Chapter 6: Lecture 25

Linear Algebra, Course 124B, Fall, 2008

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University of Vermont

Outline

The fundamental theorem of linear algebra

Approximating matrices with SVD
- The basic idea
- Guess who?
- Bonus example 1
- Bonus example 2

All the way with $A\vec{x} = \vec{b}$:

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- Applies to any $m \times n$ matrix A.
- Symmetry of A and A^{T}.

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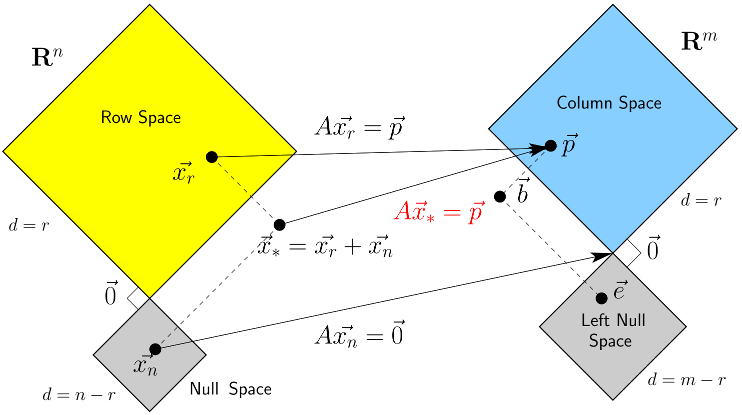
Best solution $\vec{x}_*$ when $\vec{b} = \vec{p} + \vec{e}$:



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- A is diagonal with respect to these bases and has positive entries (all $\sigma_i > 0$).
- When viewed the right way, any A is a diagonal matrix Σ.

Outline

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Idea: use SVD to approximate images

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- For color: approximate 3 matrices (RGB).

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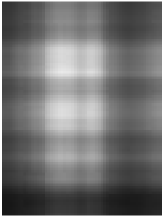
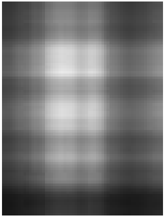
$$A = \sum_{i=1}^{\color{red}{1}} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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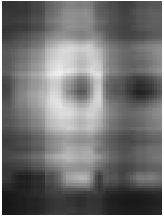
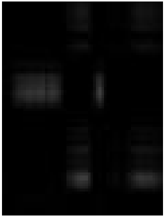
$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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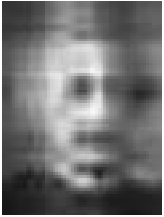

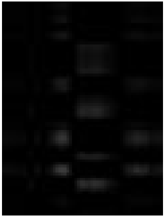

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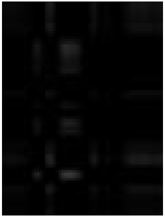

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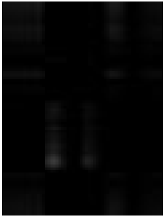

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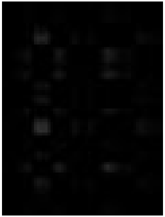

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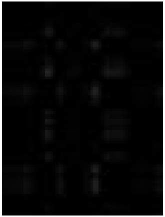

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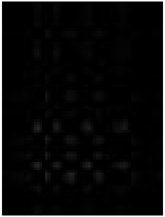
$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



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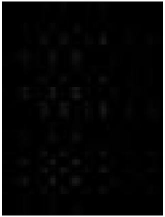

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$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

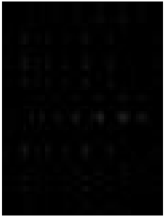
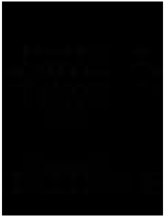


Image approximation (80x60)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



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$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

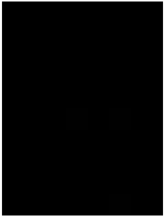


Image approximation (80x60)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

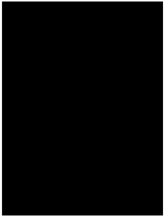


Image approximation (80x60)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

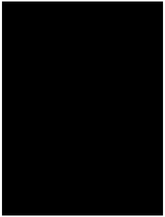
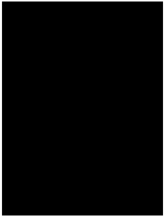


Image approximation (80x60)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



Decay of sigma values: Einstein

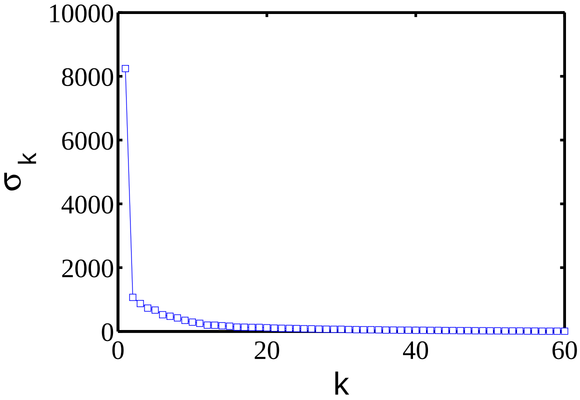
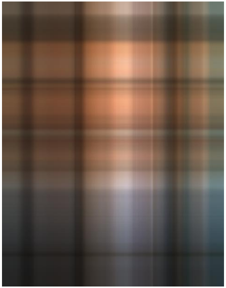
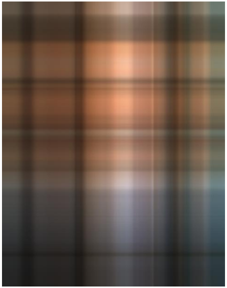


Image approximation (480x615)

$$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





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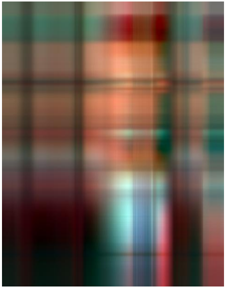

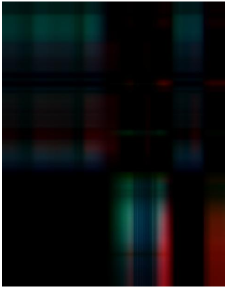

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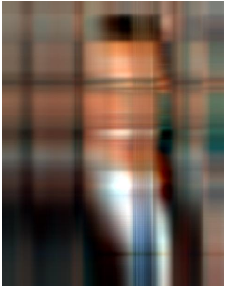

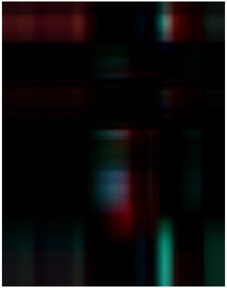

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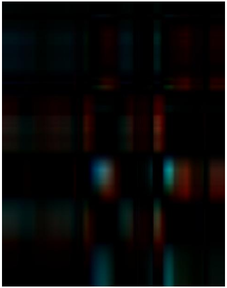

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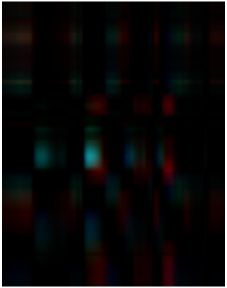

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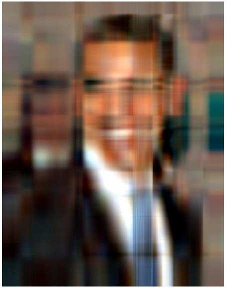

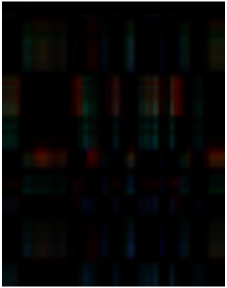

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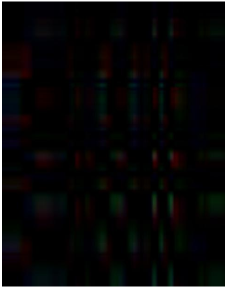

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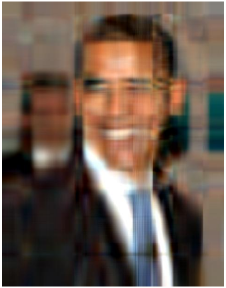

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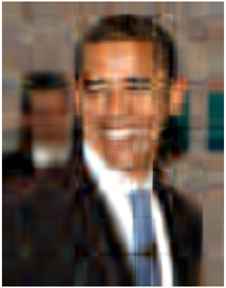


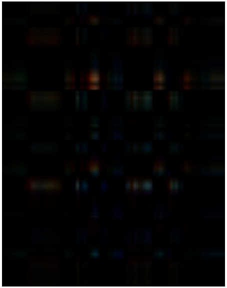
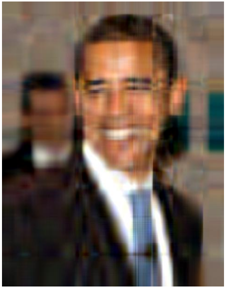


Image approximation (480x615)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



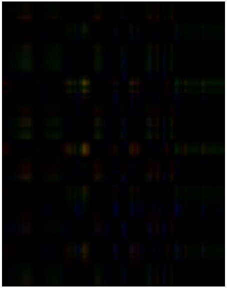


Image approximation (480x615)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

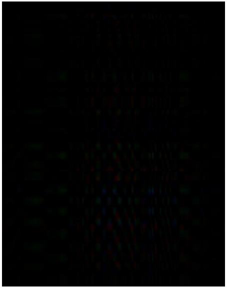


Image approximation (480x615)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

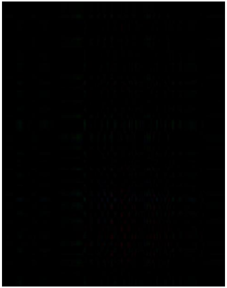


Image approximation (480x615)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

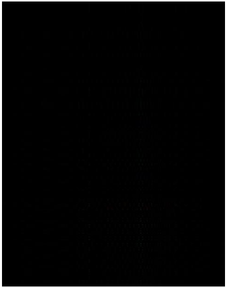


Image approximation (480x615)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

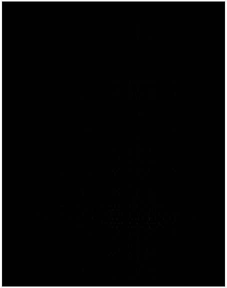


Image approximation (480x615)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

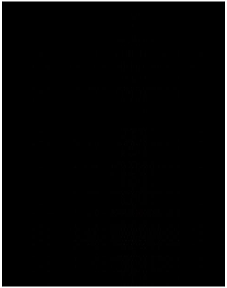
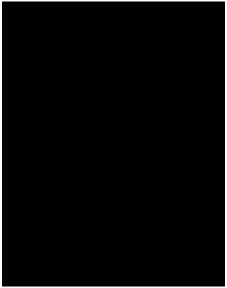


Image approximation (480x615)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



Outline

Image approximation (480x640)

$$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

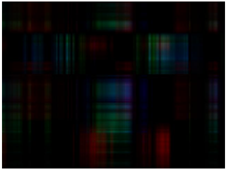
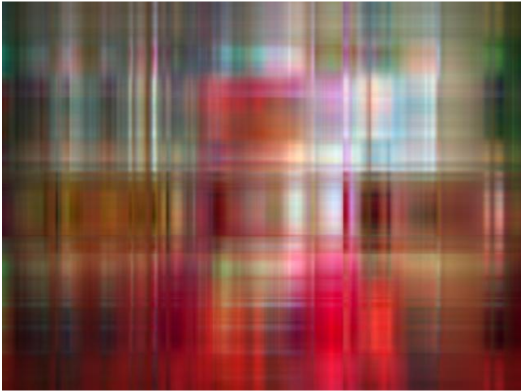


Image approximation (480x640)

$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



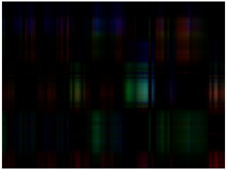


Image approximation (480x640)

$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

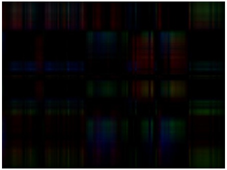


Image approximation (480x640)

$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

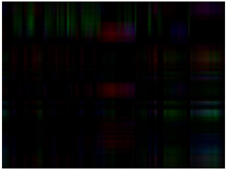


Image approximation (480x640)

$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

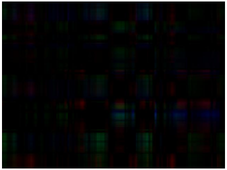


Image approximation (480x640)

$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

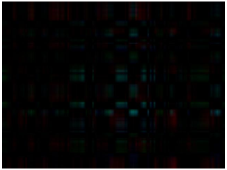


Image approximation (480x640)

$$A = \sum_{i=1}^{\color{red}{8}} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

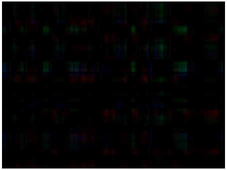


Image approximation (480x640)

$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

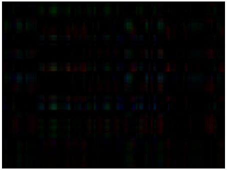


Image approximation (480x640)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

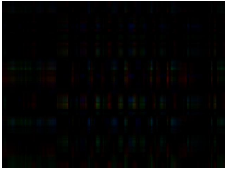


Image approximation (480x640)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

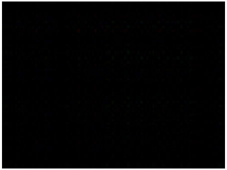


Image approximation (480x640)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

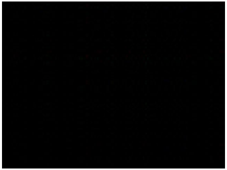


Image approximation (480x640)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

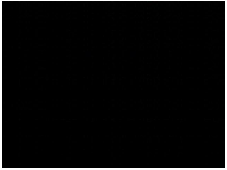


Image approximation (480x640)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

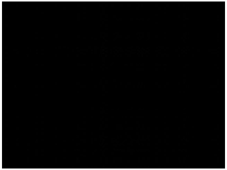


Image approximation (480x640)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

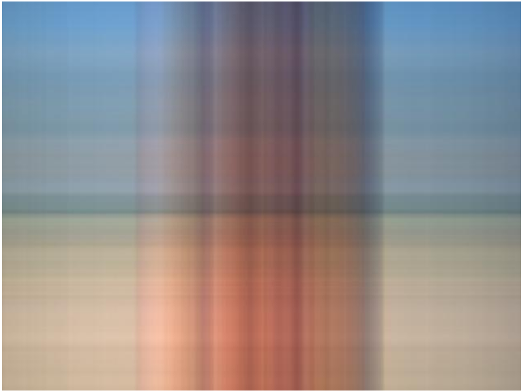
Image approximation (480x640)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Outline

Image approximation (480x640)

$$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



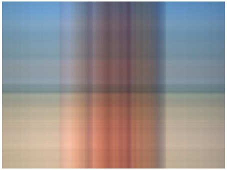


Image approximation (480x640)

$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^T$$

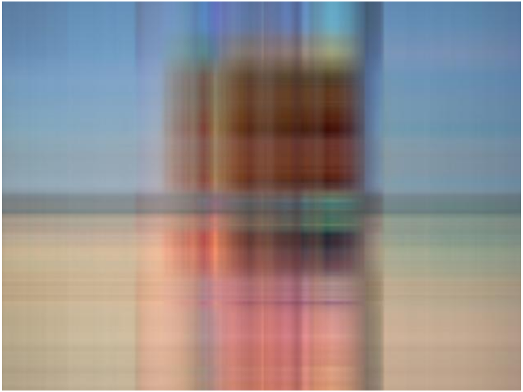


Image approximation (480x640)

$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

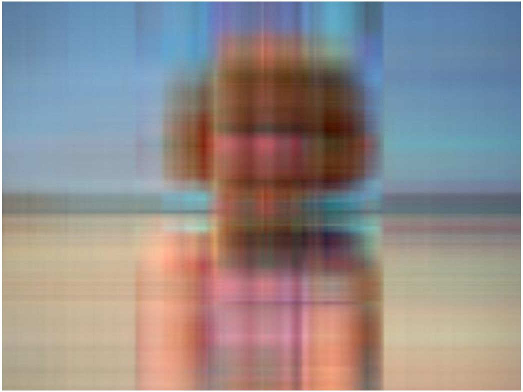
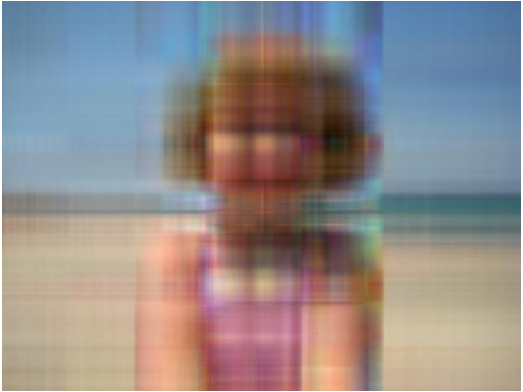


Image approximation (480x640)

$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



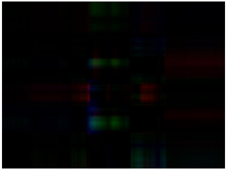
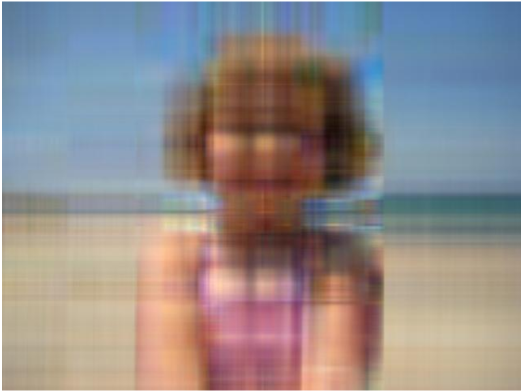


Image approximation (480x640)

$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



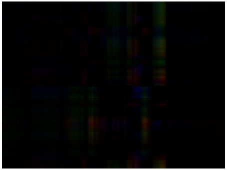


Image approximation (480x640)

$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

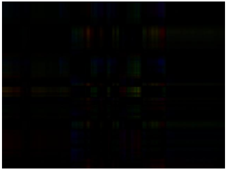


Image approximation (480x640)

$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

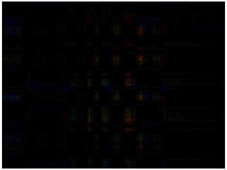


Image approximation (480x640)

$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

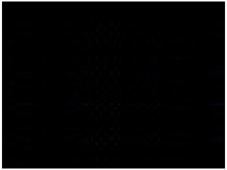


Image approximation (480x640)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

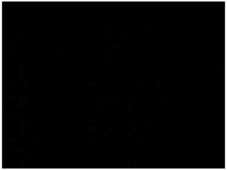


Image approximation (480x640)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

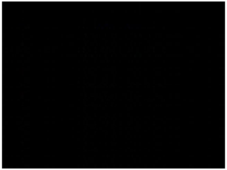


Image approximation (480x640)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

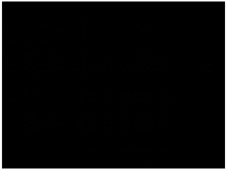


Image approximation (480x640)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

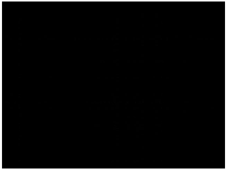


Image approximation (480x640)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{\color{red}{200}} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{320} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x640)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$