

Overview of Complex Networks

Last updated: 2023/08/22, 11:48:23 EDT

Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number,
2023–2024 | @pocsvox

Prof. Peter Sheridan Dodds | @peterdodds

Computational Story Lab | Vermont Complex Systems Center
Santa Fe Institute | University of Vermont



Licensed under the *Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License*.

The PoCVerse
Overview of
Complex
Networks
1 of 43

Complex
Networks Basics

- Etymology
- Popularity
- Graph theory?
- Basic definitions

Examples of
Complex
Networks

- Physical networks
- Interaction networks
- Relational networks

References



These slides are brought to you by:

Sealie & Lambie
Productions



The PoCverse
Overview of
Complex
Networks
2 of 43

Complex
Networks Basics

- Etymology
- Popularity
- Graph theory?
- Basic definitions

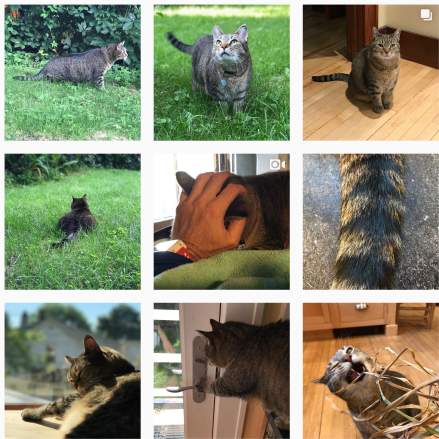
Examples of
Complex
Networks



- Physical networks
- Interaction networks
- Relational networks

References

These slides are also brought to you by:

Special Guest Executive Producer



 On Instagram at [pratchett_the_cat](https://www.instagram.com/pratchett_the_cat) 

The PoCverse
Overview of
Complex
Networks
3 of 43

Complex
Networks Basics

- Etymology
- Popularity
- Graph theory?
- Basic definitions

Examples of
Complex
Networks

- Physical networks
- Interaction networks
- Relational networks

References



Outline

The PoCSverse
Overview of
Complex
Networks
4 of 43

Complex Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Examples of
Complex
Networks

Physical networks

Interaction networks

Relational networks

Examples of Complex Networks

Physical networks

Interaction networks

Relational networks

References

References

net•work |'net,wɜrk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

- a complex system of roads, railroads, or other transportation routes : *a network of railroads.*

2 a group or system of interconnected people or things : *a trade network.*

- a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
- a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*
- a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
- a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*



Thesaurus deliciousness:


network

noun

- 1** *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- 2** *a network of lanes* MAZE, labyrinth, warren, tangle.
- 3** *a network of friends* SYSTEM, complex, nexus, web, webwork.



Ancestry:

From Keith Briggs's excellent etymological investigation: 



Opus
reticulatum:



A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network>]

The PoCverse
Overview of
Complex
Networks
9 of 43

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Examples of
Complex
Networks

Physical networks

Interaction networks

Relational networks

References








Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

-  1658–: reticulate structures in animals
-  1839–: rivers and canals
-  1869–: railways
-  1883–: distribution network of electrical cables
-  1914–: wireless broadcasting networks

The PoCverse
Overview of
Complex
Networks
10 of 43

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Examples of
Complex
Networks

Physical networks

Interaction networks


Relational networks


References

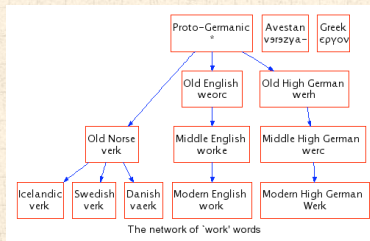
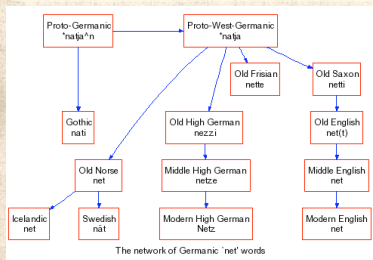



Ancestry:


Net and Work are venerable old words:

 **'Net'** first used to mean spider web (King Ælfréd, 888).

 **'Work'** appear to have long meant purposeful action.



 **'Network'** = something built based on the idea of natural, flexible lattice or web.






 c.f., ironwork, stonework, fretwork.



Key Observation:

- Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a **theoretical-physics/stat-mechish** flavor.
- Mindboggling amount of work published on complex networks since 1998 ...
- ...largely due to your typical theoretical physicist:



-  *Piranha physicus*
-  Hunt in packs.
-  Feast on new and interesting ideas (see chaos, cellular automata, ...)
-  See also: <https://xkcd.com/793/> 

The PoCverse
Overview of
Complex
Networks
12 of 43

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Examples of
Complex
Networks

Physical networks

Interaction networks

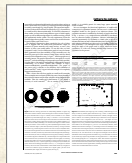
Relational networks


References




Popularity (according to Google Scholar)

The PoCVerse
Overview of
Complex
Networks
14 of 43



"Collective dynamics of 'small-world' networks" 

Watts and Strogatz,
Nature, **393**, 440–442, 1998. ^[16]

Times cited:  ~ **37,460** (as of October 24, 2018)

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

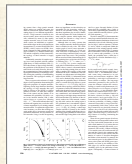
Examples of
Complex
Networks


Physical networks

Interaction networks


Relational networks

References



"Emergence of scaling in random networks" 

Barabási and Albert,
Science, **286**, 509–511, 1999. ^[2]

Times cited:  ~ **32,093** (as of October 24, 2018)



Review articles:



“Complex Networks: Structure and Dynamics” ↗

Boccaletti et al.,
Physics Reports, **424**, 175–308, 2006. [3]

Times cited: ↗ ~ **8,533** (as of October 24, 2018)



“The structure and function of complex networks” ↗

M. E. J. Newman,
SIAM Rev., **45**, 167–256, 2003. [12]

Times cited: ↗ ~ **17,782** (as of October 24, 2018)



“Statistical mechanics of complex networks” ↗

Albert and Barabási,
Rev. Mod. Phys., **74**, 47–97, 2002. [1]

Times cited: ↗ ~ **20,531** (as of October 24, 2018)

The PoCverse
Overview of
Complex
Networks
15 of 43

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Examples of
Complex
Networks

Physical networks

Interaction networks

Relational networks

References



Popularity according to textbooks:

The PoCverse
Overview of
Complex
Networks
16 of 43

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

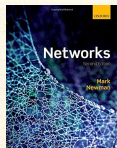
Examples of
Complex
Networks

Physical networks

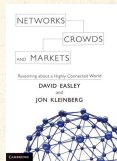
Interaction networks

Relational networks

References



“Networks” [a](#) [↗](#)
by Mark Newman (2018). ^[11]

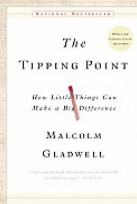


“Networks, crowds, and markets: Reasoning about a highly connected world” [a](#) [↗](#)
by Easley and Kleinberg (2010). ^[7]

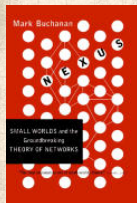
<http://cs.cornell.edu/home/kleinber/networks-book/> [↗](#)



Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [8]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

The PoCVerse
Overview of
Complex
Networks
17 of 43

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Examples of
Complex
Networks

Physical networks

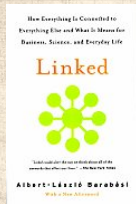
Interaction networks

Relational networks

References



Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts ^[15]

The PoCVerse
Overview of
Complex
Networks
18 of 43

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Examples of
Complex
Networks

Physical networks










Interaction networks

Relational networks

References



Numerous others ...

-  **Complex Social Networks**—F. Vega-Redondo ^[14]
-  **Fractal River Basins: Chance and Self-Organization**—I. Rodríguez-Iturbe and A. Rinaldo ^[13]
-  **Random Graph Dynamics**—R. Durrett
-  **Scale-Free Networks**—Guido Caldarelli
-  **Evolution and Structure of the Internet: A Statistical Physics Approach**—Romu Pastor-Satorras and Alessandro Vespignani
-  **Complex Graphs and Networks**—Fan Chung
-  **Social Network Analysis**—Stanley Wasserman and Kathleen Faust
-  **Handbook of Graphs and Networks**—Eds: Stefan Bornholdt and H. G. Schuster ^[5]
-  **Evolution of Networks**—S. N. Dorogovtsev and J. F. F. Mendes ^[6]

The PoCverse
Overview of
Complex
Networks
19 of 43

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Examples of
Complex
Networks

Physical networks

Interaction networks

Relational networks

References




More observations

- But surely **networks aren't new** ...
- Graph theory is well established ...
- Study of social networks started in the 1930's ...
- So why all this 'new' research on networks?
- Answer:** Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- A worthy goal: establish **mechanistic explanations**.



**If this is upsetting, maybe string theory is for you ...*





More observations

 Web-scale data sets can be overly **exciting**.


Witness:


 The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) 

 "The Unreasonable Effectiveness of Data,"
Halevy et al. ^[9].

 c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" ^[17]

But:


 For scientists, description is only part of the battle.

 We still need to **understand**.





Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

 e.g., people, forks in rivers, proteins, webpages, organisms, ...

Links = Connections between nodes

 **Links** may be directed or undirected.


 **Links** may be binary or weighted.


Other spiffing words: vertices and edges.





Super Basic definitions

Node degree = Number of links per node


 Notation: Node i 's degree = k_i .

 $k_i = 0, 1, 2, \dots$

 Notation: the average degree of a network = $\langle k \rangle$
(and sometimes z)

 Connection between number of edges m and
average degree:


$$\langle k \rangle = \frac{2m}{N}.$$


 **Defn:** \mathcal{N}_i = the set of i 's k_i neighbors




Super Basic definitions

Adjacency matrix:

 We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j) .

 e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

 (n.b., for numerical work, we always use sparse matrices.)



Examples

The PoCverse
Overview of
Complex
Networks
27 of 43

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Examples of
Complex
Networks





Physical networks

Interaction networks

Relational networks

References





So what passes for a complex network?




-  Complex networks are **large** (in node number)
-  Complex networks are **sparse** (low edge to node ratio)
-  Complex networks are usually **dynamic** and **evolving**
-  Complex networks can be social, economic, natural, informational, abstract, ...

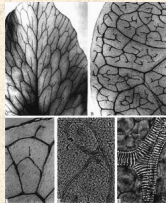
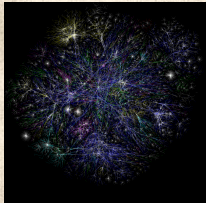



Examples

Physical networks

-  River networks
-  Neural networks
-  Trees and leaves
-  Blood networks

-  The Internet
-  Road networks
-  Power grids




 **Distribution** (branching) versus **redistribution** (cyclical)





Examples


Interaction networks


 The Blogosphere


 Biochemical networks


 Gene-protein networks

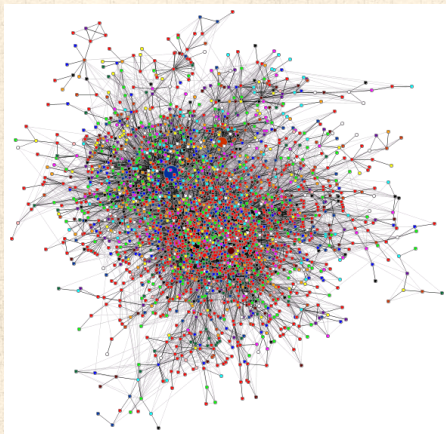
 Food webs: who eats whom


 The World Wide Web (?)

 Airline networks

 Call networks (AT&T)

 The Media



datamining.typepad.com 

The PoCVerse
Overview of
Complex
Networks
31 of 43

Complex
Networks Basics

Etymology
Popularity
Graph theory?
Basic definitions

Examples of
Complex
Networks

Physical networks
Interaction networks
Relational networks

References



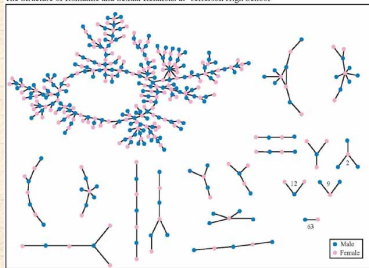
Examples

Interaction networks: social networks

- 🧱 Snogging
- 🧱 Friendships
- 🧱 Acquaintances
- 🧱 Boards and directors
- 🧱 Organizations
- 🧱 [facebook](#) ↗ [twitter](#) ↗,

🧱 'Remotely sensed' by: email activity, instant messaging, phone logs (*cough*).

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

(Bearman *et al.*, 2004)

The PoCverse
Overview of
Complex
Networks
33 of 43

Complex
Networks Basics

Etymology
Popularity
Graph theory?
Basic definitions

Examples of
Complex
Networks

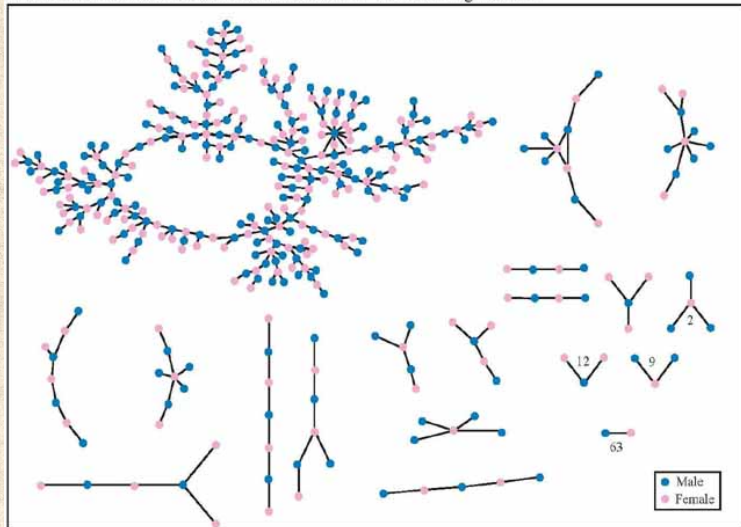
Physical networks
Interaction networks
Relational networks

References



Examples

The Structure of Romantic and Sexual Relations at "Jefferson High School"



The PoCverse
Overview of
Complex
Networks
34 of 43

Complex
Networks Basics

Etymology
Popularity
Graph theory?
Basic definitions

Examples of
Complex
Networks

Physical networks
Interaction networks
Relational networks

References



Examples

Relational networks



Consumer purchases
(Walmart, Target, Amazon, ...)





Thesauri: Networks of words generated by meanings



Knowledge/Databases/Ideas



Metadata—Tagging: bit.ly  [flickr](http://flickr.com) 

common tags cloud | [list](#)

community daily dictionary education **encyclopedia**
english free imported info information internet knowledge
learning news **reference** research resource
resources search tools useful web web2.0 **wiki**
wikipedia

The PoCverse
Overview of
Complex
Networks
36 of 43

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Examples of
Complex
Networks

Physical networks

Interaction networks

Relational networks

References




References I

- [1] R. Albert and A.-L. Barabási.
Statistical mechanics of complex networks.
[Rev. Mod. Phys., 74:47–97, 2002. pdf](#) ↗
- [2] A.-L. Barabási and R. Albert.
Emergence of scaling in random networks.
[Science, 286:509–511, 1999. pdf](#) ↗
- [3] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang.
Complex networks: Structure and dynamics.
[Physics Reports, 424:175–308, 2006. pdf](#) ↗



References II

- [4] J. Bollen, H. Van de Sompel, A. Hagberg, L. Bettencourt, R. Chute, M. A. Rodriguez, and B. Lyudmila.
Clickstream data yields high-resolution maps of science.
[PLoS ONE, 4:e4803, 2009. pdf](#) 
- [5] S. Bornholdt and H. G. Schuster, editors.
Handbook of Graphs and Networks.
Wiley-VCH, Berlin, 2003.
- [6] S. N. Dorogovtsev and J. F. F. Mendes.
Evolution of Networks.
Oxford University Press, Oxford, UK, 2003.



References III

- [7] D. Easley and J. Kleinberg.
Networks, crowds, and markets: Reasoning about a highly connected world.
Cambridge University Press, 2010.
- [8] M. Gladwell.
The Tipping Point.
Little, Brown and Company, New York, 2000.
- [9] A. Halevy, P. Norvig, and F. Pereira.
The unreasonable effectiveness of data.
IEEE Intelligent Systems, 24:8–12, 2009. [pdf](#) 
- [10] C. A. Hidalgo, B. Klinger, A.-L. Barabási, and R. Hausman.
The product space conditions the development of nations.
Science, 317:482–487, 2007. [pdf](#) 

The PoCSverse
Overview of
Complex
Networks
41 of 43

Complex
Networks Basics

Etymology
Popularity
Graph theory?
Basic definitions


Examples of
Complex
Networks

Physical networks
Interaction networks
Relational networks

References



References IV

- [11] M. Newman.
Networks.
Oxford university press, 2nd edition, 2018.
- [12] M. E. J. Newman.
The structure and function of complex networks.
SIAM Rev., 45(2):167–256, 2003. pdf 
- [13] I. Rodríguez-Iturbe and A. Rinaldo.
Fractal River Basins: Chance and Self-Organization.
Cambridge University Press, Cambridge, UK, 1997.
- [14] F. Vega-Redondo.
Complex Social Networks.
Cambridge University Press, 2007.

The PoCSverse
Overview of
Complex
Networks
42 of 43

Complex
Networks Basics

Etymology

Popularity

Graph theory?

Basic definitions

Examples of
Complex
Networks

Physical networks


Interaction networks

Relational networks

References



References V

- [15] D. J. Watts.
Six Degrees.
Norton, New York, 2003.
- [16] D. J. Watts and S. J. Strogatz.
Collective dynamics of 'small-world' networks.
Nature, 393:440–442, 1998. [pdf](#) 
- [17] E. Wigner.
The unreasonable effectiveness of mathematics in
the natural sciences.
Communications on Pure and Applied
Mathematics, 13:1–14, 1960. [pdf](#) 