Overview of Complex Networks

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Outline

net•work |ˈnetˌwərk|
noun
1 an arrangement of intersecting horizontal and vertical lines.
- a complex system of roads, railroads, or other transportation routes *: a network of railroads.*

2 a group or system of interconnected people or things *: a trade network.*
- a group of people who exchange information, contacts, and experience for professional or social purposes *: a support network.*
- a group of broadcasting stations that connect for the simultaneous broadcast of a program *: the introduction of a second TV network* | [as adj.] *network television.*
- a number of interconnected computers, machines, or operations *: specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
- a system of connected electrical conductors.

verb [trans.]
connect as or operate with a network *: the stock exchanges have proven to be resourceful in networking these deals.*
- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career *: the skills of networking, bargaining, and negotiation.*

Ancestry:

From Keith Briggs's excellent etymological investigation:

- Opus reticulatum:
- A Latin origin?

[http://serialconsign.com/2007/11/we-put-net-network]

Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

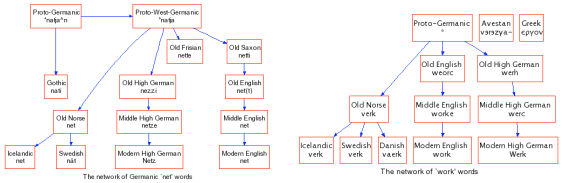
From the OED via Briggs:

- 1658–: reticulate structures in animals
- 1839–: rivers and canals
- 1869–: railways
- 1883–: distribution network of electrical cables
- 1914–: wireless broadcasting networks

Ancestry:

Net and Work are venerable old words:

- 'Net' first used to mean spider web (King Ælfréd, 888).
- 'Work' appear to have long meant purposeful action.



- 'Network' = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

Key Observation:

- Many complex systems can be viewed as complex networks of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.
- Mindboggling amount of work published on complex networks since 1998 ...
- ...largely due to your typical theoretical physicist:

- *Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)
- See also: https://xkcd.com/793/

Popularity (according to Google Scholar)

"Collective dynamics of 'small-world' networks"
Watts and Strogatz,
Nature, **393**, 440–442, 1998. [16]

Times cited: ~ 37, 460 (as of October 24, 2018)

"Emergence of scaling in random networks"
Barabási and Albert,
Science, **286**, 509–511, 1999. [2]

Times cited: ~ 32, 093 (as of October 24, 2018)

Review articles:

"Complex Networks: Structure and Dynamics"
Boccaletti et al.,
Physics Reports, **424**, 175–308, 2006. [3]

Times cited: ~ 8,533 (as of October 24, 2018)

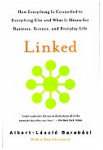
"The structure and function of complex networks"
M. E. J. Newman,
SIAM Rev., **45**, 167–256, 2003. [12]

Times cited: ~ 17,782 (as of October 24, 2018)

"Statistical mechanics of complex networks"
Albert and Barabási,
Rev. Mod. Phys., **74**, 47–97, 2002. [1]

Times cited: ~ 20,531 (as of October 24, 2018)

Popularity according to books:

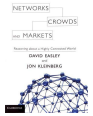


Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási

Six Degrees: The Science of a Connected Age—Duncan Watts [15]

Popularity according to textbooks:

"Networks"
by Mark Newman (2018). [11]



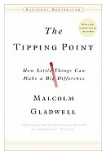
"Networks, crowds, and markets: Reasoning about a highly connected world"
by Easley and Kleinberg (2010). [7]

http://cs.cornell.edu/home/kleinber/networks-book/

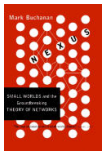
Numerous others ...

- Complex Social Networks—F. Vega-Redondo [14]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [13]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [5]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [6]

Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [8]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

More observations

- But surely networks aren't new ...
- Graph theory is well established ...
- Study of social networks started in the 1930's ...
- So why all this 'new' research on networks?
- Answer: Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- A worthy goal: establish mechanistic explanations.

**If this is upsetting, maybe string theory is for you ...*

More observations

- Web-scale data sets can be overly exciting.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired)
- "The Unreasonable Effectiveness of Data," Halevy et al. [9].
- c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" [17]

But:

- For scientists, description is only part of the battle.
- We still need to understand.

Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

- e.g., people, forks in rivers, proteins, webpages, organisms, ...

Links = Connections between nodes

- Links may be directed or undirected.
- Links may be binary or weighted.

Other spiffing words: vertices and edges.

Super Basic definitions

Node degree = Number of links per node

- Notation: Node i's degree = k_i.
- k_i = 0,1,2,....
- Notation: the average degree of a network = $\langle k \rangle$ (and sometimes z)
- Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

- Defn: N_i = the set of i's k_i neighbors

Super Basic definitions

Adjacency matrix:

- We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j).
- e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

- (n.b., for numerical work, we always use sparse matrices.)

Examples

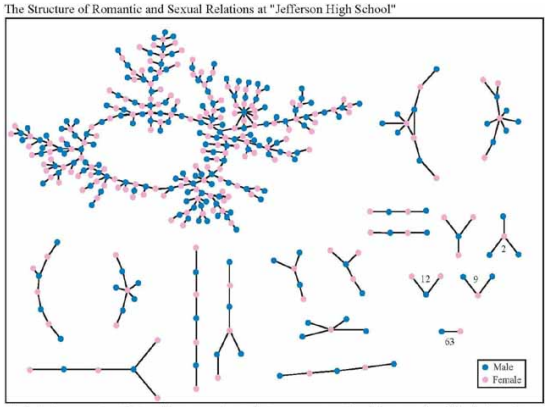
Interaction networks

- The Blogosphere (RIP)
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The internet[1]
- Airline networks
- Call networks (AT&T)
- The Media
- Social Media

datamining.typepad.com

[1] What was the World Wide Web, then the Internet, then the internet

Examples



Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

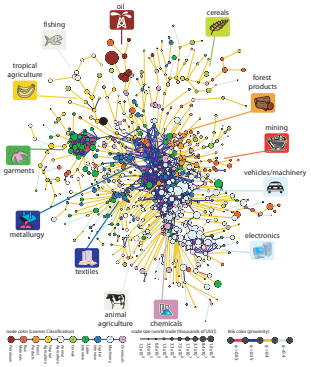
Examples

So what passes for a complex network?

- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...

topics:

- Hidalgo et al.'s "The Product Space Conditions the Development of Nations" [10]
- How do products depend on each other, and how does this network evolve?
- How do countries depend on each other for water, energy, people (immigration), investments?



Examples

Relational networks

- Consumer purchases (Walmart, Target, Amazon, ...)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: bit.ly flickr

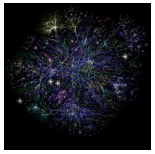
common tags cloud | list

community daily dictionary education encyclopedia english free imported info information internet knowledge learning news reference research resource resources search tools useful web web2.0 wiki wikipedia

Examples

Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids

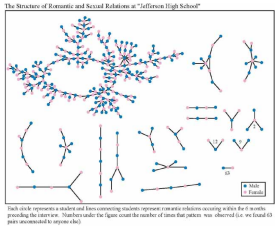


- Distribution (branching) versus redistribution (cyclical)

Examples

Interaction networks: social networks

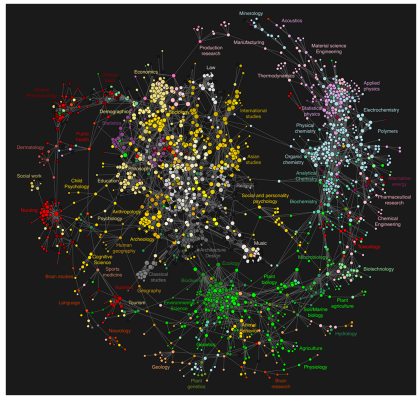
- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- facebook, twitter,



(Bearman *et al.*, 2004)

- 'Remotely sensed' by: email activity, instant messaging, phone logs (*cough*).

Clickworthy Science:



"Clickstream Data Yields High-Resolution Maps of Science", Bollen et al. [4], 2009.

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Rev. Mod. Phys., 74:47–97, 2002. pdf

[2] A.-L. Barabási and R. Albert.
Emergence of scaling in random networks.
Science, 286:509–511, 1999. pdf

[3] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang.
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Physics Reports, 424:175–308, 2006. pdf

[4] J. Bollen, H. Van de Sompel, A. Hagberg, L. Bettencourt, R. Chute, M. A. Rodriguez, and B. Lyudmila.
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PLoS ONE, 4:e4803, 2009. pdf

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[10] C. A. Hidalgo, B. Klinger, A.-L. Barabási, and R. Hausman.
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Communications on Pure and Applied Mathematics, 13:1–14, 1960. pdf