

Biological Contagion

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Contagion

A confusion of contagions:

- Did Harry Potter spread like a virus?
- Can disinformation be “infectious”?
- Suicide, violence?
- Morality? Evil? Laziness? Stupidity? Happiness?
- Religion?
- Democracy ...?
- Language? The alphabet? ^[10]
- Stories?

Contagion

Naturomorphisms

- “The feeling was contagious.”
- “The news spread like wildfire.”
- “Freedom is the most contagious virus known to man.”
—Hubert H. Humphrey, Johnson’s vice president
- “Nothing is so contagious as enthusiasm.”
—Samuel Taylor Coleridge

Optimism according to Ambrose Bierce:

The doctrine that everything is beautiful, including what is ugly, everything good, especially the bad, and everything right that is wrong. ... **It is hereditary, but fortunately not contagious.**

Social contagion

Eric Hoffer, 1902–1983

There is a grandeur in the uniformity of the mass. When a fashion, a dance, a song, a slogan or a joke sweeps like **wildfire** from one end of the continent to the other, and a hundred million people roar with laughter, sway their bodies in unison, **hum one song** or **break forth in anger and denunciation**, there is the overpowering feeling that in this country we have come nearer the brotherhood of man than ever before.

Hoffer was an interesting fellow...

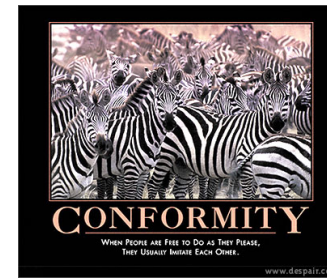
The spread of fanaticism

Hoffer’s most famous work: “**The True Believer: Thoughts On The Nature Of Mass Movements**” (1951) ^[12]

Aphorisms-aplenty:

- “We can be absolutely certain only about things we do not understand.”
- “Mass movements can rise and spread without belief in a God, but never without belief in a devil.”
- “Where freedom is real, equality is the passion of the masses. Where equality is real, freedom is the passion of a small minority.”

Imitation

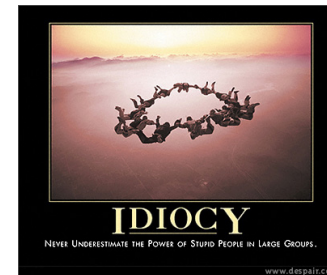


despair.com

“When people are free to do as they please, they usually imitate each other.”

—Eric Hoffer
“The Passionate State of Mind” ^[13]

The collective...



despair.com

“Never Underestimate the Power of Stupid People in Large Groups.”

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An awful recording: Wikipedia’s list of epidemics from 430 BC on.

Year	Location	Date	Comment	Disease	Reference
430–404 BC	Greece	430–404 BC	Known as Plague of Athens, believed to be primarily typhoid.	unknown, similar to typhoid	
165–180	Europe, Western Asia, Northern Africa	165–180	Known as Antonine Plague, due to the name of the Roman emperor emperor at the time.	unknown, symptoms similar to smallpox	
200–206 AD	Europe	200–206 AD	Known as the Plague of Cyprus, named after Cyprus, which after the Plague, became an island.	unknown, possibly anthrax	
541–542	Europe	541–542	Known as Plague of Justinian, due to the name of the Byzantine emperor at the time.	Bubonic plague	[1]
1346–1353	Europe	1346–1353	Known as Black Death or Second plague pandemic, that return of the plague to Europe after the Justinian plague of the 6th century.	plague	[2]
1545–1548	Mexico	1545–1548		vir hemorrhagic fever	[3][4]
1576	Mexico	1576		vir hemorrhagic fever	[5][6]
1582–1586	Swedish nation	1582–1586		measles	[7]

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Examples of non-disease spreading:

Interesting infections:

☞ Spreading of certain buildings in the US:

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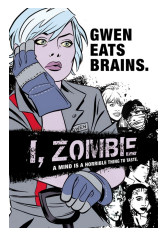
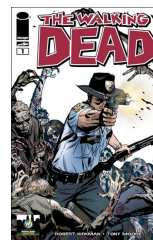
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Contagion

Definitions

- ☞ (1) The spreading of a quality or quantity between individuals in a population.
- ☞ (2) A disease itself: the plague, a blight, the dreaded lurgi, ...
- ☞ from Latin: *con* = 'with' + *tangere* 'to touch.'
- ☞ Contagion has unpleasant overtones...
- ☞ Just **Spreading** might be a more neutral word
- ☞ But **contagion** is kind of exciting...

Contagions

Two main classes of contagion

1. **Infectious diseases:** tuberculosis, HIV, ebola, SARS, influenza, zombification, ...
2. **Social contagion:** fashion, word usage, rumors, uprisings, religion, stories about zombies, ...

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Community—S2E6: Epidemiology

Mathematical Epidemiology

The standard SIR model^[18]

☞ = basic model of disease contagion

☞ Three states:

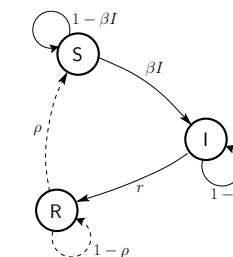
1. S = Susceptible
2. I = Infective/Infectious
3. R = Recovered or Removed or Refractory

$$S(t) + I(t) + R(t) = 1$$

- ☞ Presumes random interactions (mass-action principle)
- ☞ Interactions are independent (no memory)
- ☞ Discrete and continuous time versions

Mathematical Epidemiology

Discrete time automata example:



Transition Probabilities:

β for being infected given contact with infected

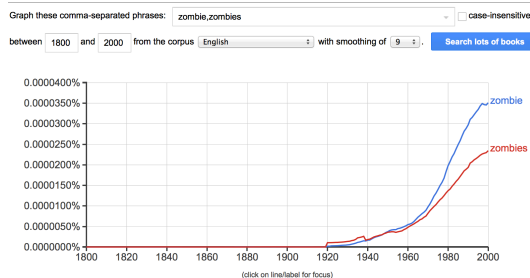
r for recovery

ρ for loss of immunity

Marbleization of the US:

The most terrifying contagious outbreak?

Google books Ngram Viewer



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Mathematical Epidemiology

Original models attributed to

- 1920's: Reed and Frost
- 1920's/1930's: Kermack and McKendrick [14, 16, 15]
- Coupled differential equations with a mass-action principle

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Independent Interaction models

Differential equations for continuous model

$$\frac{d}{dt}S = -\beta IS + \rho R$$

$$\frac{d}{dt}I = \beta IS - rI$$

$$\frac{d}{dt}R = rI - \rho R$$

β , r , and ρ are now **rates**.

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Reproduction Number R_0

Reproduction Number R_0

- R_0 = expected number of infected individuals resulting from a single initial infective
- Epidemic threshold: If $R_0 > 1$, 'epidemic' occurs.
- Exponential take off: R_0^n where n is the number of generations.
- Fantastically awful notation convention: R_0 and the R in SIR .

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Reproduction Number R_0

Discrete version:

- Set up: One Infective in a randomly mixing population of Susceptibles
- At time $t = 0$, single infective random bumps into a Susceptible
- Probability of transmission = β
- At time $t = 1$, single Infective remains infected with probability $1 - r$
- At time $t = k$, single Infective remains infected with probability $(1 - r)^k$

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Reproduction Number R_0

Discrete version:

Expected number infected by original infective:

$$R_0 = \beta + (1 - r)\beta + (1 - r)^2\beta + (1 - r)^3\beta + \dots$$

$$= \beta (1 + (1 - r) + (1 - r)^2 + (1 - r)^3 + \dots)$$

$$= \beta \frac{1}{1 - (1 - r)} = \beta/r$$

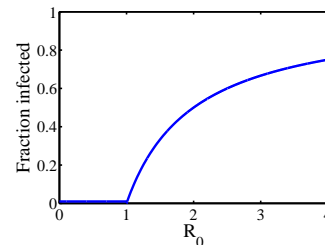
For $S(0) \approx 1$ initial susceptibles
($1 - S(0) = R(0) =$ fraction initially immune):

$$R_0 = S(0)\beta/r$$

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Independent Interaction models

Example of epidemic threshold:



- Continuous phase transition.
- Fine idea from a simple model.

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Independent Interaction models

For the continuous version

Second equation:

$$\frac{d}{dt}I = \beta SI - rI$$

$$\frac{d}{dt}I = (\beta S - r)I$$

Number of infectives grows initially if

$$\beta S(0) - r > 0 \Rightarrow \beta S(0) > r \Rightarrow \beta S(0)/r > 1$$

where $S(0) \approx 1$.

Same story as for discrete model.

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Independent Interaction models

Many variants of the SIR model:

- SIS: susceptible-infective-susceptible
- SIRS: susceptible-infective-recovered-susceptible
- compartment models (age or gender partitions)
- more categories such as 'exposed' (SEIRS)
- recruitment (migration, birth)

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Watch someone else pretend to save the world:



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For novel diseases:

1. Can we predict the size of an epidemic?
2. How important is the reproduction number R_0 ?

R_0 approximately same for all of the following:

- 1918-19 "Spanish Flu" ~ 75,000,000 world-wide, 500,000 deaths in US.
- 1957-58 "Asian Flu" ~ 2,000,000 world-wide, 70,000 deaths in US.
- 1968-69 "Hong Kong Flu" ~ 1,000,000 world-wide, 34,000 deaths in US.
- 2003 "SARS Epidemic" ~ 800 deaths world-wide.



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Size distributions

As we know, heavy-tailed size distributions are somewhat prevalent in complex systems:

- earthquakes (Gutenberg-Richter law)
- city sizes, forest fires, war fatalities
- wealth distributions
- 'popularity' (books, music, websites, ideas)
- Epidemics?**

Power law distributions are common but not obligatory...

Really, what about epidemics?

- Simply hasn't attracted much attention.
- Data not as clean as for other phenomena.

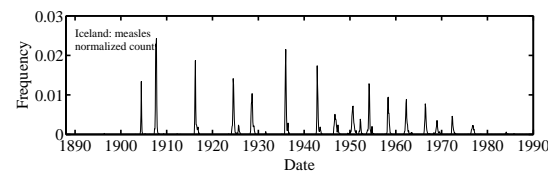


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Feeling Ill in Iceland

Caseload recorded monthly for range of diseases in Iceland, 1888-1990



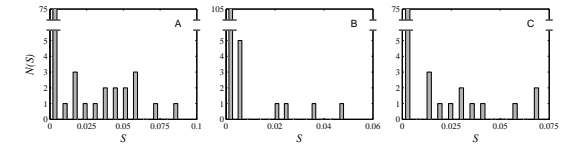
- Treat outbreaks separated in time as 'novel' diseases.



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Epidemic size distributions $N(S)$ for Measles, Rubella, and Whooping Cough.



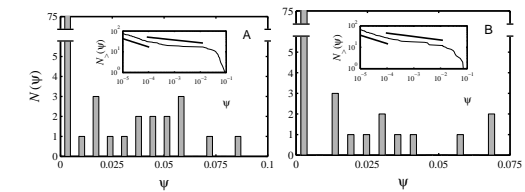
Spike near $S = 0$, relatively flat otherwise.



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Measles & Pertussis



Insert plots:

Complementary cumulative frequency distributions:

$$N(\Psi' > \Psi) \propto \Psi^{-\gamma+1}$$

Limited scaling with a possible break.



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Power law distributions

Measured values of γ :

- measles: 1.40 (low Ψ) and 1.13 (high Ψ)
- pertussis: 1.39 (low Ψ) and 1.16 (high Ψ)

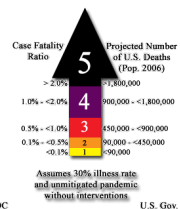
- Expect $2 \leq \gamma < 3$ (finite mean, infinite variance)
- When $\gamma < 1$, can't normalize
- Distribution is quite flat.



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Pandemic severity index (PSI)

Classification during/post pandemic:



- Category based.
- 1-5 scale.
- Modeled on the Saffir-Simpson hurricane scale .



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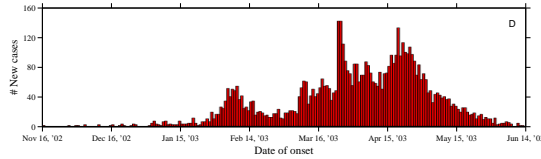
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Resurgence—example of SARS



- Epidemic slows... then an infective moves to a new context.
- Epidemic discovers new 'pools' of susceptibles: **Resurgence.**
- Importance of rare, stochastic events.

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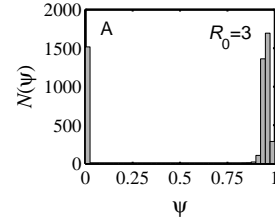
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Size distributions



Simple models typically produce **bimodal** or **unimodal** size distributions.

- This **includes** network models: random, small-world, scale-free, ...
- Exceptions:
 - Forest fire models
 - Sophisticated metapopulation models

Burning through the population

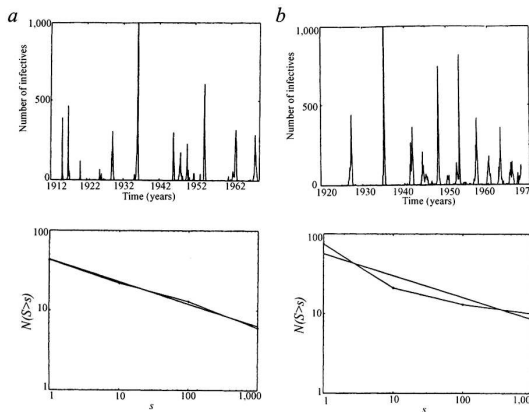
Forest fire models: [19]

- Rhodes & Anderson, 1996
- The physicist's approach: **"if it works for magnets, it'll work for people..."**

A bit of a stretch:

- Epidemics ≡ forest fires spreading on 3-d and 5-d lattices.
- Claim Iceland and Faroe Islands exhibit power law distributions for outbreaks.
- Original forest fire model not completely understood.

Size distributions



From Rhodes and Anderson, 1996.

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Sophisticated metapopulation models:

- Multiscale models suggested earlier by others but not formalized (Bailey^[1], Cliff and Haggett^[6], Ferguson et al.)
- Community based mixing (two scales)—Longini.^[17]
- Eubank et al.'s EpiSims/TRANSIMS—city simulations.^[9]
- Spreading through countries—Airlines: Germann et al., Colizza et al.^[7]



GLEAM: Global pandemic simulations by Vespignani et al.



"The hidden geometry of complex, network-driven contagion phenomena" Brockmann and Helbing, Science, **342**, 1337-1342, 2013. [5]

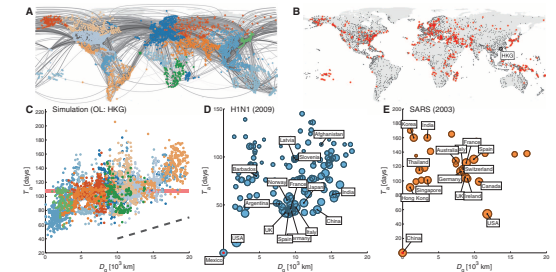


Fig. 1. Complexity in global, network-driven contagion phenomena. (A) The global mobility network (GMN). Gray lines represent passenger flows along direct connections between 4069 airports worldwide. Geographic regions are distinguished by color [classified according to network modularity maximization (39)]. (B) Temporal snapshot of a simulated global pandemic with initial outbreak location (OU) in Hong Kong (HK). The simulation is based on the metapopulation model defined by Eq. 3 with parameters $R_0 = 1.5$, $\beta = 0.285 \text{ day}^{-1}$, $\gamma = 2.8 \times 10^{-4} \text{ day}^{-1}$, $\epsilon = 10^{-4}$. Red symbols depict locations with epidemic arrival times in the time window 105 days $\leq T_a \leq 110$ days. Because of the multiscale structure of the underlying network, the spatial distribution of disease prevalence (i.e., the fraction of infected individuals) lacks geometric coherence. No clear wave front is visible, and based on this dynamic state, the OU cannot be easily detected. (C) For the same simulation as in (B), the panel depicts arrival times T_a as a function of geographic distance D_g from the OU (nodes are colored according to geographic region as in (A)) for each of the 4069 nodes in the network. On a global scale, T_a weakly correlates with geographic distance D_g ($r^2 = 0.34$). A linear fit yields an average global spreading speed of $v_g = 321 \text{ km/day}$ (see also Fig. S7). Using D_g and v_g to estimate arrival times for specific locations, however, does not work well owing to the strong variability of the arrival times for a given geographic distance. The red horizontal bar corresponds to the arrival time window shown in (B). (D) Arrival times versus geographic distance from the source (Mexico) for the 2009 H1N1 pandemic. Symbols represent 140 affected countries, and symbol size quantifies total traffic per country. Arrival times are defined as the date of the first confirmed case in a given country after the initial outbreak on 17 March 2009. As in the simulated scenario, arrival time and geographic distance are only weakly correlated ($r^2 = 0.0394$). (E) In analogy to (D), the panel depicts the arrival times versus geographic distance from the source (China) of the 2003 SARS epidemic for 29 affected countries worldwide. Arrival times are taken from WHO published data (2). As in (C) and (D), arrival time correlates weakly with geographic distance.

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Community—S2E6: Epidemiology

The challenge

- So... can a simple model produce
 - broad epidemic distributions** and
 - resurgence?**

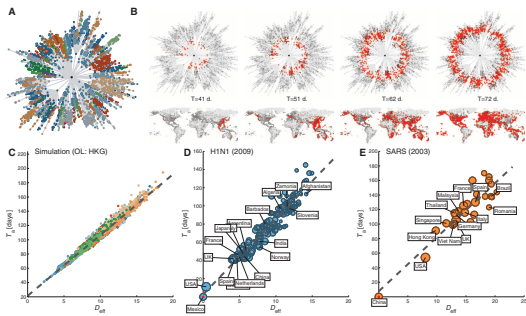


Fig. 2. Understanding global contagion phenomena using effective distance. (A) The structure of the shortest path tree (in gray) from Hong Kong (central node). Radial distance represents effective distance D_{eff} as defined by Eq. 4 and 5. Nodes are colored according to the same scheme as in Fig. 1A. (B) The sequence from left to right of panels depicts the time course of a simulated model disease with initial outbreak in Hong Kong (HKG), for the same parameter set as used in Fig. 1B. Prevalence is reflected by the redness of the symbols. Each panel compares the size of the system in the conventional geographic representation (bottom) with the effective distance representation (top). The complex spatial pattern in the conventional view is equivalent to a homogeneous wave that propagates outwards at constant effective speed in the effective distance representation. (C) Epidemic arrival time T_a versus effective distance D_{eff} for the same simulated epidemic as in (B). In contrast to geographic distance (Fig. 1C), effective distance correlates strongly with arrival time ($R^2 = 0.973$). I.e., effective distance is an excellent predictor of arrival times. (D and E) Linear relationship between effective distance and arrival time for the 2009 H1N1 pandemic (D) and the 2003 SARS epidemic (E). The arrival time data are the same as in Fig. 1, D and E. The effective distance was computed from the projected global mobility network between countries. As in the model system, we observe a strong correlation between arrival time and effective distance.

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Size distributions

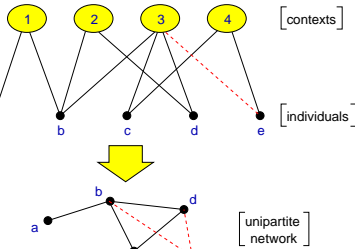
- 🧩 Vital work but perhaps hard to generalize from...
- 🧩 ⇒ Create a simple model involving multiscale travel
- 🧩 Very big question: **What is N ?**
- 🧩 Should we model SARS in Hong Kong as spreading in a neighborhood, in Hong Kong, Asia, or the world?
- 🧩 For simple models, we need to know the final size beforehand...



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Improving simple models

Contexts and Identities—Bipartite networks



- 🧩 boards of directors
- 🧩 movies
- 🧩 transportation modes (subway)



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Improving simple models

Idea for social networks: incorporate identity

Identity is formed from attributes such as:

- 🧩 Geographic location
- 🧩 Type of employment
- 🧩 Age
- 🧩 Recreational activities

Groups are crucial...

- 🧩 formed by people with at least one similar attribute
- 🧩 Attributes ⇔ Contexts ⇔ Interactions ⇔ Networks. [23]



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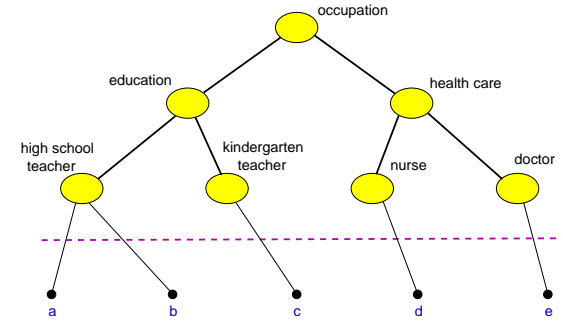
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Infer interactions/network from identities

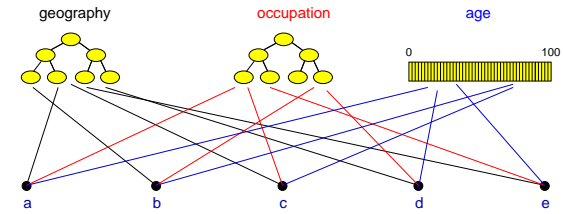


Distance makes sense in identity/context space.



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Generalized context space



(Blau & Schwartz [3], Simmel [20], Breiger [4])



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A toy agent-based model:



“Multiscale, resurgent epidemics in a hierarchical metapopulation model”
Watts et al.,
Proc. Natl. Acad. Sci., **102**, 11157–11162, 2005. [24]

Geography: allow people to move between contexts

- 🧩 Locally: standard SIR model with random mixing
- 🧩 discrete time simulation
- 🧩 β = infection probability
- 🧩 γ = recovery probability
- 🧩 P = probability of travel
- 🧩 **Movement distance:** $\Pr(d) \propto \exp(-d/\xi)$
- 🧩 ξ = typical travel distance



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Community—S2E6: Epidemiology



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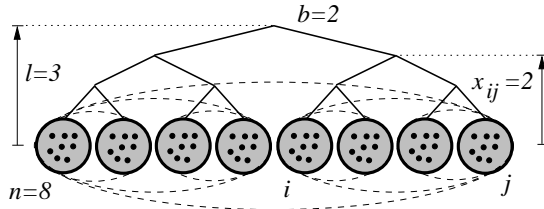
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A toy agent-based model

Schematic:



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Model output

- Define P_0 = Expected number of infected individuals **leaving** initially infected context.
- Need $P_0 > 1$ for disease to spread (independent of R_0).
- Limit epidemic size by restricting frequency of travel and/or range

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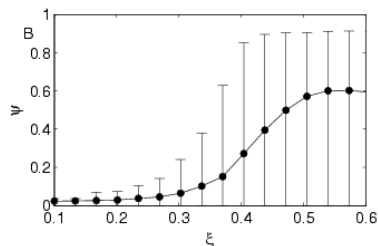
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Model output

Varying ξ :



- Transition in expected final size based on typical movement distance (**sensible**)

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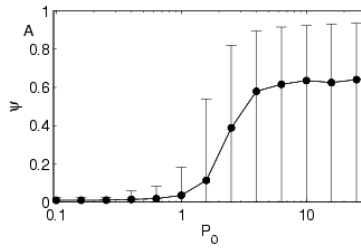
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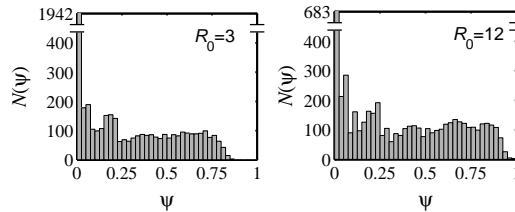
Model output

Varying P_0 :



- Transition in expected final size based on typical number of infectives leaving first group (**also sensible**)
- Travel advisories: ξ has larger effect than P_0 .

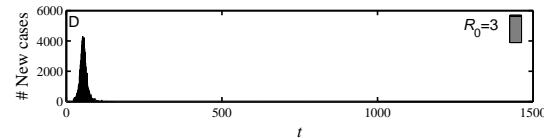
Example model output: size distributions



- Flat distributions are possible for certain ξ and P_0 .
- Different R_0 's may produce similar distributions
- Same epidemic sizes may arise from different R_0 's

Model output—resurgence

Standard model:



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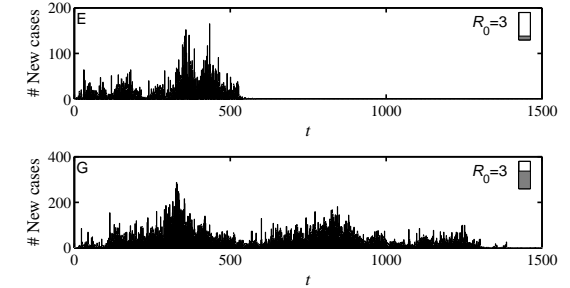
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Model output—resurgence

Standard model with transport:



The upshot

- Simple multiscale population structure + stochasticity
- leads to resurgence + broad epidemic size distributions

Nutshelling

- For the hierarchical movement model, epidemic size is highly unpredictable
- Model is more complicated than SIR but still simple.
- We haven't even included normal social responses such as travel bans and self-quarantine.
- The reproduction number R_0 is not terribly useful.
- R_0 , however measured, is not informative about
 - how likely the observed epidemic size was,
 - and how likely future epidemics will be.
- Problem: R_0 summarises **one** epidemic after the fact and enfold movement, the price of bananas, everything.

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Conclusions

- ☞ Disease's spread is highly sensitive to population structure.
- ☞ Rare events may matter enormously: e.g., an infected individual taking an international flight.
- ☞ More support for controlling population movement: e.g., travel advisories, quarantine

Nutshelling

What to do:

- ☞ Need to separate movement from disease
- ☞ R_0 needs a friend or two.
- ☞ Need $R_0 > 1$ and $P_0 > 1$ and ξ sufficiently large for disease to have a chance of spreading
- ☞ And in general: keep building up the kitchen sink models.

More wondering:

- ☞ Exactly how important are rare events in disease spreading?
- ☞ Again, what is N ?

Krugman, 1998: "Why most economists' predictions are wrong."



"The growth of the Internet will slow drastically, as the flaw in "Metcalfe's law"—which states that the number of potential connections in a network is proportional to the square of the number of participants—becomes apparent: most people have nothing to say to each other! By 2005 or so, it will become clear that the Internet's impact on the economy has been no greater than the fax machine's."¹

¹<http://www.redherring.com/mag/issue55/economics.html>

Economics, Schmeconomics

Alan Greenspan (September 18, 2007):

"I've been dealing with these big mathematical models of forecasting the economy ...

If I could figure out a way to determine whether or not people are more fearful or changing to more euphoric, I don't need any of this other stuff.

I could forecast the economy better than any way I know."



<http://wikipedia.org>

Economics, Schmeconomics

Greenspan continues:

"The trouble is that we can't figure that out. I've been in the forecasting business for 50 years. I'm no better than I ever was, and nobody else is. Forecasting 50 years ago was as good or as bad as it is today. And the reason is that human nature hasn't changed. We can't improve ourselves."

Jon Stewart:

"You just bunned the @*!# out of me."



wildbluffmedia.com

☞ From the Daily Show (September 18, 2007)

☞ The full episode is here:

<http://www.cc.com/video-clips/cenrt5/the-daily-show-with-jon-stewart>

Predicting social catastrophe isn't easy...

"Greenspan Concedes Error on Regulation"

- ☞ ...humbled Mr. Greenspan admitted that he had put too much faith in the self-correcting power of free markets ...
- ☞ "Those of us who have looked to the self-interest of lending institutions to protect shareholders' equity, myself included, are in a state of shocked disbelief"
- ☞ Rep. Henry A. Waxman: "Do you feel that your ideology pushed you to make decisions that you wish you had not made?"
- ☞ Mr. Greenspan conceded: "Yes, I've found a flaw. I don't know how significant or permanent it is. But I've been very distressed by that fact."

New York Times, October 23, 2008

Economics, Schmeconomics

James K. Galbraith:

NYT But there are at least 15,000 professional economists in this country, and you're saying only two or three of them foresaw the mortgage crisis? [JKG] Ten or 12 would be closer than two or three.

NYT What does that say about the field of economics, which claims to be a science? [JKG] It's an enormous blot on the reputation of the profession. There are thousands of economists. Most of them teach. And most of them teach a theoretical framework that has been shown to be fundamentally useless.

From the New York Times, 11/02/2008

Other attempts to use SIR and co. elsewhere:

- ☞ Adoption of ideas/beliefs (Goffman & Newell, 1964)^[1]
- ☞ Spread of rumors (Daley & Kendall, 1965)^[8]
- ☞ Diffusion of innovations (Bass, 1969)^[2]
- ☞ Spread of fanatical behavior (Castillo-Chávez & Song, 2003)
- ☞ Spread of Feynmann diagrams (Bettencourt et al., 2006)

Social contagion:

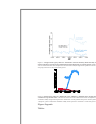
- ☞ SIR may apply sometimes ...
- ☞ But we need new fundamental models.
- ☞ Next up: Thresholds.

We really should know social contagion is different but ...



"It's contagious: Rethinking a metaphor dialogically" Warren and Power, Culture & Psychology, 21, 359–379, 2015. [22]

☞ "Facebook will lose 80% of users by 2017, say Princeton researchers" (Guardian, 2014)



"Epidemiological modeling of online social network dynamics" Spechler and Cannarella, Available online at <http://arxiv.org/abs/1401.4208>, 2014. [21]

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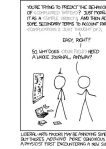
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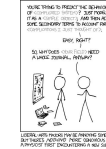
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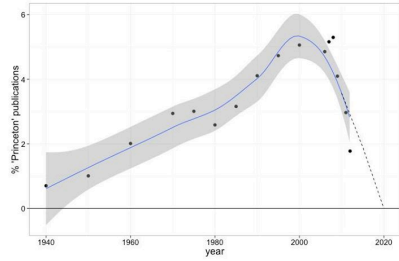
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The Facebook Data Science team's response ↗



Mike Develin, Lada Adamic, and Sean Taylor.

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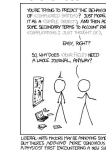
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