PoCS What's The Story?

Principles of Complex Systems, CSYS/MATH 300
University of Vermont, Fall 2015
Assignment 8 • code name: A sort of greenish-purple

Dispersed: Saturday, November 7, 2015.
Due: Friday, November 20, 2015.
Some useful reminders:
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Course website: http://www.uvm.edu/~pdodds/teaching/courses/2015-08UVM-300

All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

Email submission: PDF only! Please name your file as follows (where the number is to be padded by a 0 if less than 10 and names are all lowercase):
`CSYS300assignment%02d$firstname-$lastname.pdf` as in
`CSYS300assignment06michael-palin.pdf`

Please submit your project's current draft in pdf format via email. Please use this file name format (all lowercase after CSYS):
`CSYS300project-$firstname-$lastname-YYYY-MM-DD.pdf` as in
`CSYS300project-lisa-simpson-1989-12-17.pdf`

1. (3 + 3)

   Determine the clustering coefficient for toy model small-world networks [1] as a function of the rewiring probability p. Find C_1, the average local clustering coefficient:

   $$C_1(p) = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i = \frac{1}{N} \sum_{i=1}^{N} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}$$

   where N is the number of nodes, $a_{ij} = 1$ if nodes i and j are connected, and $\mathcal{N}_i$ indicates the neighborhood of i.

   As per the original model, assume a ring network with each node connected to a fixed, even number m local neighbors ($m/2$ on each side). Take the number of nodes to be $N \gg m$.

Start by finding $C_1(0)$ and argue for a $(1-p)^3$ correction factor to find an approximation of $C_1(p)$.

Hint 1: you can think of finding C_1 as averaging over the possibilities for a single node.

Hint 2: assume that the degree of individual nodes does not change with rewiring but rather stays fixed at m. In other words, take the average degree of individuals as the degree of a randomly selected individual.

For what value of p is $C_1 \simeq 1/2$?

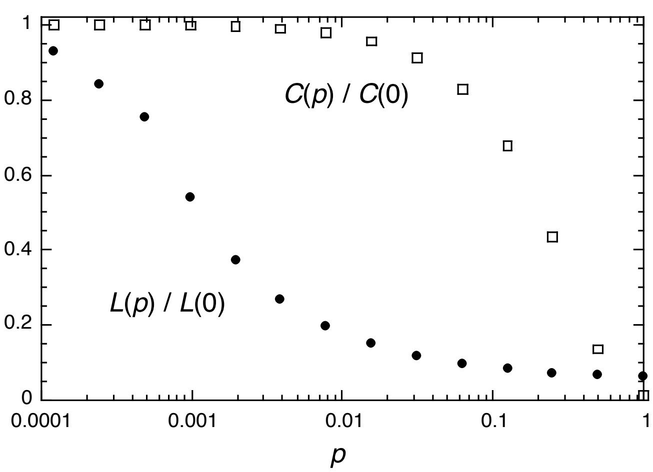
(3 points for set up, 3 for solving.)

2. Simulate the small-world model and reproduce Fig. 2 from the 1998 Watts-Strogatz paper showing how clustering and average shortest path behave with rewiring probability p [1].

   Please find and use any suitable code online, and feel free to Share with each other via Slack.

   Use $N = 1000$ nodes and $k = 10$ for average degree, and vary p from 0.0001 to 1, evenly spaced on a logarithmic scale (there are only 14 values used in the paper).

   Here's the figure you're trying to produce:



3. (3 + 3 + 3 + 3 + 3) *More on the power law stuff:*

   We take a look at the 80/20 rule, 1 per centers, and similar concepts.

   Take x to be the wealth held by an individual in a population of n people, and the number of individuals with wealth between x and $x + \mathrm{d}x$ to be approximately $N(x)\mathrm{d}x$.

   Given a power-law size frequency distribution $N(x) = cx^{-\gamma}$ where $x_{\min} \ll x \ll \infty$, determine the value of γ for which the so-called 80/20 rule holds.

In other words, find γ for which the bottom 4/5 of the population holds 1/5 of the overall wealth, and the top 1/5 holds the remaining 4/5.

Assume the mean is finite, i.e., $\gamma > 2$.

(a) Determine the total wealth W in the system given $\int_{x_{\min}}^{\infty} \mathrm{d}x N(x) = n$.

(b) Imagine that $100q$ percent of the population holds $100(1-r)$ percent of the wealth.

Show γ depends on q and r as

$$\gamma = 1 + \frac{\ln \frac{1}{(1-q)}}{\ln \frac{1}{(1-q)} - \ln \frac{1}{r}}.$$

(c) Given the above, is every pairing of q and r possible?

(d) Find γ for the 80/20 requirement ($q = r = 4/5$).

(e) For the "80/20" γ you find, determine how much wealth $100q$ percent of the population possesses as a function of q and plot the result.

References

[1] D. J. Watts and S. J. Strogatz. Collective dynamics of 'small-world' networks. *Nature*, 393:440–442, 1998. pdf