

## The Amusing Law of Benford

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## Outline

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## Benford's Law —The Law of First Digits



$$P(\text{first digit} = d) \propto \log_b \left( 1 + \frac{1}{d} \right)$$

for certain sets of ‘naturally’ occurring numbers in base  $b$




Around 30.1% of first digits are ‘1’,  
compared to only 4.6% for ‘9’.



First observed by [Simon Newcomb](#) <sup>[4]</sup> in 1881  
“[Note on the Frequency of Use of the Different Digits in Natural Numbers](#)”



Independently discovered in 1938 by [Frank Benford](#)  <sup>[1]</sup>











Newcomb almost always noted but Benford gets the stamp,  
according to [Stigler's Law of Eponymy](#). 

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## Benford's Law—The Law of First Digits

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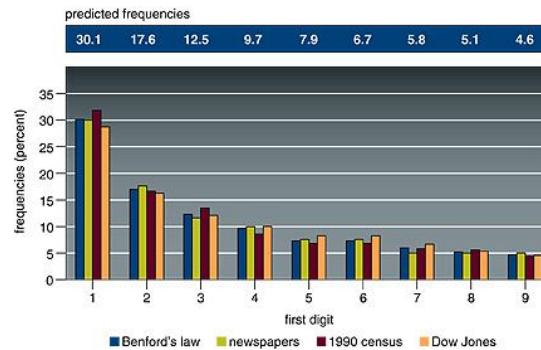
Observed for

-  Fundamental constants (electron mass, charge, etc.)
-  Utility bills
-  Numbers on tax returns (ha!)
-  Death rates
-  Street addresses
-  Numbers in newspapers
-  Cited as [evidence of fraud](#)  in the 2009 Iranian elections.

## Benford's Law—The Law of First Digits

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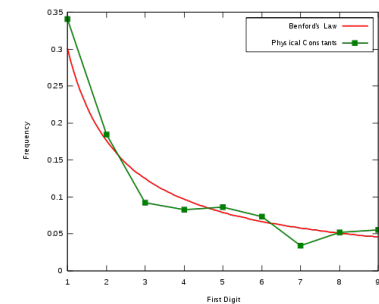
Real data:




From ‘The First-Digit Phenomenon’ by T. P. Hill (1998) <sup>[2]</sup>

## Benford's Law—The Law of First Digits

Physical constants of the universe:



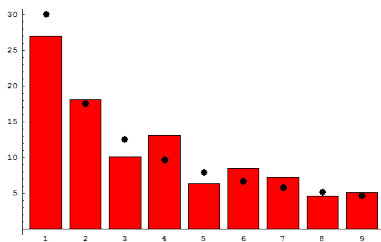
Taken from [here](#) .


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## Benford's Law—The Law of First Digits

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Population of countries:



Taken from [here](#) .

## Essential story



$$\begin{aligned} P(\text{first digit} = d) &\propto \log_b \left( 1 + \frac{1}{d} \right) \\ &= \log_b \left( \frac{d+1}{d} \right) \\ &= \log_b (d+1) - \log_b (d) \end{aligned}$$



Observe this distribution if numbers are distributed  
uniformly in log-space:

$$P(\log_e x) d(\log_e x) \propto 1 \cdot d(\log_e x) = x^{-1} dx = P(x) dx$$



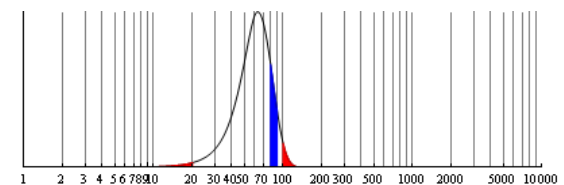
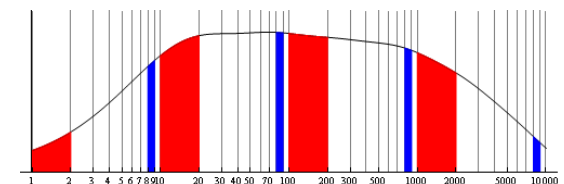
Power law distributions at work again...




Extreme case of  $\gamma \approx 1$ .

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## Benford's law



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“Citations to Articles citing Benford’s law: A Benford analysis”  
Tariq Ahmad Mir,  
Preprint available at  
<https://arxiv.org/abs/1602.01205>, 2016. [3]

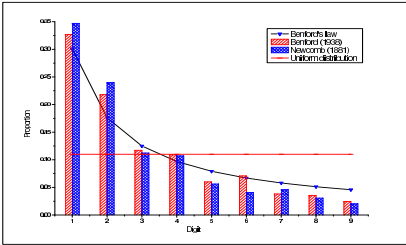


Fig. 1: The observed proportions of first digits of citations received by the articles citing FB and SN on September 30, 2012. For comparison the proportions expected from BL and uniform distributions are also shown.

References II

On counting and logarithms:



Earlier: Listen to Radiolab's  
“Numbers.”  
Now: Benford's Law

References I

[1] F. Benford.  
The law of anomalous numbers.  
[Proceedings of the American Philosophical Society](#),  
78(4):551–572, 1938. pdf

[2] T. P. Hill.  
The first-digit phenomenon.  
[American Scientist](#), 86:358–, 1998.

[3] T. A. Mir.  
Citations to articles citing Benford's law: A Benford analysis,  
2016.  
Preprint available at <https://arxiv.org/abs/1602.01205>. pdf

[4] S. Newcomb.  
Note on the frequency of use of the different digits in natural  
numbers.  
[American Journal of Mathematics](#), 4:39–40, 1881. pdf

