

**Complex Networks, CSYS/MATH 303—Assignment 4**  
**University of Vermont, Spring 2011**

**Dispersed:** Tuesday, February 22, 2011.

**Due:** By start of lecture, 2:30 pm, Thursday, March 3, 2011.

*Some useful reminders:*

**Instructor:** Peter Dodds

**Office:** Farrell Hall, second floor, Trinity Campus

**E-mail:** peter.dodds@uvm.edu

**Office hours:** 11:45 am to 12:15 pm and 1:00 pm to 3:00 pm, Wednesday

**Course website:** <http://www.uvm.edu/~pdodds/teaching/courses/2011-01UVM-303>

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All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use  $\LaTeX$  (or related variant).

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**Size-density laws:**

1. For a uniformly distributed population, to minimize the average distance between individuals and their nearest facility, we've made a claim that facilities would be placed at the centres of the tiles on a hexagonal lattice (or the vertices of a triangular lattice). Why is this?
2. In two dimensions, the size-density law for distributed source density  $D(\vec{x})$  given a sink density  $\rho(\vec{x})$  states that  $D \propto \rho^{2/3}$ . We showed in class that an approximate argument that minimizes the average distance between sinks and nearest sources gives the 2/3 exponent ([1]; also see Supply Networks lecture notes).

Repeat this argument for the  $d$ -dimensional case and find the general form of the exponent  $\beta$  in  $D \propto \rho^\beta$ .

3. Following Um et al.'s approach [2], obtain a more general scaling for mixed public-private facilities in two dimensions. Use the cost function:

$$c_i = n_i \langle r_i \rangle^\beta \text{ with } 0 \leq \beta \leq 1,$$

where, respectively,  $n_i$  and  $\langle r_i \rangle$  are population and the average 'source to sink' distance for the population of the  $i$ th Voronoi cell (which surrounds the  $i$ th facility).

Note that  $\beta = 0$  corresponds to purely commercial facilities, and  $\beta = 1$  to strongly social ones.

## References

- [1] M. T. Gastner and M. E. J. Newman. Optimal design of spatial distribution networks. *Phys. Rev. E*, 74:016117, 2006. [pdf](#) (⊞)
- [2] J. Um, S.-W. Son, S.-I. Lee, H. Jeong, and B. J. Kim. Scaling laws between population and facility densities. *Proc. Natl. Acad. Sci.*, 106:14236–14240, 2009. [pdf](#) (⊞)