Chapter 2: Lecture 7 Linear Algebra, Course 124B, Fall, 2008

Prof. Peter Dodds

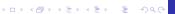
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Frame 1/8





Frame 2/8



Sections covered on first midterm:

► Chapter 1 and Chapter 2 (Sections 2.1–2.7)



Basics:

Review for Exam 1

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- ► Knowledge of Chapter 1 as needed for Chapter 2 = solving $A\vec{x} = \vec{b}$.
- Want 'understanding' and 'doing' abilities.





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Row, Column, & Matrix Pictures of Linear Systems $(A\vec{x} = \vec{b})$

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- ▶ How to convert between the three pictures.



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Review for Exam 1





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- 4. Factor A as A = LU
 - Solve two triangular systems by forward and back substitution
 - First $L\vec{c} = \vec{b}$ then $U\vec{x} = \vec{c}$.



Solving $A\vec{x} = \vec{b}$ by elimination

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Understand number of solutions business:

▶ 0, 1, or ∞: why, when, ...





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Frame 6/8

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- Understand how inverses of elimination matrices are simply related to elimination matrices.



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$$(AB)^{-1} = B^{-1}A^{-1}$$



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- $(A^{-1})^{\mathrm{T}} = (A^{\mathrm{T}})^{-1}$



