

Mechanisms for Generating Power-Law Size Distributions, Part 4

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Principles of Complex Systems,
Vols. 1, 2, 3D, 4 Fourever, V for Vendetta

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University of Vermont | Santa Fe Institute



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The PoCVerse
Power-Law
Mechanisms, Pt. 4
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Optimization

Minimal Cost

Mandelbrot vs. Simon

Assumptions

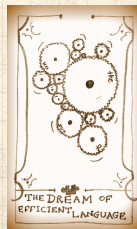
Model

Analysis

And the winner is ...?

Nutshell

References



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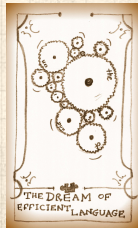
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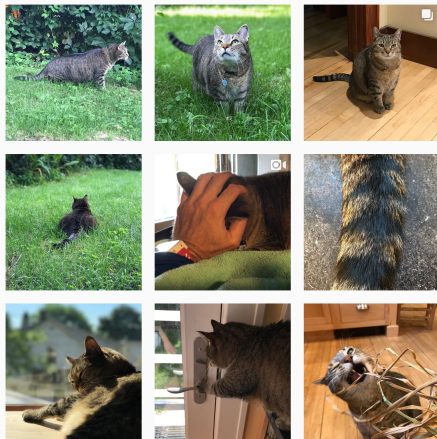
Nutshell



References



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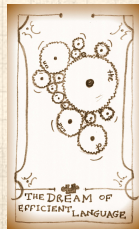
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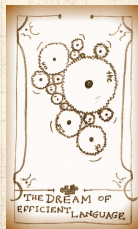
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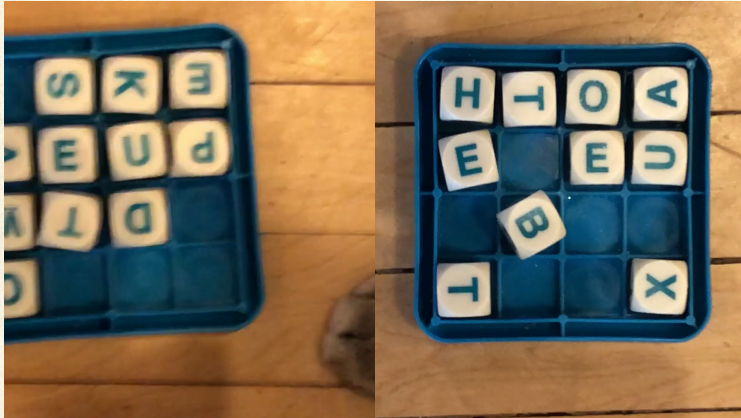
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The Boggoracle Speaks:



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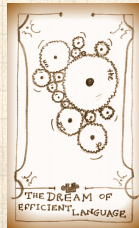
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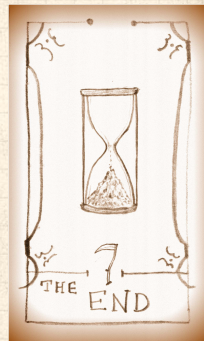
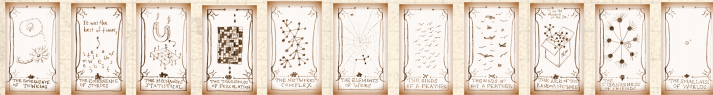
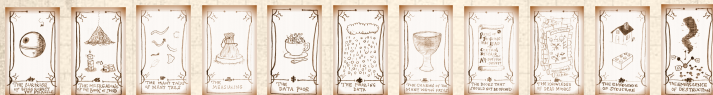
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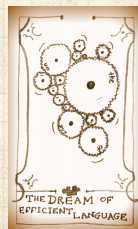
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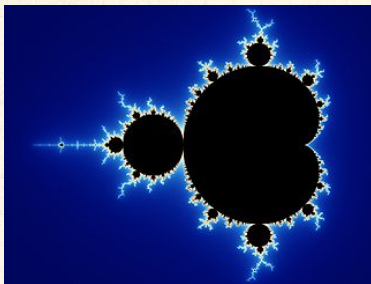
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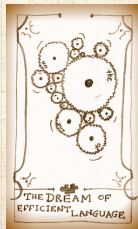
Mandelbrot = father of fractals



Mandelbrot = almond bread



Bonus Mandelbrot set action: [here](#) .



Another approach:

Benoît Mandelbrot



Derived Zipf's law through optimization [8]

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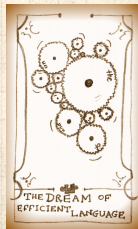
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 **Idea:** Language is efficient

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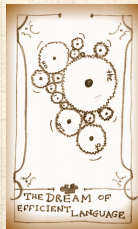
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- Idea:** Language is efficient
- Communicate as much information as possible for as little cost

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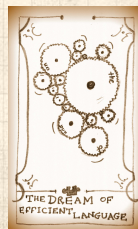
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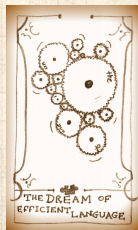
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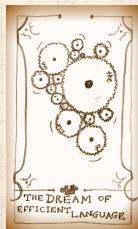
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- Need measures of information (H) and average cost (C) ...



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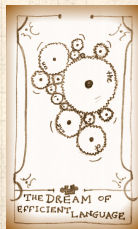
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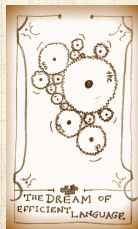
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- Need measures of information (H) and average cost (C) ...
- Language evolves to maximize H/C , the amount of information per average cost.
- Equivalently: minimize C/H .
- Recurring theme:** what role does optimization play in complex systems?



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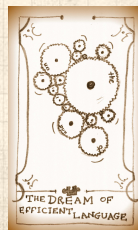
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The Quicken~~ing~~ ↗ —Mandelbrot v. Simon:

There Can Be Only One: ↗



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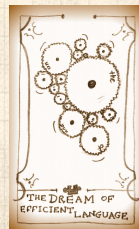
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The Quickenings ↗ —Mandelbrot v. Simon:

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Things there should be only one of: Theory, Highlander Films.

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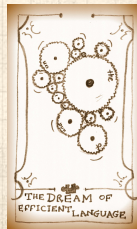
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The Quickening ↗ —Mandelbrot v. Simon:

There Can Be Only One: ↗



Things there should be only one of: Theory, Highlander Films.



Feel free to play Queen's It's a Kind of Magic ↗ in your head (funding remains tight).

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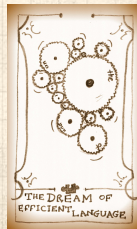
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Now let us “enjoy” the Trailer for Highlander:  



Or: Two theories enter, one theory leaves 

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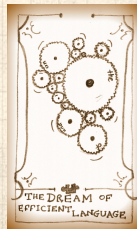
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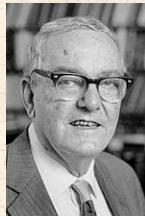
References



We were born to be Princes of the Universe



VS.



Mandelbrot vs. Simon:

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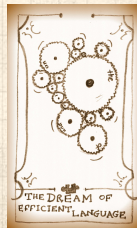
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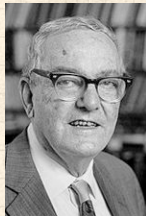
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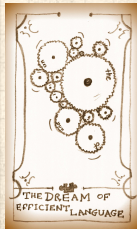
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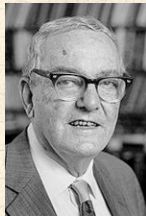
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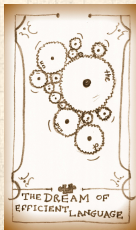
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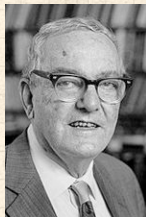
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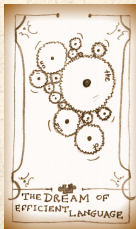


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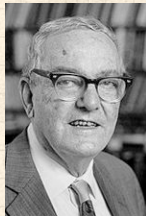
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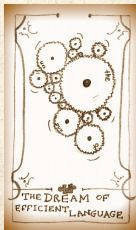


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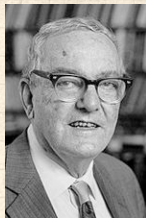
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-  Simon (1960): “Some further notes on a class of skew distribution functions” [15]




I have no rival, No man can be my equal



VS.



Mandelbrot vs. Simon:

 Mandelbrot (1961): “Final note on a class of skew distribution functions: analysis and critique of a model due to H.A. Simon” ^[10]

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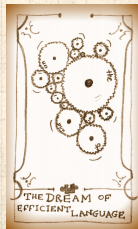
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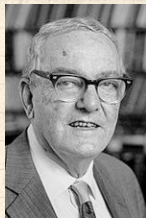
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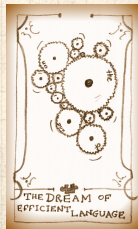
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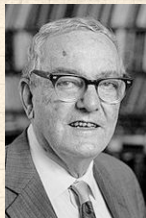
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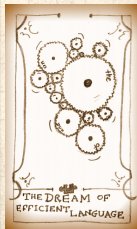
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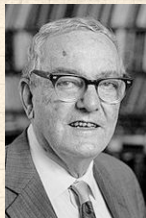
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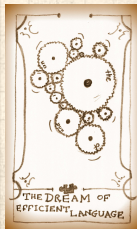
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I am immortal, I have inside me blood of kings

Mandelbrot:

“We shall restate in detail our 1959 objections to Simon’s 1955 model for the Pareto-Yule-Zipf distribution. Our objections are valid quite irrespectively of the sign of $p-1$, so that most of Simon’s (1960) reply was irrelevant.” ^[10]

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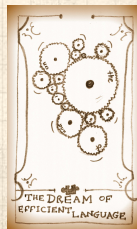
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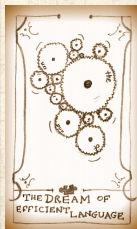
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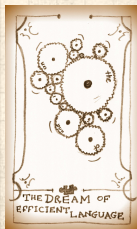
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Plankton:



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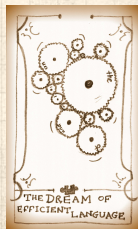
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Plankton:

“You can’t do this to me, **I WENT TO COLLEGE!**”



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Assumptions

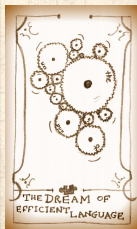
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Analysis

And the winner is ...?

Nutshell

References



I am immortal, I have inside me blood of kings

Mandelbrot:

“We shall restate in detail our 1959 objections to Simon’s 1955 model for the Pareto-Yule-Zipf distribution. Our objections are valid quite irrespectively of the sign of $p-1$, so that most of Simon’s (1960) reply was irrelevant.” [10]

Simon:

“Dr. Mandelbrot has proposed a new set of objections to my 1955 models of the Yule distribution. Like his earlier objections, these are invalid.” [17]

Plankton:



“You can’t do this to me, **I WENT TO COLLEGE!**”
“You weak minded fool!”

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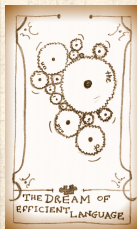
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“You can’t do this to me, **I WENT TO COLLEGE!**”
“You weak minded fool!”
“You just lost your brain privileges,” etc.

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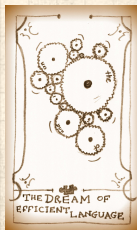
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Dan T. said,

August 5, 2010 @ 10:58 am

And even half a century after publication, those papers are locked up behind a paywall; academic publishing sucks.

[(myl) Weird, isn't it? Simon's 1955 paper is available here, and his first response to Mandelbrot is here, but I haven't been able to find accessible copies of the other episodes. Penn doesn't have an online subscription to Information & Control, and the physical copies of the bound journals are in remote storage. 25 years ago I made xerox copies of the exchange in the Bell Labs library, but lord knows which dusty box or folder those are in. I once took the trouble to get the relevant issues fetched for me at Penn, and made xeroxes, but I can't remember where I put those either. I would have liked to be able to find them today, in order to quote some of the extraordinary examples of formal academic invective in the exchange. As I recall, one of them accuses the other of an undergraduate error in calculus, and the other responds with a counter-accusation of a high-school level error in algebra. Or something like that.

You can get some of the flavor of the exchange from the abstracts of the last three contributions, which are available without paying \$31.50 a shot:





A note on a class of skew distribution functions: Analysis and critique of a paper by H. A. Simon

Author: Benoit Mandelbrot

Publication: Information and Control

Publisher: Elsevier

Date: April 1959

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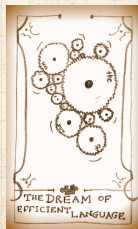
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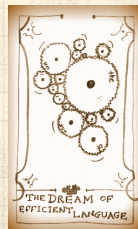
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Zipfarama via Optimization:

Mandelbrot's Assumptions:

 Language contains n words: w_1, w_2, \dots, w_n .

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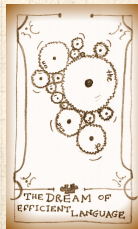
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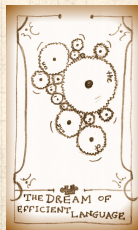
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Mandelbrot's Assumptions:

 Language contains n words: w_1, w_2, \dots, w_n .

 i th word appears with probability p_i



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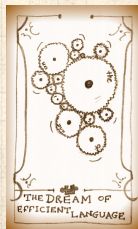
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Nutshell

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Mandelbrot's Assumptions:

-  Language contains n words: w_1, w_2, \dots, w_n .
-  i th word appears with probability p_i
-  Words appear randomly according to this distribution (obviously not true ...)



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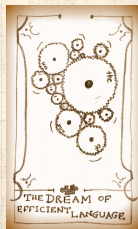
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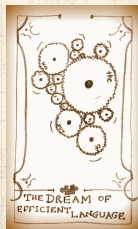
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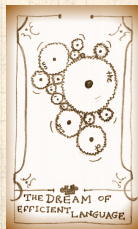
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-  i th word appears with probability p_i
-  Words appear randomly according to this distribution (obviously not true ...)
-  Words = composition of letters is important
-  Alphabet contains m letters
-  Words are ordered by length (shortest first)



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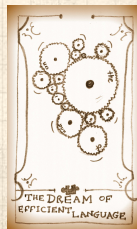
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 Length of word (plus a space)

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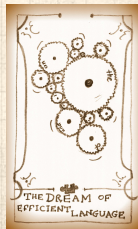
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Zipfarama via Optimization:

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Length of word (plus a space)



Word length was irrelevant for Simon's method

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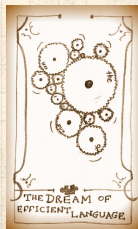
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Objection



Real words don't use all letter sequences

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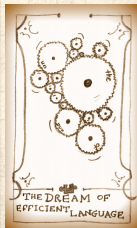
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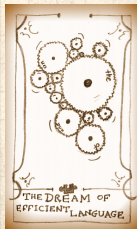
-  Length of word (plus a space)
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Objection

-  Real words don't use all letter sequences

Objections to Objection

-  Maybe real words roughly follow this pattern (?)



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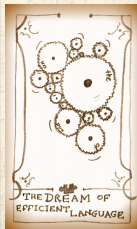
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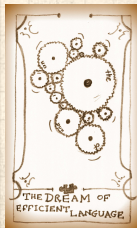
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-  Words can be encoded this way
-  Na na na-na naaaaa ...



Zipfarama via Optimization:

Binary alphabet plus a space symbol

i	1	2	3	4	5	6	7	8
word	1	10	11	100	101	110	111	1000
length	1	2	2	3	3	3	3	4
$1 + \log_2 i$	1	2	2.58	3	3.32	3.58	3.81	4

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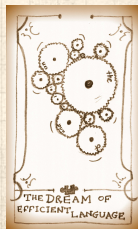
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 Word length of 2^k th word: $= k + 1$

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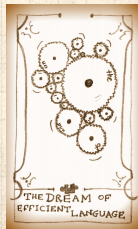
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 Word length of 2^k th word: $= k + 1 = 1 + \log_2 2^k$

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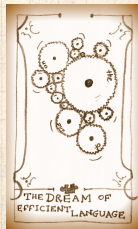
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 Word length of 2^k th word: $= k + 1 = 1 + \log_2 2^k$

 Word length of i th word $\simeq 1 + \log_2 i$

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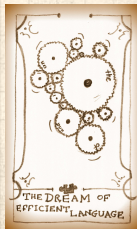
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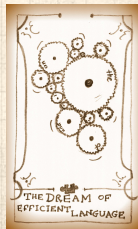
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 Word length of 2^k th word: $= k + 1 = 1 + \log_2 2^k$

 Word length of i th word $\simeq 1 + \log_2 i$

 For an alphabet with m letters,
word length of i th word $\simeq 1 + \log_m i$.



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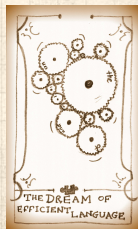
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Zipfarama via Optimization:

Total Cost C

 Cost of the i th word: $C_i \simeq 1 + \log_m i$

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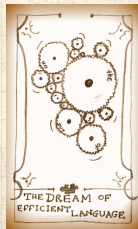
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Zipfarama via Optimization:

Total Cost C

 Cost of the i th word: $C_i \simeq 1 + \log_m i$

 Cost of the i th word plus space: $C_i \simeq 1 + \log_m (i + 1)$

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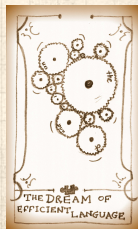
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Cost of the i th word plus space: $C_i \simeq 1 + \log_m (i + 1)$



Subtract fixed cost: $C'_i = C_i - 1 \simeq \log_m (i + 1)$

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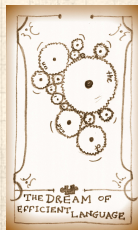
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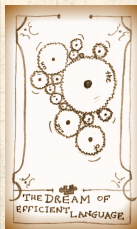


Subtract fixed cost: $C'_i = C_i - 1 \simeq \log_m (i + 1)$



Simplify base of logarithm:

$$C'_i \simeq \log_m (i + 1) = \frac{\log_e (i + 1)}{\log_e m}$$



Zipfarama via Optimization:

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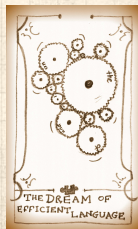


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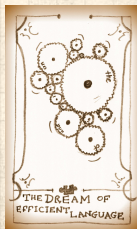
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 Simplify base of logarithm:

$$C'_i \simeq \log_m (i + 1) = \frac{\log_e (i + 1)}{\log_e m} \propto \log_e (i + 1)$$

 Total Cost:

$$C \sim \sum_{i=1}^n p_i C'_i \propto \sum_{i=1}^n p_i \log_e (i + 1)$$



Zipfarama via Optimization:

Information Measure



Use Shannon's Entropy (or Uncertainty):

$$H = \sum_{i=1}^n p_i \frac{1}{\log_2 p_i}$$

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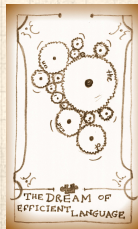
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 Use Shannon's Entropy (or Uncertainty):

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 (allegedly) von Neumann suggested 'entropy' ...

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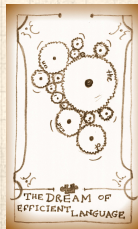
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$$H = \sum_{i=1}^n p_i \frac{1}{\log_2 p_i}$$

 (allegedly) von Neumann suggested 'entropy' ...

 Proportional to average number of bits needed to encode each 'word' based on frequency of occurrence

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Optimization

Minimal Cost

Mandelbrot vs. Simon

Assumptions

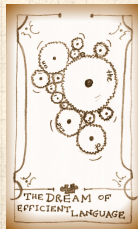
Model

Analysis

And the winner is ...?


Nutshell

References





Zipfarama via Optimization:


Information Measure

 Use Shannon's Entropy (or Uncertainty):

$$H = \sum_{i=1}^n p_i \frac{1}{\log_2 p_i}$$

 (allegedly) von Neumann suggested 'entropy' ...

 Proportional to average number of bits needed to encode each 'word' based on frequency of occurrence

 $-\log_2 p_i = \log_2 1/p_i =$ minimum number of bits needed to distinguish event i from all others

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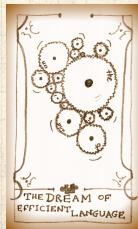
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Zipfarama via Optimization:

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And the winner is ...?


Nutshell


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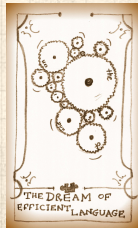
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 If $p_i = 1/2$, **need only 1 bit** ($\log_2 1/p_i = 1$)





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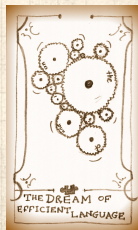
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 $-\log_2 p_i = \log_2 1/p_i$ = minimum number of bits needed to distinguish event i from all others


 If $p_i = 1/2$, **need only 1 bit** ($\log_2 1/p_i = 1$)

 If $p_i = 1/64$, **need 6 bits** ($\log_2 1/p_i = 6$)



Zipfarama via Optimization:

Information Measure

 Use a slightly simpler form:

$$H = - \sum_{i=1}^n p_i \log_e p_i / \log_e 2$$

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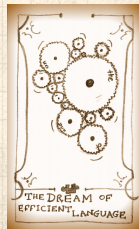
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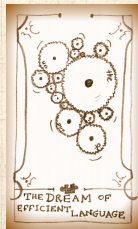
Information Measure



Use a slightly simpler form:

$$H = - \sum_{i=1}^n p_i \log_e p_i / \log_e 2 = -g \sum_{i=1}^n p_i \log_e p_i$$

where $g = 1/\log_e 2$



Zipfarama via Optimization:



Minimize

$$F(p_1, p_2, \dots, p_n) = C/H$$

subject to constraint

$$\sum_{i=1}^n p_i = 1$$

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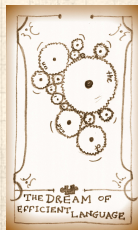
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....
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Minimize

$$F(p_1, p_2, \dots, p_n) = C/H$$

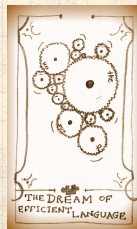
subject to constraint

$$\sum_{i=1}^n p_i = 1$$



Tension:

(1) Shorter words are cheaper



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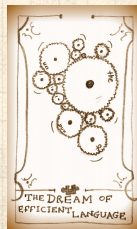
subject to constraint

$$\sum_{i=1}^n p_i = 1$$



Tension:

- (1) Shorter words are cheaper
- (2) Longer words are more informative (rarer)



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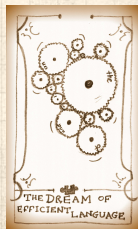
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Zipfarama via Optimization:

Time for Lagrange Multipliers:



Minimize

$$\Psi(p_1, p_2, \dots, p_n) = \\ F(p_1, p_2, \dots, p_n) + \lambda G(p_1, p_2, \dots, p_n)$$

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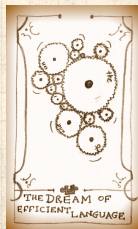
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where

$$F(p_1, p_2, \dots, p_n) = \frac{C}{H} = \frac{\sum_{i=1}^n p_i \log_e (i+1)}{-g \sum_{i=1}^n p_i \log_e p_i}$$

and the constraint function is

$$G(p_1, p_2, \dots, p_n) = \sum_{i=1}^n p_i - 1 (= 0)$$

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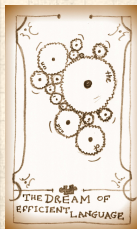
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Nutshell

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Insert assignment question

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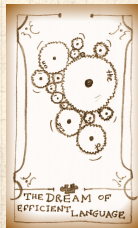
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And the winner is ...?

Nutshell

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Zipfarama via Optimization:

Some mild suffering leads to:



$$p_j = e^{-1-\lambda H^2/gC} (j+1)^{-H/gC}$$

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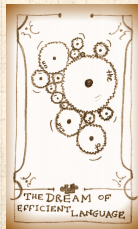
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$$p_j = e^{-1-\lambda H^2/gC} (j+1)^{-H/gC} \propto (j+1)^{-H/gC}$$

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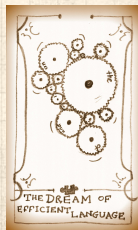
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A power law appears [applause]: $\alpha = H/gC$

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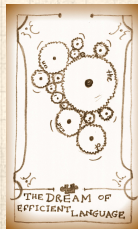
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Next: sneakily deduce λ in terms of g , C , and H .

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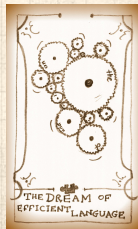
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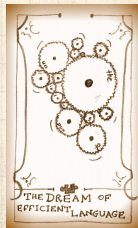


Next: sneakily deduce λ in terms of g , C , and H .



Find

$$p_j = (j+1)^{-H/gC}$$



Zipfarama via Optimization:

Finding the exponent



Now use the normalization constraint:

$$1 = \sum_{j=1}^n p_j$$

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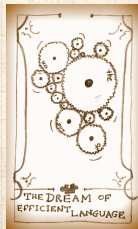
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Nutshell

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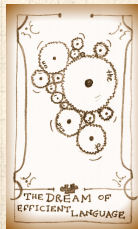
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And the winner is ...?

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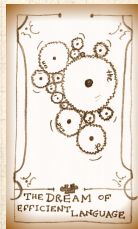
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where ζ is the Riemann Zeta Function

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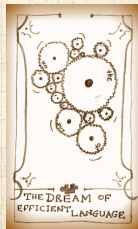
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 Gives $\alpha \simeq 1.73$ (> 1 , too high) or $\gamma = 1 + \frac{1}{\alpha} \simeq 1.58$ (very wild)

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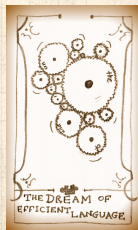
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And the winner is ...?


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Zipfarama via Optimization:


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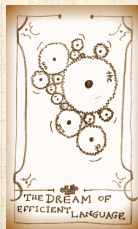
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 If cost function **changes** ($j+1 \rightarrow j+a$) then exponent is tunable





Zipfarama via Optimization:


Finding the exponent


 Now use the normalization constraint:

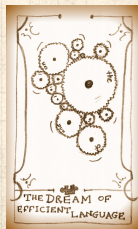
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 Gives $\alpha \simeq 1.73$ (> 1 , too high) or $\gamma = 1 + \frac{1}{\alpha} \simeq 1.58$ (very wild)

 If cost function **changes** ($j+1 \rightarrow j+a$) then exponent is tunable

 Increase a , decrease α



Zipfarama via Optimization:

All told:



Reasonable approach: Optimization is at work in evolutionary processes

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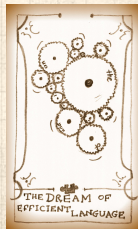
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And the winner is ...?

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Zipfarama via Optimization:

All told:



Reasonable approach: Optimization is at work in evolutionary processes



But optimization can involve many incommensurate elephants: monetary cost, robustness, happiness, ...

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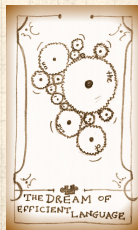
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And the winner is ...?




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References



Zipfarama via Optimization:

All told:

-  Reasonable approach: Optimization is at work in evolutionary processes
-  But optimization can involve many incommensurate elephants: monetary cost, robustness, happiness, ...
-  Mandelbrot's argument is not super convincing

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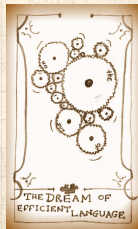
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Zipfarama via Optimization:

All told:

-  Reasonable approach: Optimization is at work in evolutionary processes
-  But optimization can involve many incommensurate elephants: monetary cost, robustness, happiness, ...
-  Mandelbrot's argument is not super convincing
-  Exponent depends too much on a loose definition of cost

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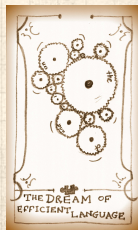
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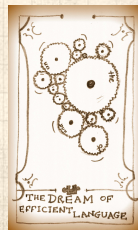
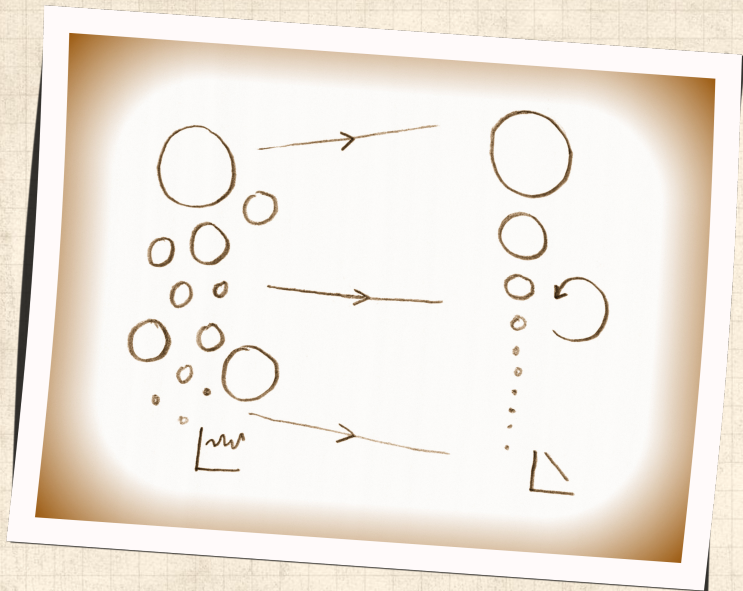
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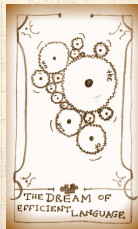
References



From the discussion at the end of Mandelbrot's paper:

 A. S. C. Ross: "M. Mandelbrot states that 'the actual direction of evolution (sc. of language) is, in fact, towards fuller and fuller utilization of places'. We are, in fact, completely without evidence as to the existence of any 'direction of evolution' in language, and it is axiomatic that we shall remain so. Many philologists would deny that a 'direction of evolution' could be theoretically possible; thus I myself take the view that a language develops in what is essentially a purely random manner."

 Mandelbrot: "As to the 'fundamental linguistic units being the least possible differences between pairs of utterances' this is a logical consequence of the fact that two is the least integer greater than one."



More:

Reconciling Mandelbrot and Simon



Mixture of local optimization and randomness

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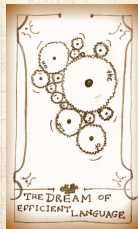
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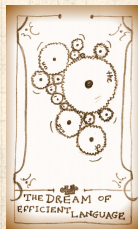


Reconciling Mandelbrot and Simon


 Mixture of local optimization and randomness


 Numerous efforts ...

1. Carlson and Doyle, 1999:
Highly Optimized Tolerance (HOT)—Evolved/Engineered
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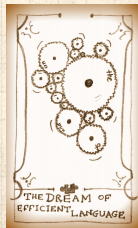


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2. Ferrer i Cancho and Solé, 2002:
Zipf's Principle of Least Effort ^[6]

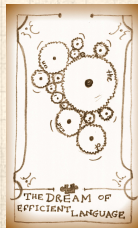


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 Numerous efforts ...

1. Carlson and Doyle, 1999:
Highly Optimized Tolerance (HOT)—Evolved/Engineered
Robustness ^[2, 3]
2. Ferrer i Cancho and Solé, 2002:
Zipf's Principle of Least Effort ^[6]
3. D'Souza et al., 2007:
Scale-free networks ^[4]



More

Other mechanisms:

 Much argument about whether or not monkeys typing could produce Zipf's law ... (Miller, 1957) ^[12]

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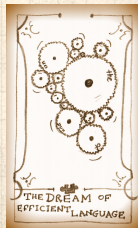
Model

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References



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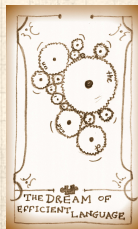
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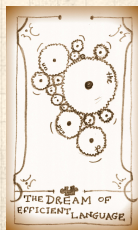
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- ❏ Side note: Miller mentions "Genes of Language."

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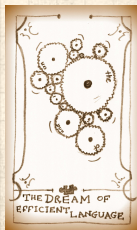
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- ❏ Let us now slap Miller around by simply reading his words out (see next slides):



- ❏ Side note: Miller mentions "Genes of Language."
- ❏ Still fighting: "Random Texts Do Not Exhibit the Real Zipf's Law-Like Rank Distribution" [5] by Ferrer-i-Cancho and Elvevåg, 2010.

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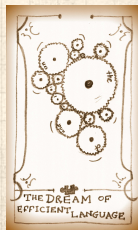
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INTRODUCTION

The Psycho-Biology of Language is not calculated to please every taste. Zipf was the kind of man who would take roses apart to count their petals; if it violates your sense of values to tabulate the different words in a Shakespearean sonnet, this is not a book for you. Zipf took a scientist's view of language — and for him that meant the statistical analysis of language as a biological, psychological, social process. If such analysis repels you, then leave your language alone and avoid George Kingsley Zipf like the plague. You will be much happier reading Mark Twain: "There are liars, damned liars, and statisticians." Or W. H. Auden: "Thou shalt not sit with statisticians nor commit a social science."

However, for those who do not flinch to see beauty murdered in a good cause, Zipf's scientific exertions yielded some wonderfully unexpected results to boggle the mind and tease the imagination. Language *is* — among other things — a biological, psychological, social process; to apply statistics to it merely acknowledges its essential unpredictability, without which it would be useless. But who would have thought that in the very heart of all the freedom language allows us Zipf would find an invariant as solid and reliable as the law of gravitation?

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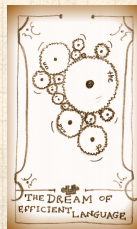
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Put it this way. Suppose that we acquired a dozen monkeys and chained them to typewriters until they had produced some very long and random sequence of characters. Suppose further that we defined a “word” in this monkey-text as any sequence of letters occurring between successive spaces. And suppose finally that we counted the occurrences of these “words” in just the way Zipf and others counted the occurrences of real words in meaningful texts. When we plot our results in the same manner, we will find exactly the same “Zipf curves” for the monkeys as for the human authors. Since we are not likely to argue that the poor monkeys were searching for some equilibrium between uniformity and diversity in expressing their ideas, such explanations seem equally inappropriate for human authors.

A mathematical rationalization for this result has been provided by Benoit Mandelbrot. The crux of it is that if we assume that word-boundary markers (spaces) are scattered randomly through a text, then there will necessarily be more occurrences of short than long words. Add to this fact the further observation that the variety of different words available increases exponentially with their length and the phenomenon Zipf reported becomes inescapable: a few short words will be used an enormous number of times while a vast number of longer words will occur infrequently or not at all.

So Zipf was wrong. His facts were right enough, but not his explanations. In a broader sense he was right, however, for he called attention to a stochastic process that is frequently seen in the social sciences, and by accumulating statistical data that cried out for some better explanation he challenged his colleagues and his successors to explore an important new type of probability distribution. Zipf belongs among those rare but stimulating men whose failures are more profitable than most men’s successes.

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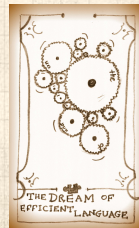
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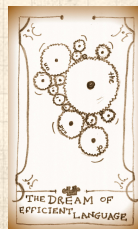
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So who's right?

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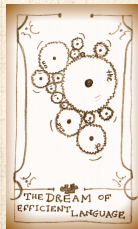
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Show Simon’s model fares well.

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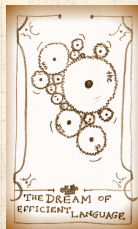
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Show Simon’s model fares well.



Recall ρ = probability new flavor appears.

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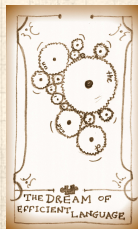
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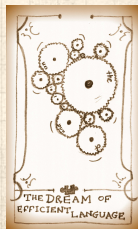
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give $\rho \simeq 0.10$



So who's right?

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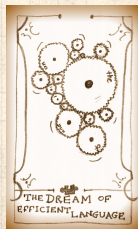
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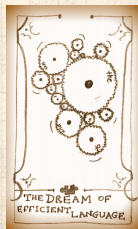
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-  Leads to $\gamma = 1 + \frac{1}{1-\rho} \simeq 2.1$ for in-link distribution.
-  Cite direct measurement of γ at the time: 2.1 ± 0.1 and 2.09
in two studies.



So who's right?

Recent evidence for Zipf's law ...

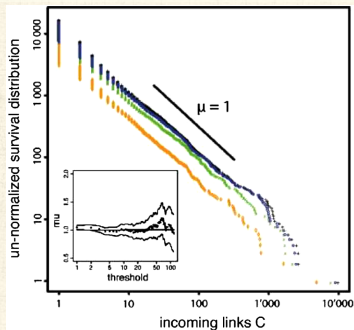


FIG. 1 (color online). (Color Online) Log-log plot of the number of packages in four Debian Linux Distributions with more than C in-directed links. The four Debian Linux Distributions are Woody (19.07.2002) (orange diamonds), Sarge (06.06.2005) (green crosses), Etch (15.08.2007) (blue circles), Lenny (15.12.2007) (black+'s). The inset shows the maximum likelihood estimate (MLE) of the exponent μ together with two boundaries defining its 95% confidence interval (approximately given by $1 \pm 2/\sqrt{n}$, where n is the number of data points using in the MLE), as a function of the lower threshold. The MLE has been modified from the standard Hill estimator to take into account the discreteness of C .

Maillart et al., PRL, 2008:

“Empirical Tests of Zipf's Law Mechanism in Open Source Linux Distribution” [7]

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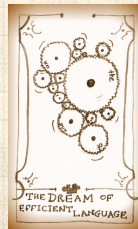
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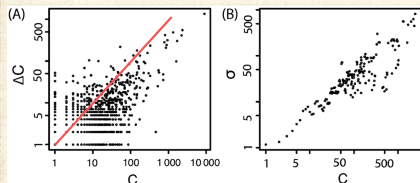


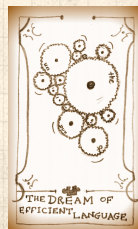
FIG. 2. Left panel: Plots of ΔC versus C from the Etch release (15.08.2007) to the latest Lenny version (05.05.2008) in double logarithmic scale. Only positive values are displayed. The linear regression $\Delta C = R \times C + C_0$ is significant at the 95% confidence level, with a small value $C_0 = 0.3$ at the origin and $R = 0.09$. Right panel: same as left panel for the standard deviation of ΔC .



Red line added.



Rough, approximately linear relationship between C number of in-links and ΔC .¹



¹Breaks down for large C . These tick marks are a human rights violation.

So who's right?

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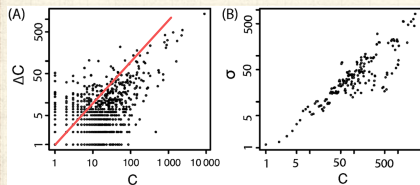


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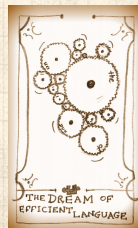
Red line added.



Rough, approximately linear relationship between C number of in-links and ΔC .¹



More complicated mechanism: Packages are deleted, merge, get renamed.



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Nutshell:

Talking points:

- Simonish random ‘rich-get-richer’ models agree in detail with empirical observations.

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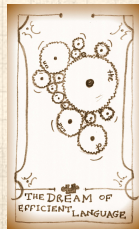
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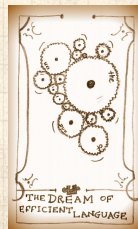
And the winner is ...?

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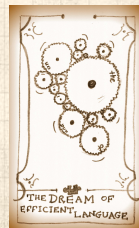
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- Simonish random ‘rich-get-richer’ models agree in detail with empirical observations.
- But it completely fails in the limit it is supposed to work (more later).



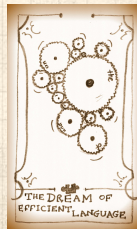
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- Simonish random ‘rich-get-richer’ models agree in detail with empirical observations.
- But it completely fails in the limit it is supposed to work (more later).
- Power-lawfulness**: Mandelbrot’s optimality is still apparent.
- Maybe optimality arises for free in **Random Competitive Replication** rich-get-richer models.



Neural reboot: Walking with a baby robin



Tsubasa.

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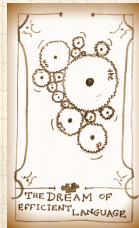
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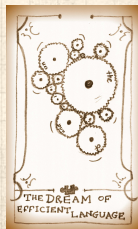
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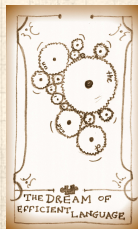
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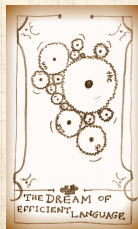
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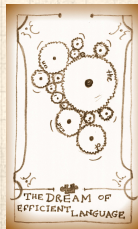
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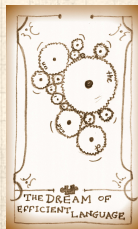
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