA 109



📷 Aurich Lawson



Many people  are within three degrees from a random person ...

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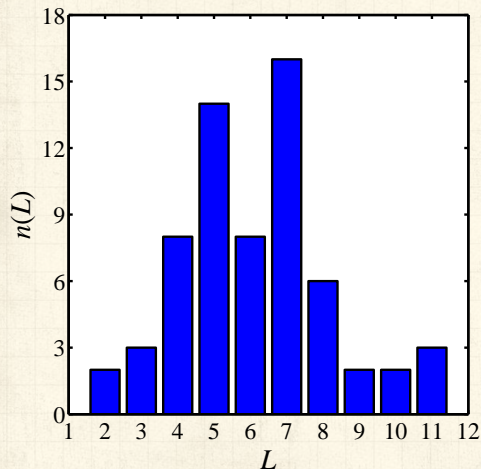


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Lengths of successful chains:



From Travers and
Milgram (1969) in
Sociometry: ^[9]
“An Experimental
Study of the Small
World Problem.”



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Two features characterize a social 'Small World':

1. Short paths exist, (= Geometric piece)
and
2. People are good at finding them. (= Algorithmic piece)



Social Search

Milgram's small world experiment with email:



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"An Experimental study of Search in Global Social Networks" ↗

Dodds, Muhamad, and Watts,
Science, **301**, 827–829, 2003. [4]



Social search—the Columbia experiment

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 60,000+ participants in 166 countries

 18 targets in 13 countries including


 a professor at an Ivy League university,


 an archival inspector in Estonia,

 a technology consultant in India,


 a policeman in Australia,

and

 a veterinarian in the Norwegian army.

 24,000+ chains

We were lucky and contagious (more later):

“Using E-Mail to Count Connections” , Sarah Milstein, New York Times, Circuits Section (December, 2001)



All targets:

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Table S1

Target	City	Country	Occupation	Gender	N	N _c (%)	r (n ₀)	<L>
1	Novosibirsk	Russia	PhD student	F	8234	20(0.24)	64 (76)	4.05
2	New York	USA	Writer	F	6044	31 (0.51)	65 (73)	3.61
3	Bandung	Indonesia	Unemployed	M	8151	0	66 (76)	n/a
4	New York	USA	Journalist	F	5690	44 (0.77)	60 (72)	3.9
5	Ithaca	USA	Professor	M	5855	168 (2.87)	54 (71)	3.84
6	Melbourne	Australia	Travel Consultant	F	5597	20 (0.36)	60 (71)	5.2
7	Bardufoss	Norway	Army veterinarian	M	4343	16 (0.37)	63 (76)	4.25
8	Perth	Australia	Police Officer	M	4485	4 (0.09)	64 (75)	4.5
9	Omaha	USA	Life Insurance Agent	F	4562	2 (0.04)	66 (79)	4.5
10	Welwyn Garden City	UK	Retired	M	6593	1 (0.02)	68 (74)	4
11	Paris	France	Librarian	F	4198	3 (0.07)	65 (75)	5
12	Tallinn	Estonia	Archival Inspector	M	4530	8 (0.18)	63(79)	4
13	Munich	Germany	Journalist	M	4350	32 (0.74)	62 (74)	4.66
14	Split	Croatia	Student	M	6629	0	63 (77)	n/a
15	Gurgaon	India	Technology Consultant	M	4510	12 (0.27)	67 (78)	3.67
16	Managua	Nicaragua	Computer analyst	M	6547	2 (0.03)	68 (78)	5.5
17	Katikati	New Zealand	Potter	M	4091	12 (0.3)	62 (74)	4.33
18	Elderton	USA	Lutheran Pastor	M	4438	9 (0.21)	68 (76)	4.33
Totals					98,847	384 (0.4)	63 (75)	4.05



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Milgram's participation rate was roughly 75%



Email version: Approximately 37% participation rate.



Probability of a chain of length 10 getting through:

$$.37^{10} \simeq 5 \times 10^{-5}$$



\Rightarrow 384 completed chains (1.6% of all chains).



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 Motivation/Incentives/Perception matter.

 If target *seems* reachable
⇒ participation more likely.

 Small changes in attrition rates
⇒ large changes in completion rates

 e.g., ↘ 15% in attrition rate
⇒ ↗ 800% in completion rate



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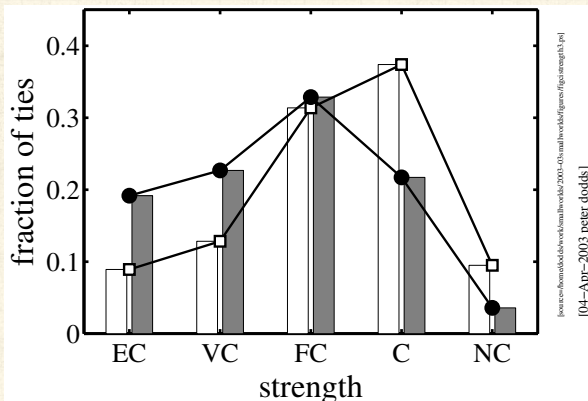
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Comparing successful to unsuccessful chains:



Successful chains used relatively weaker ties:



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Successful chains disproportionately used:

-  Weak ties, Granovetter [5]
-  Professional ties (34% vs. 13%)
-  Ties originating at work/college
-  Target's work (65% vs. 40%)

...and disproportionately avoided

-  hubs (8% vs. 1%) (+ no evidence of funnels)
-  family/friendship ties (60% vs. 83%)

Geography → Work



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Senders of successful messages showed
little absolute dependency on

-  age, gender
-  country of residence
-  income
-  religion
-  relationship to recipient

Range of completion rates for subpopulations:

30% to 40%



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Mildly bad for continuing chain:

choosing recipients because “they have lots of friends” or because they will “likely continue the chain.”

Why:



Specificity important



Successful links used relevant information.

(e.g. connecting to someone who shares same profession as target.)



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Basic results:

-  $\langle L \rangle = 4.05$ for all completed chains
-  L_* = Estimated ‘true’ median chain length (zero attrition)
-  Intra-country chains: $L_* = 5$
-  Inter-country chains: $L_* = 7$
-  All chains: $L_* = 7$
-  Milgram: $L_* \simeq 9$



Usefulness:

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Harnessing social search:

-  Can distributed social search be used for something big/good?
-  What about something evil? (Good idea to check.)
-  What about socio-inspired algorithms for information search? (More later.)
-  For real social search, we have an incentives problem.
-  Which kind of influence mechanisms/algorithms would help propagate search?
-  Fun, money, prestige, ...?
-  Must be 'non-gameable.'



Red balloons:










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A Grand Challenge:

-  1969: The Internet is born 
(the ARPANET —four nodes!).
-  Originally funded by DARPA who created a grand Network Challenge  for the 40th anniversary.
-  Saturday December 5, 2009: DARPA puts 10 red weather balloons up during the day.
-  Each 8 foot diameter balloon is anchored to the ground somewhere in the United States.
-  Challenge: Find the latitude and longitude of each balloon.
-  Prize: **\$40,000**.

*DARPA = Defense Advanced Research Projects Agency .



Where the balloons were:



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Finding red balloons:

The winning team and strategy:

-  MIT's Media Lab  won in less than 9 hours. ^[7]
-  Pickard et al. "Time-Critical Social Mobilization," ^[7] Science Magazine, 2011.
-  People were virally recruited online to help out.
-  Idea: Want people to both (1) find the balloons, and (2) involve more people.
-  Recursive incentive structure with exponentially decaying payout:
 -  \$2000 for correctly reporting the coordinates of a balloon.
 -  \$1000 for recruiting a person who finds a balloon.
 -  \$500 for recruiting a person who recruits the balloon finder, ...
 -  (Not a Ponzi scheme.)
-  True victory: Colbert interviews Riley Crane 




Finding balloons:

Clever scheme:

- Max payout = \$4000 per balloon.
- Individuals have clear incentives to both
 - involve/source more people (spread), and
 - find balloons (goal action).
- Gameable?
- Limit to how much money a set of bad actors can extract.

Extra notes:

- MIT's brand helped greatly.
- MIT group first heard about the competition a few days before. **Ouch.**
- A number of other teams did well .
- Worthwhile looking at these competing strategies. [7]

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


References



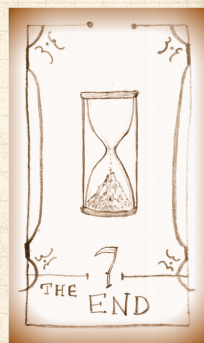
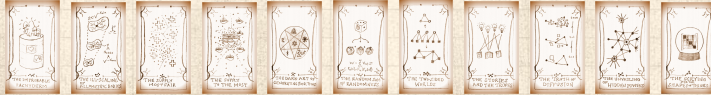
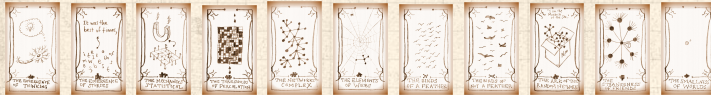
Collective Detective:

Once again, social media proved to be a powerful dragnet. Around 1:15 p.m., a Washingtonian posted a picture on Twitter of Rusty in a patch of weeds in the Adams Morgan district, not far from the 163-acre zoo, which was created in 1889 by an act of Congress. "Red panda in our neighborhood," [wrote Ashley Foughty](#), who identified herself as a singer, actress and traveler. "Please come save him!"

Another neighbor posted [a photograph](#) of two zoo workers, one in safari shorts standing on a rooftop, one holding a giant butterfly net. Soon the zoo announced: "Rusty the red panda has been recovered, crated & is headed safely back to the National Zoo!"

-  Finding an errant panda 
-  Nature News: "Crowdsourcing in manhunts can work: Despite mistakes over the Boston bombers, social media can help to find people quickly"  by Philip Ball (April 26, 2013)
-  Motherboard, Vice: One Degree of Separation in the Forever War  by Brian Castner (November 11, 2015)





The social world appears to be small ...why?

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Theory: how do we understand the small world property?

 Connected **random networks** have short average path lengths:

$$\langle d_{AB} \rangle \sim \log(N)$$

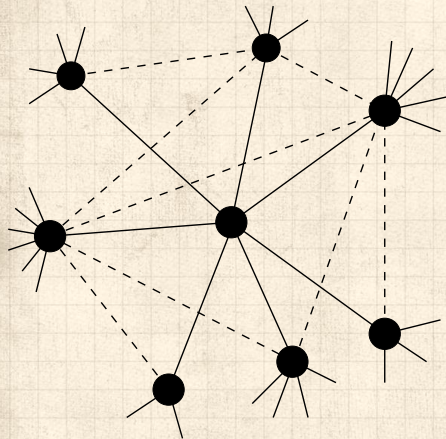
N = population size,

d_{AB} = distance between nodes A and B .

 **But: social networks aren't random ...**



Simple socialness in a network:



Need “clustering” (your friends are likely to know each other):

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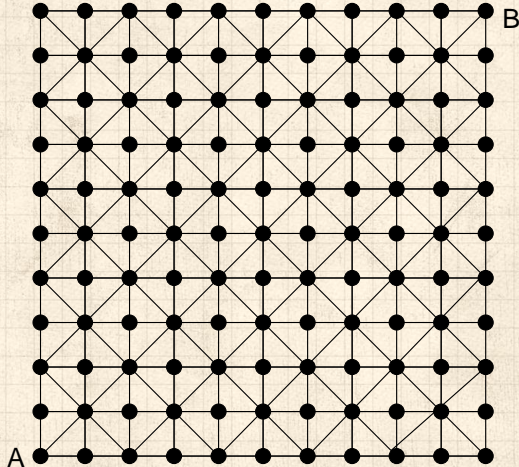
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Non-randomness gives clustering:



$d_{AB} = 10 \rightarrow$ too many long paths.

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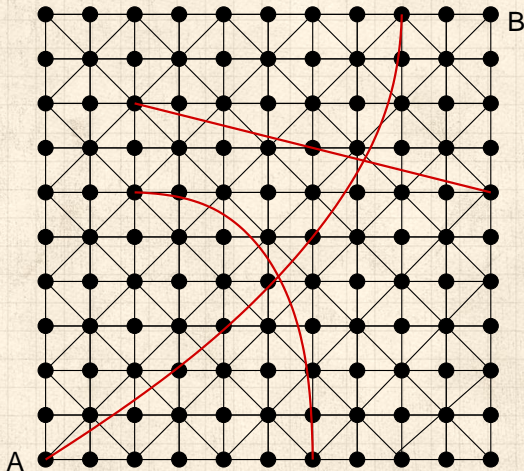
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Randomness + regularity



Now have $d_{AB} = 3$

$\langle d \rangle$ decreases overall

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Introduced by Watts and Strogatz (Nature, 1998) ^[11]

“Collective dynamics of ‘small-world’ networks.”

Small-world networks were found everywhere:

-  neural network of C. elegans,
-  semantic networks of languages,
-  actor collaboration graph,
-  food webs,
-  social networks of comic book characters, ...

Very weak requirements:

-  local regularity + random short cuts



Papers should be apps:

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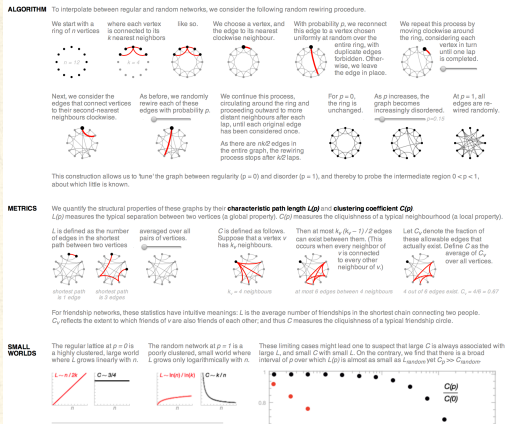
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 Bret Victor's Scientific Communication As Sequential Art 

 Interactive figures and tables = windows into large data sets (empirical or simulated).



The structural small-world property:

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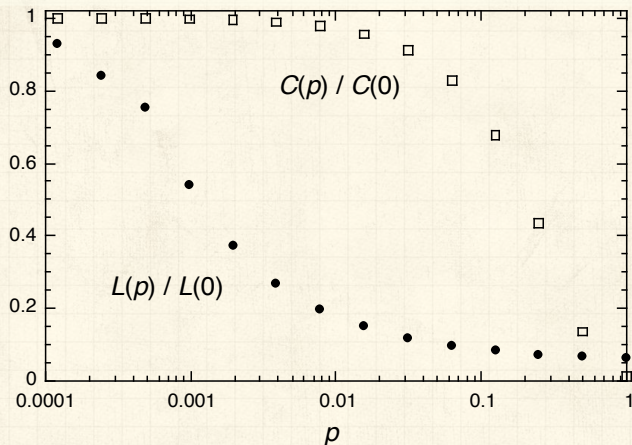
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$L(p)$ = average shortest path length as a function of p



$C(p)$ = average clustering as a function of p



Previous work—finding short paths

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But are these short cuts findable?

Nope. [6]

Nodes **cannot** find each other quickly
with **any** local search method.

Need a more sophisticated model ...



Previous work—finding short paths

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What can a local search method reasonably use?



How to find things without a map?



Need some measure of distance between friends and the target.

Some possible knowledge:



Target's identity



Friends' popularity



Friends' identities



Where message has been



Previous work—finding short paths

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Jon Kleinberg (Nature, 2000) ^[6]
“Navigation in a small world.”

Allowed to vary:

1. local search algorithm
and
2. network structure.



Previous work—finding short paths

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Kleinberg's Network:

1. Start with regular d -dimensional cubic lattice.
2. Add local links so nodes know all nodes within a distance q .
3. Add m short cuts per node.
4. Connect i to j with probability

$$p_{ij} \propto x_{ij}^{-\alpha}.$$



$\alpha = 0$: random connections.



α large: reinforce local connections.



$\alpha = d$: connections grow logarithmically in space.



Previous work—finding short paths

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Theoretical optimal search:



“Greedy” algorithm.



Number of connections grow logarithmically (slowly) in space: $\alpha = d$.



Social golf.

Search time grows slowly with system size (like $\log^2 N$).

But: social networks aren't lattices plus links.



Previous work—finding short paths

- ❏ If networks have hubs can also search well: Adamic et al. (2001)^[1]

$$P(k_i) \propto k_i^{-\gamma}$$

where k = degree of node i (number of friends).

- ❏ Basic idea: get to hubs first (airline networks).

- ❏ But: hubs in social networks are limited.



The problem

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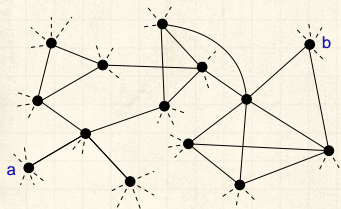
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If there are no hubs and no underlying lattice, how can search be efficient?



Which friend of **a** is closest to the target **b**?





What does 'closest' mean?

What is 'social distance'?



One approach: incorporate **identity**.

Identity is formed from attributes such as:

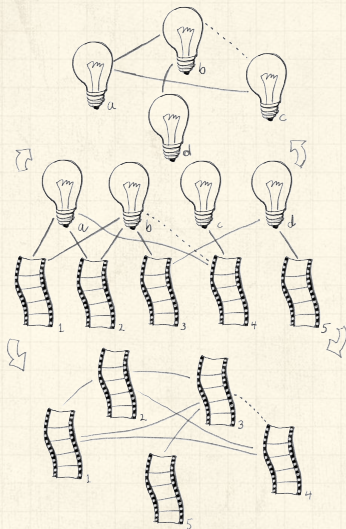
-  Geographic location
-  Type of employment
-  Religious beliefs
-  Recreational activities.

Groups are formed by people with at least one similar attribute.

Attributes \Leftrightarrow Contexts \Leftrightarrow Interactions \Leftrightarrow Networks.



Bipartite affiliation structures:



Many real-world networks have an underlying multi-partite structure.

- Stories-tropes.
- Boards and directors.
- Films-actors-directors.
- Classes-teachers-students.
- Upstairs-downstairs.



Unipartite networks may be induced or co-exist.



Social distance—Context distance

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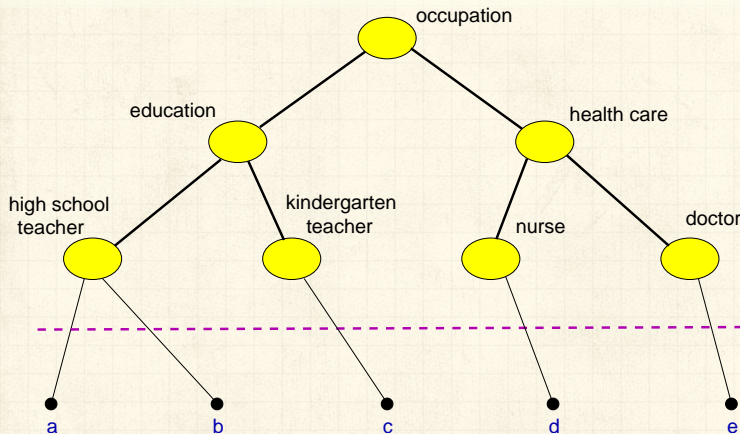
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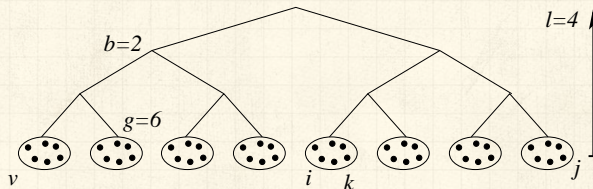
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Distance between two individuals x_{ij} is the height of lowest common ancestor.




$$x_{ij} = 3, x_{ik} = 1, x_{iv} = 4.$$




 Individuals are more likely to know each other the closer they are within a hierarchy.

 Construct z connections for each node using

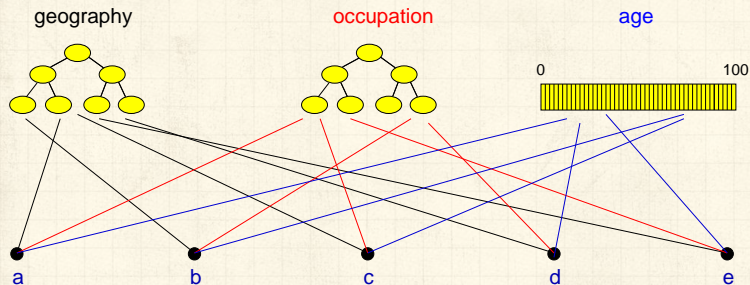
$$p_{ij} = c \exp\{-\alpha x_{ij}\}.$$


 $\alpha = 0$: random connections.

 α large: local connections.



Generalized affiliation networks



 Blau & Schwartz ^[2], Simmel ^[8], Breiger ^[3], Watts *et al.* ^[10]; see also Google+ Circles.



The model

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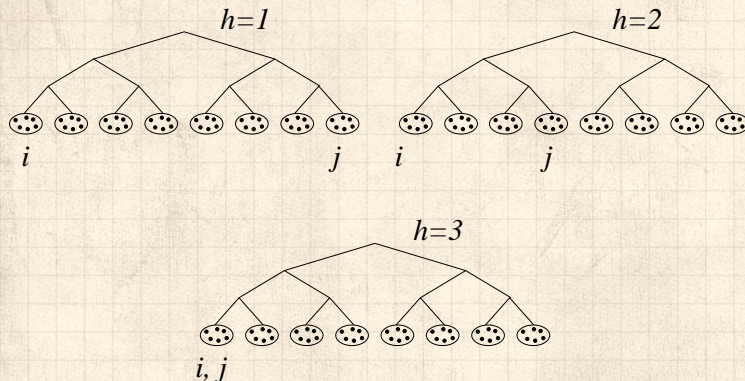
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$$\vec{v}_i = [1 \ 1 \ 1]^T, \vec{v}_j = [8 \ 4 \ 1]^T$$

$$x_{ij}^1 = 4, x_{ij}^2 = 3, x_{ij}^3 = 1.$$

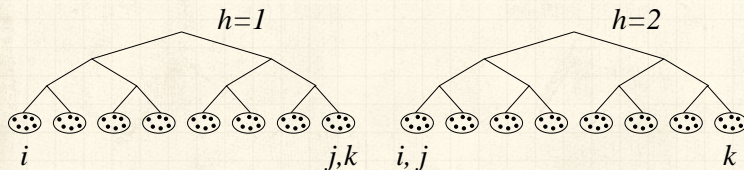
Social distance:

$$y_{ij} = \min_h x_{ij}^h.$$



The model

Triangle inequality doesn't hold:



$$y_{ik} = 4 > y_{ij} + y_{jk} = 1 + 1 = 2.$$



The model

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Individuals know the identity vectors of

1. themselves,
2. their friends,
and
3. the target.



Individuals can estimate the social distance between their friends and the target.



Use a greedy algorithm + allow searches to fail randomly.



The model-results—searchable networks

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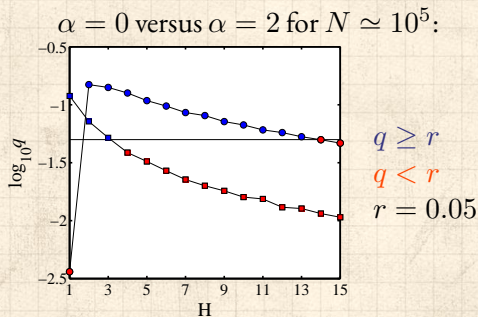
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q = probability an arbitrary message chain reaches a target.



A few dimensions help.



Searchability decreases as population increases.

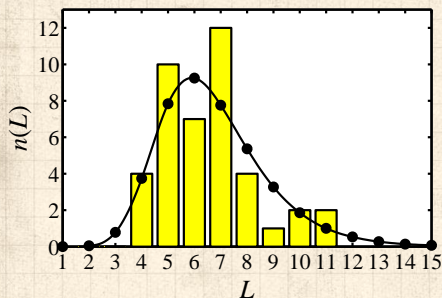


Precise form of hierarchy largely doesn't matter.








The model-results


Milgram's Nebraska-Boston data:



Model parameters:

-  $N = 10^8$,
-  $z = 300, g = 100$,
-  $b = 10$,
-  $\alpha = 1, H = 2$;

 $\langle L_{\text{model}} \rangle \simeq 6.7$

 $L_{\text{data}} \simeq 6.5$



References

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Adamic and Adar (2003)

-  For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.
-  Probability of connection as function of real distance $\propto 1/r$.



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-  Tags create identities for objects
-  Website tagging: bitly.com 
-  (e.g., Wikipedia)
-  Photo tagging: flickr.com 
-  Dynamic creation of metadata plus links between information objects.
-  Folksonomy: collaborative creation of metadata



Social Search—Real world uses







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Recommender systems:

-  Amazon uses people's actions to build effective connections between books.
-  Conflict between 'expert judgments' and tagging of the hoi polloi.



Nutshell for Small-World Networks:

-  Bare networks are typically unsearchable.
-  Paths are findable if nodes understand how network is formed.
-  Importance of identity (interaction contexts).
-  Improved social network models.
-  Construction of peer-to-peer networks.
-  Construction of searchable information databases.



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