Overview of Complex Networks

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Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number,
2023–2024 | @pocsvox

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Outline

Plan

Basic definitions

Popularity

Examples of Complex Networks

Properties of Complex Networks

Nutshell

References

Something of a plan:

- Lecture 1: Overview; Background
- Lecture 2: Random, Scale-free, and Small-World networks
- Lecture 3: Models of Contagion
- Lecture 4: Transportation networks; Discovering structure

Exciting details regarding these slides:

- Three versions (all in pdf):
  1. Presentation,
  2. Flat Presentation,
  3. Handout (2x2).
- Presentation versions are navigable and hyperlinks are clickable.
- Web links look like this.
- References in slides link to full citation at end. [2]
- Citations contain links to papers in pdf (if available).
- 50 hours of lectures → 5 hours.
- Brought to you by a concoction of LaTeX, Beamer, perl, and madness.

Bonus materials:

Graduate Course Websites:

- SFI Summer School Course (this one!):
  https://pdodds.w3.uvm.edu/teaching/courses/2023-2024pocsverse
- Principles of Complex Systems, University of Vermont
- Complex Networks, University of Vermont

Textbooks:

- Mark Newman (Physics, Michigan)
  "Networks: An Introduction"
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
  "Networks, Crowds, and Markets: Reasoning About a Highly Connected World"

Bonus materials:

Review articles:

- S. Boccaletti et al.
  "Complex networks: structure and dynamics" [4]
  Times cited: 1,028 (as of June 7, 2010)
- M. Newman
  "The structure and function of complex networks" [15]
  Times cited: 2,559 (as of June 7, 2010)
- R. Albert and A.-L. Barabási
  "Statistical mechanics of complex networks" [1]
  Times cited: 3,995 (as of June 7, 2010)

Basic definitions:

Complex: (Latin = with + fold/weave (com + plex))

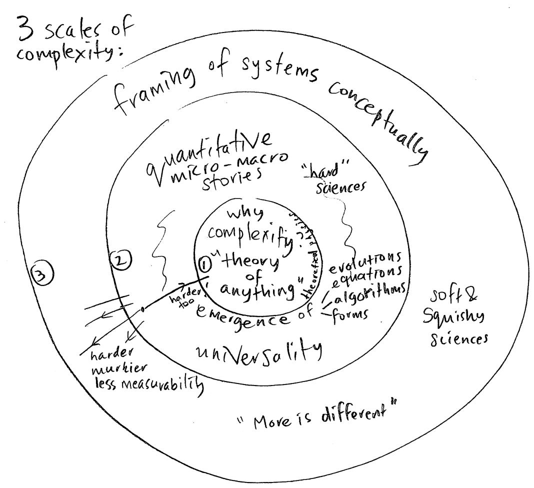
Adjective

- Made up of multiple parts; intricate or detailed.
- Not simple or straightforward.

Basic definitions:

Complex System—Some ingredients:

- Distributed system of many interrelated parts
- No centralized control
- Nonlinear relationships
- Existence of feedback loops
- Complex systems are open (out of equilibrium)
- Presence of Memory
- Modular (nested)/multiscale structure
- Opaque boundaries
- Emergence—'More is Different' [2]



Thesaurus deliciousness:

network
noun

1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.

2 *a network of lanes* MAZE, labyrinth, warren, tangle.

3 *a network of friends* SYSTEM, complex, nexus, web, webwork.

Ancestry:

From Keith Briggs's excellent etymological investigation:

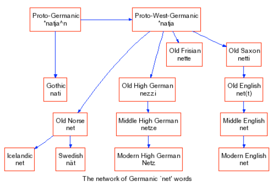
- Opus reticulatum:
- A Latin origin?

[http://serialconsign.com/2007/11/we-put-net-network]

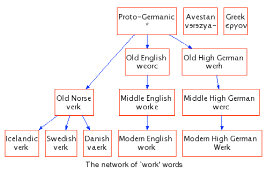
Ancestry:

Net and Work are venerable old words:

- 'Net' first used to mean spider web (King Ælfréd, 888).
- 'Work' appears to have long meant purposeful action.



The network of Germanic 'net' words



The network of 'work' words

- 'Network' = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

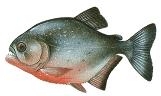
From the OED via Briggs:

- 1658–: reticulate structures in animals
- 1839–: rivers and canals
- 1869–: railways
- 1883–: distribution network of electrical cables
- 1914–: wireless broadcasting networks

- Natural $\rightarrow$ man-made
- Physical connections $\rightarrow$ Wire-less connections $\rightarrow$ abstract connections

Key Observation:

- Many complex systems can be viewed as complex networks of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.
- Mindboggling amount of work published on complex networks since 1998...
- ... largely due to your typical theoretical physicist:



- *Piranha physicus*
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)

Popularity (according to ISI Web of Knowledge)

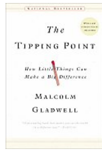
"Collective dynamics of 'small-world' networks" [21]

- Watts and Strogatz
  Nature, 1998
- Cited ≈ 4325 times (as of June 7, 2010)
- Over 1100 citations in 2008.

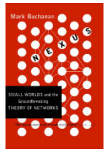
"Emergence of scaling in random networks" [3]

- Barabási and Albert
  Science, 1999
- Cited ≈ 4769 times (as of June 7, 2010)
- Over 1100 citations in 2008.

Popularity according to books:

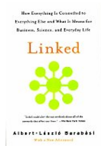


The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [10]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási

Six Degrees: The Science of a Connected Age—Duncan Watts [20]

Numerous others:

- Complex Social Networks—F. Vega-Redondo [19]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [16]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [6]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [9]

More observations

- But surely networks aren't new...
- Graph theory is well established...
- Study of social networks started in the 1930's...
- So why all this 'new' research on networks?
- Answer: Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- A worthy goal: establish mechanistic explanations.

**If this is upsetting, maybe string theory is for you...*

More observations

- Web-scale data sets can be overly exciting.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired)
- "The Unreasonable Effectiveness of Data," Halevy et al. [11].

But:

- For scientists, description is only part of the battle.
- We still need to understand.

Super Basic definitions:

Nodes = A collection of entities which have properties that are somehow related to each other

- e.g., people, forks in rivers, proteins, webpages, organisms,...

Links = Connections between nodes

- Links may be directed or undirected.
- Links may be binary or weighted.

Other spiffing words: vertices and edges.

Super Basic definitions:

Node degree = Number of links per node

- Notation: Node i's degree = k_i.
- k_i = 0,1,2,....
- Notation: the average degree of a network = $\langle k \rangle$ (and sometimes z)
- Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

- Defn: $\mathcal{N}_i$ = the set of i's k_i neighbors

Super Basic definitions:

Adjacency matrix:

- We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i,j).
- e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

- (n.b., for numerical work, we always use sparse matrices.)

Examples

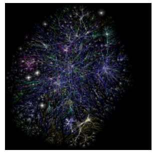
So what passes for a complex network?

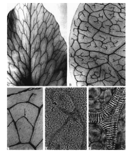
- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...

Examples

Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids





- Distribution (branching) vs. redistribution (cyclical)

Examples

Interaction networks

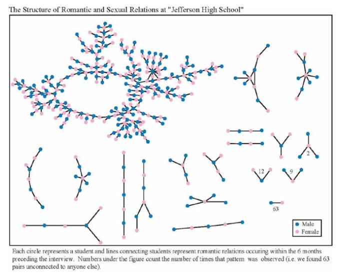
- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
- Airline networks
- Call networks (AT&T)
- The Media
- Paper citations

datamining.typepad.com

Examples

Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- facebook.com, twitter.com

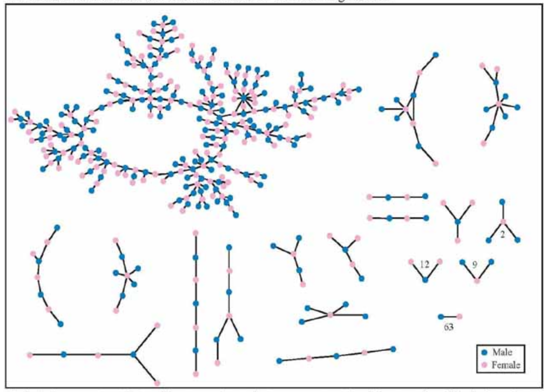


(Bearman *et al.*, 2004)

- 'Remotely sensed' by: email activity, instant messaging, phone logs (*cough*).

Examples

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occuring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

Examples

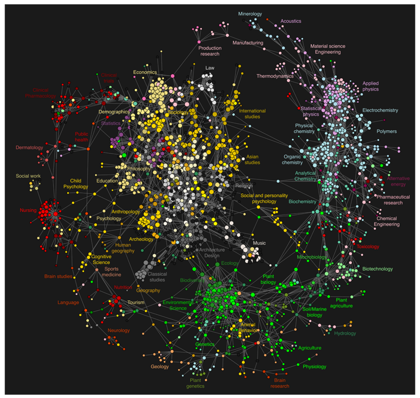
Relational networks

- Consumer purchases (Wal-Mart: ≈ 1 petabyte $= 10^{15}$ bytes)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: del.icio.us, flickr

common tags cloud | list

community daily dictionary education encyclopedia english free imported info information internet knowledge learning news reference research resource resources search tools useful web web2.0 wiki wikipedia

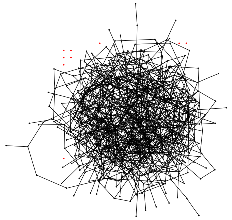
Clickworthy Science:



Bollen et al. [5]

A notable feature of large-scale networks:

- Graphical renderings are often just a big mess.



⇐ Typical hairball
- number of nodes $N = 500$
- number of edges $m = 1000$
- average degree $\langle k \rangle = ?4$

- And even when renderings somehow look good: "That is a very graphic analogy which aids understanding wonderfully while being, strictly speaking, wrong in every possible way" said Ponder [Stibbons] —*Making Money*, T. Pratchett.
- We need to extract digestible, meaningful aspects.

Properties

Some key features of real complex networks:

- Degree distribution
- Assortativity
- Homophily
- Clustering
- Motifs
- Modularity
- Concurrency
- Hierarchical scaling
- Network distances
- Centrality
- Efficiency
- Robustness

- Coevolution of network structure and processes on networks.

Properties

1. Degree distribution P_k

- P_k is the probability that a randomly selected node has degree k
- Big deal: Form of P_k key to network's behavior
- ex 1: Erdős-Rényi random networks have a Poisson distribution:

$$P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$$

- ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'
- We'll come back to this business soon...

Properties

2. Assortativity/3. Homophily:

- Social networks: Homophily = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network: [14] similar degree nodes connecting to each other.
  - *Often social: company directors, coauthors, actors.*
- Disassortative network: high degree nodes connecting to low degree nodes.
  - *Often techological or biological: Internet, protein interactions, neural networks, food webs.*

Properties

4. Clustering:

- Your friends tend to know each other.
- Two measures:

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$ due to Watts & Strogatz [21]

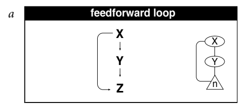
$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$ due to Newman [15]

- C_1 is the average fraction of pairs of neighbors who are connected.
- Interpret C_2 as probability two of a node's friends know each other.

Properties

5. Motifs:

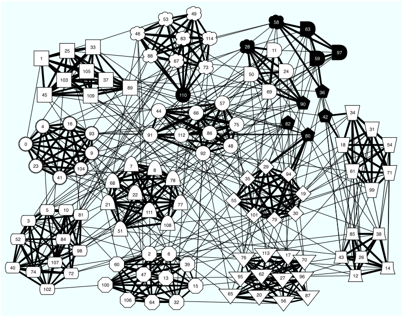
- Small, recurring functional subnetworks
- e.g., Feed Forward Loop:



Shen-Orr, Uri Alon, *et al.* [17]

Properties

6. modularity:



Clauset *et al.*, 2006 [7]: NCAA football

Properties

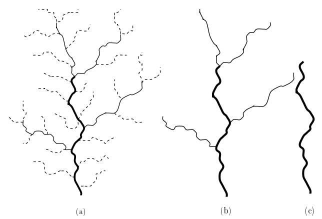
7. Concurrency:

- Transmission of a contagious element only occurs during contact [13]
- Rather obvious but easily missed in a simple model
- Dynamic property—static networks are not enough
- Knowledge of previous contacts crucial
- Beware cumulated network data!

Properties

8. Horton-Strahler stream ordering:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Reveals fractal nature of natural branching networks
  - Hierarchy is not pure but mixed (Tokunaga). [18, 8]
  - Major examples: rivers and blood networks.



- Beautifully described but poorly explained.

Properties

9. Network distances:

(a) shortest path length d_{ij}:

- Fewest number of steps between nodes i and j.
- (Also called the chemical distance between i and j.)

(b) average path length $\langle d_{ij} \rangle$:

- Average shortest path length in whole network.
- Good algorithms exist for calculation.
- Weighted links can be accommodated.

Properties

9. Network distances:

(c) Network diameter d_{max}:

- Maximum shortest path length in network.

(d) Closeness $d_{\text{cl}} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$:

- Average 'distance' between any two nodes.
- Closeness handles disconnected networks ($d_{ij} = \infty$)
- $d_{\text{cl}} = \infty$ only when all nodes are isolated.

Properties

10. Centrality:

- Many such measures of a node's 'importance.'
- ex 1: Degree centrality: k_i.
- ex 2: Node i's betweenness = fraction of shortest paths that pass through i.
- ex 3: Edge ℓ's betweenness = fraction of shortest paths that travel along ℓ.
- ex 4: Recursive centrality: Hubs and Authorities (Jon Kleinberg [12])

Nutshell:

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
- Three main (blurred) categories:
  1. Physical (e.g., river networks),
  2. Interactional (e.g., social networks),
  3. Abstract (e.g., thesauri).

Nutshell:

Overview Key Points (cont.):

- Obvious connections with the vast extant field of graph theory.
- But focus on dynamics is more of a physics/stat-mech/comp-sci flavor.
- Two main areas of focus:
  1. Description: Characterizing very large networks
  2. Explanation: Micro story ⇒ Macro features
- Some essential structural aspects are understood: degree distribution, clustering, assortativity, group structure, overall structure,...
- Still much work to be done, especially with respect to dynamics...

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