Chapter 2: Lecture 7

Linear Algebra, Course 124B, Fall, 2008

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Outline

Review for Exam 1

Basics:

Sections covered on first midterm:

- Chapter 1 and Chapter 2 (Sections 2.1–2.7)

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- Want 'understanding' and 'doing' abilities.

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- How to convert between the three pictures.

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Understand number of solutions business:

- 0, 1, or ∞: why, when, ...

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- Understand how L is made up of the l_{ij} multipliers.
- Understand how inverses of elimination matrices are simply related to elimination matrices.

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