Chapter 6: Lecture 25 Linear Algebra, Course 124B, Fall, 2008

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Department of Mathematics & Statistics University of Vermont



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The fundamental theorem of linear algebra

Approximating matrices with SVD

The basic idea

Guess who?

Bonus example 2

Frame 1/16





Outline

The fundamental theorem of linear algebra

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The fundamental theorem of linear algebra

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Guess who?

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Bonus example 2



- ▶ Applies to any $m \times n$ matrix A.
- ► Symmetry of A and A^{T} .

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Guess who?

Bonus example 2



- ▶ Applies to any $m \times n$ matrix A.
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Where \vec{x} lives:

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Bonus example 1



- ▶ Applies to any $m \times n$ matrix A.
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Where \vec{x} lives:

- ▶ Row space $C(A^T) \subset R^n$.
- ▶ (Right) Nullspace $N(A) \subset R^n$.

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The basic idea Guess who?

Guess who? Bonus example



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Bonus example 2



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Where \vec{b} lives:

- ▶ Column space $C(A) \subset R^m$.
- ▶ Left Nullspace $N(A^{T}) \subset R^{m}$.

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Where \vec{x} lives:

- ▶ Row space $C(A^T) \subset R^n$.
- ▶ (Right) Nullspace $N(A) \subset R^n$.
- ▶ dim $C(A^{T})$ + dim N(A) = r + (n r) = n

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Bonus example 1



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- ▶ Orthogonality: $C(A^T) \otimes N(A) = R^n$

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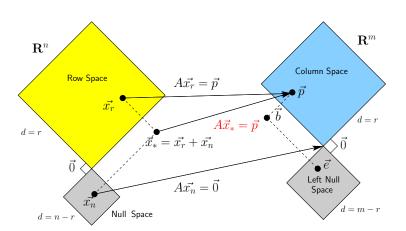
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Best solution \vec{x}_* when $\vec{b} = \vec{p} + \vec{e}$:



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Now we see:

 Each of the four fundamental subspaces has a 'best' orthonormal basis The fundamental theorem of linear algebra

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Bonus example 2



Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ▶ The \hat{v}_i span R^n

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> Bonus example 1 Bonus example 2



Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ▶ The \hat{v}_i span R^n
- ▶ We find the \hat{v}_i as eigenvectors of A^TA .

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- ▶ We find the \hat{v}_i as eigenvectors of A^TA .
- ▶ The \hat{u}_i span R^m

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- ▶ The \hat{v}_i span R^n
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- ▶ The \hat{u}_i span R^m
- ▶ We find the \hat{u}_i as eigenvectors of AA^T .

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Ronus example 1

Bonus example 1
Bonus example 2



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- ▶ We find the \hat{v}_i as eigenvectors of A^TA .
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Happy bases

• $\{\hat{v}_1, \dots, \hat{v}_r\}$ span Row space

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Now we see:

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Happy bases

- $ightharpoonup \{\hat{v}_1,\ldots,\hat{v}_r\}$ span Row space
- $\{\hat{v}_{r+1}, \dots, \hat{v}_n\}$ span Null space

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Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
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- ▶ We find the \hat{u}_i as eigenvectors of AA^T .

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- ▶ $\{\hat{v}_1, \dots, \hat{v}_r\}$ span Row space
- $\{\hat{v}_{r+1}, \dots, \hat{v}_n\}$ span Null space
- $\{\hat{u}_1, \dots, \hat{u}_r\}$ span Column space

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- Each of the four fundamental subspaces has a 'best' orthonormal basis
- ▶ The \hat{v}_i span R^n
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- ▶ The \hat{u}_i span R^m
- ▶ We find the \hat{u}_i as eigenvectors of AA^T .

Happy bases

- ▶ $\{\hat{v}_1, \dots, \hat{v}_r\}$ span Row space
- $\{\hat{v}_{r+1}, \dots, \hat{v}_n\}$ span Null space
- ▶ $\{\hat{u}_1, \dots, \hat{u}_r\}$ span Column space
- $\{\hat{u}_{r+1}, \dots, \hat{u}_m\}$ span Left Null space

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How $A\vec{x}$ works:

 $ightharpoonup A = U\Sigma V^{\mathrm{T}}$



How $A\vec{x}$ works:

- $\rightarrow A = U\Sigma V^{\mathrm{T}}$
- ▶ A sends each $\vec{v}_i \in C(A^T)$ to its partner $\vec{u}_i \in C(A)$ with a stretch/shrink factor $\sigma_i > 0$.

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How $A\vec{x}$ works:

- $ightharpoonup A = U\Sigma V^{\mathrm{T}}$
- ▶ A sends each $\vec{v}_i \in C(A^T)$ to its partner $\vec{u}_i \in C(A)$ with a stretch/shrink factor $\sigma_i > 0$.
- ▶ A is diagonal with respect to these bases and has positive entries (all $\sigma_i > 0$).

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Bonus example 1



How $A\vec{x}$ works:

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- ▶ A sends each $\vec{v}_i \in C(A^T)$ to its partner $\vec{u}_i \in C(A)$ with a stretch/shrink factor $\sigma_i > 0$.
- ▶ A is diagonal with respect to these bases and has positive entries (all $\sigma_i > 0$).
- When viewed the right way, any A is a diagonal matrix Σ.

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Interpret elements of matrix A as color values of an image. The fundamental theorem of linear algebra

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Guess who?

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- Interpret elements of matrix A as color values of an image.
- Truncate series SVD representation of A:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_{i} \hat{u}_{i} \hat{v}_{i}^{\mathrm{T}}$$

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Bonus example

Bonus example 2



- Interpret elements of matrix A as color values of an image.
- Truncate series SVD representation of A:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_{i} \hat{u}_{i} \hat{v}_{i}^{\mathrm{T}}$$

• Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.

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- Interpret elements of matrix A as color values of an image.
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- Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.
- ▶ Rank $r = \min(m, n)$.

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- Interpret elements of matrix A as color values of an image.
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- Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.
- ▶ Rank $r = \min(m, n)$.
- Rank r = # of pixels on shortest side.

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Guess who?

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Bonus example 2



Image approximation (80x60)

Idea: use SVD to approximate images

- Interpret elements of matrix A as color values of an image.
- Truncate series SVD representation of A:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_{i} \hat{u}_{i} \hat{v}_{i}^{\mathrm{T}}$$

- Use fact that $\sigma_1 > \sigma_2 > \ldots > \sigma_r > 0$.
- ▶ Rank $r = \min(m, n)$.
- Rank r = # of pixels on shortest side.
- ► For color: approximate 3 matrices (RGB).

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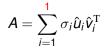
The fundamental theorem of linear algebra

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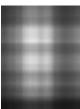
Guess who?

Bonus exampl

Bonus example 2







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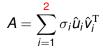


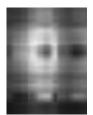
Approximating natrices with SVD

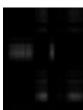
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Bonus example







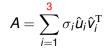


Approximating natrices with SVD

Guess who?

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Bonus example :







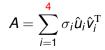


Approximating matrices with SVE

Guess who?

Guess who?

Bonus example







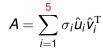


Approximating matrices with SVE

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Bonus example :









Approximating natrices with SVD

Guess who?

Guess wno?

Bonus example







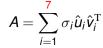


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Guess who?

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Bonus example









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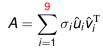




Approximating natrices with SVD

Guess who?

Bonus example 1







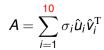


Approximating matrices with SVE

Guess who?

Bonus example

Bonus example :







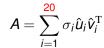


Approximating matrices with SVE

Guess who?

Guess who?

Bonus example :







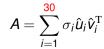


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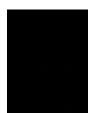
Guess who?

Bonus exampl

Bonus example :







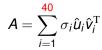


Approximating natrices with SVD

Guess who?

Bonus exampl

Bonus example :







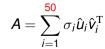


Approximating natrices with SVD

Guess who?

Bonus example

Bonus example :





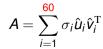




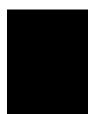
Approximating natrices with SVD

Guess who?

Bonus example

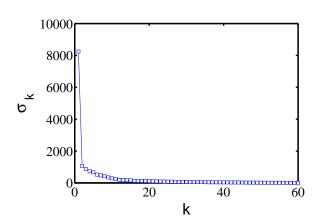








Decay of sigma values: Einstein



The fundamental theorem of linear algebra

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Bonus example 1
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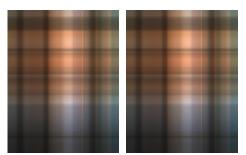
Approximating natrices with SVD

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Guess who?

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Bonus example :









Approximating matrices with SVE

Guess who?

Bonus example









Approximating matrices with SVE

Guess who?

Guess who?

Bonus example 2









Approximating matrices with SVE

Guess who?

Bonus example 1





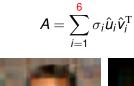




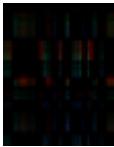
Approximating matrices with SVE

Guess who?

Bonus example









Approximating matrices with SVE

Guess who?

Bonus example :









Approximating matrices with SVE

Guess who?

Guess who?

Bonus example 2





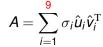




Approximating natrices with SVD

Guess who?

Bonus example









Approximating matrices with SVD

Guess who?

Bonus example

 $A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$







$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$





The fundamental algebra

Guess who?



The fundamental

theorem of line algebra Approximating

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Guess who?

Bonus example 2





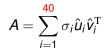




Approximating matrices with SVE

Guess who?

Bonus example 1







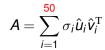


Approximating matrices with SVE

Guess who?

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Bonus example :







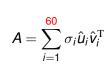


Approximating matrices with SVE

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Bonus example 2







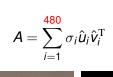


Approximating matrices with SVE

Guess who?

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$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$





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Image approximation (480x640)

$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

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Approximating natrices with SVD

Guess who?

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Image approximation (480x640)

$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Image approximation (480x640)

$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who

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$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Bonus example 1



$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?
Bonus example 1

Bonus example 1
Bonus example 2



$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





Ch. 6: Lec. 25

The fundamental theorem of linear algebra

Approximating matrices with SVD

Guess who?
Bonus example 1

Bonus example 1
Bonus example 2



$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Approximating matrices with SVD

Guess who?
Bonus example 1

Bonus example 1



$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Bonus example 1



$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?

Bonus example 1



$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Approximating matrices with SVE

Guess who?

Bonus example 1



$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$





The fundamental theorem of linear algebra

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Guess who?

Bonus example 1



$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?

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Bonus example 1



$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$





The fundamental theorem of linear algebra

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Guess who?

Bonus example 1



$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?

Bonus example 1



$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Approximating matrices with SVD

Guess who?

Bonus example 1



$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Outline

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Bonus example 2

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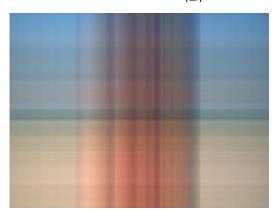
he basic idea Suess who? Ionus example 1

Bonus example 1

Frame 15/16



$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$





The fundamental theorem of linear algebra

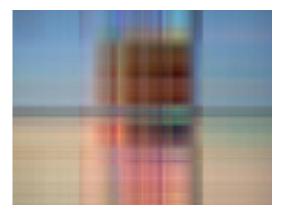
Approximating matrices with SVD

Guess who?

Bonus example 1
Bonus example 2



$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?

Bonus example 2



$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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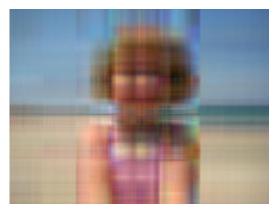
Approximating matrices with SVD

Guess who?

Bonus example 1
Bonus example 2



$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?

Bonus example 2



$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?

Bonus example 2



$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?

Bonus example 2



$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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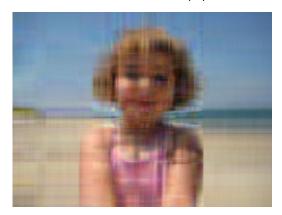
Approximating matrices with SVD

Guess who?

Bonus example 1
Bonus example 2



$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?

Bonus example 1
Bonus example 2



$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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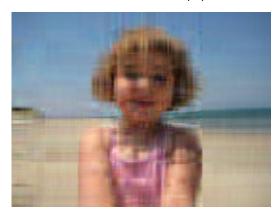
Approximating matrices with SVD

Guess who?

Bonus example 2



$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?

Bonus example 2



$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?

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Guess who?

Bonus example 2



$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Guess who?

Bonus example 1 Bonus example 2



$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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Bonus example 1
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$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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