P o C S What's The Story?

Principles of Complex Systems, CSYS/MATH 300
University of Vermont, Fall 2015
Assignment 1 • code name: The Wailing

Dispersed: Saturday, September 12, 2015.
Due: By start of lecture, 1:15 pm, Thursday, September 17, 2015.
Some useful reminders:
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Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2015-08UVM-300

All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

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1. Use a back-of-an-envelope scaling argument to show that maximal rowing speed V increases as the number of oarspeople n as $V \propto N^{1/9}$.

   Assume the following:

   (a) Rowing shells are geometrically similar (isometric). The table below taken from McMahon and Bonner [1] shows that shell width is roughly proportional to shell length ℓ.

   Shell dimensions and performances.

No. of oarsmen	Modifying description	Length, l (m)	Beam, b (m)	l/b	Boat mass per oarsman (kg)	Time for 2000 m (min) I	II	III	IV
8	Heavyweight	18.28	0.610	30.0	14.7	5.87	5.92	5.82	5.73
8	Lightweight	18.28	0.598	30.6	14.7				
4	With coxswain	12.80	0.574	22.3	18.1				
4	Without coxswain	11.75	0.574	21.0	18.1	6.33	6.42	6.48	6.13
2	Double scull	9.76	0.381	25.6	13.6				
2	Pair-oared shell	9.76	0.356	27.4	13.6	6.87	6.92	6.95	6.77
1	Single scull	7.93	0.293	27.0	16.3	7.16	7.25	7.28	7.17

   (b) The resistance encountered by a shell is due largely to drag on its wetted surface.

   (c) Drag is proportional to the product of the square of the shell's speed (V^2) and the area of the wetted surface ($\propto \ell^2$ due to shell isometry).

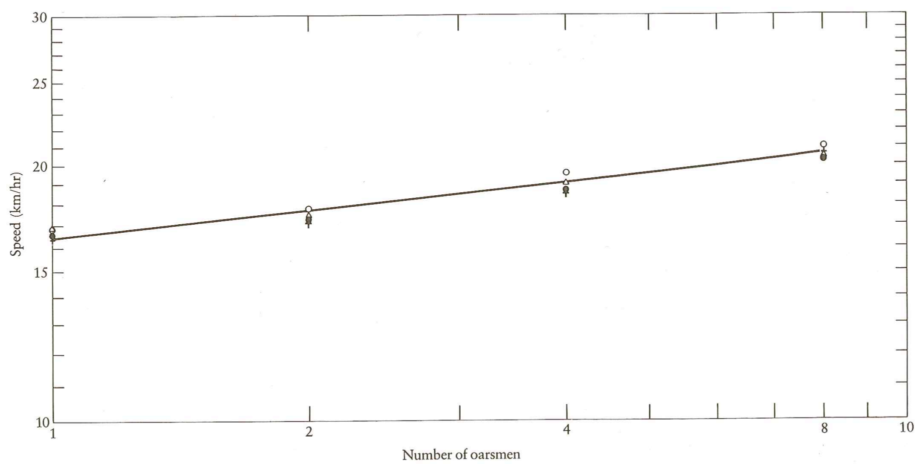
(d) Power $\propto$ drag force $\times$ speed (in symbols: $P \propto D_f \times V$).

(e) Volume displacement of water by a shell is proportional to the number of oarspeople N (i.e., the team's combined weight).

(f) Assume the depth of water displacement by the shell grows isometrically with boat length ℓ.

(g) Power is proportional to the number of oarspeople N.

2. Find the modern day world record times for 2000 metre races and see if this scaling still holds up. Of course, our relationship is approximate as we have neglected numerous factors, the range is extremely small (1–8 oarspeople), and the scaling is very weak (1/9). But see what you can find. The figure below shows data from McMahon and Bonner.



3. Check current weight lifting records for the snatch, clean and jerk, and the total for scaling with body mass (three regressions).

   For weight classes, take the upper limit for the mass of the lifter.

   (a) Does 2/3 scaling hold up?

   (b) Normalized by the appropriate scaling, who holds the overall, rescaled world record?

4. Finish the calculation for the platypus on a pendulum problem so show that a simple pendulum's period τ is indeed proportional to $\sqrt{\ell/g}$.

   Basic plan from lectures: Create a matrix $\mathbf{A}$ where ijth entry is the power of dimension i in the jth variable, and solve by row reduction to find basis null vectors.

In lectures, we arrived at:

$$\mathbf{A}\vec{x} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

You only have to take a few steps from here.

Help—Buckingham π theorem (runs from 25:05 to 46:20, length 21:15):

Note: Video embed only works in Adobe Reader. Direct link: http://www.youtube.com/watch?v=sMfgxVpClvk&rel=0&start=1505&end=2780

5. Show that the maximum speed of animals $V_{\max}$ is proportional to their length L ([2]).

   Here are five dimensionful parameters:

   - $V_{\max}$, maximum speed.
   - L, animal length.
   - ρ, organismal density.
   - σ, maximum applied force per unit area of tissue.
   - b, maximum metabolic rate per unit mass (b has the dimensions of power per unit mass).

   And here are the three dimensions: L, M, and T.

Use a back-of-the-envelope calculation to express $V_{\max}/L$ in terms of ρ, σ, and b.

Note: It's argued in [2] that these latter three parameters vary little across all organisms (we're mostly thinking about running organisms here), and so finding $V_{\max}/L$ as a function of them indicates that $V_{\max}/L$ is also roughly constant.

6. Use the Buckingham π theorem to reproduce G. I. Taylor's finding the energy of an atom bomb E is related to the density of air ρ and the radius of the blast wave R at time t:

$$E = \text{constant} \times \rho R^5/t^2.$$

   In constructing the matrix, order parameters as E, ρ, R, and t and dimensions as L, T, and M.

7. Surface area of allometrically growing Minecraft organisms:

   Let's consider animals as parallelepipeds (e.g., the well known box cow), with dimensions L_1, L_2, and L_3 and volume $V = L_1 \times L_2 \times L_3$.

   As we vary in scale of organism, let's assume the lengths scale with volume as $L_i = c_i^{-1} V^{\gamma_i}$ where the exponents satisfy $\gamma_1 + \gamma_2 + \gamma_3 = 1$ and the c_i are prefactors such that $c_1 \times c_2 \times c_3 = 1$. Let's also arrange our organisms so that $\gamma_1 \le \gamma_2 \le \gamma_3$.

   (a) Show that our definitions mean that indeed $L_1 \times L_2 \times L_3 = V$.
   (b) Write down the γ_i corresponding to isometric scaling.
   (c) Calculate the surface area S of our imaginary beings.
   (d) Show how S behaves as V becomes large (i.e., which term(s) dominate).
   (e) Which sets of γ_i give the fastest and slowest possible scaling?

   Note: surface area is a big deal for organisms and this calculation will matter later in PoCS and/or CoNKS.

8. (a) A parent has two children, not twins, and one is a girl born on a Tuesday. What's the probability that both children are girls?

   See if you can produce both a calculation of probabilities and a visual explanation with shapes (e.g., discs and pie pieces).

   Once you have the answer, can you improve our intuition here? Why does adding the more detailed piece of information of the Tuesday birth change the probability from $1/3$?

   (Assume 50/50 birth probabilities.)

   (b) Same as the previous question but we now know that one is a girl born on December 31. Again, what's the probability that both are girls?

References

[1] T. A. McMahon and J. T. Bonner. *On Size and Life*. Scientific American Library, New York, 1983.

[2] N. Meyer-Vernet and J.-P. Rospars. How fast do living organisms move: Maximum speeds from bacteria to elephants and whales. *American Journal of Physics*, pages 719–722, 2015. pdf