P o C S What's The Story?

Principles of Complex Systems, CSYS/MATH 300
University of Vermont, Fall 2014
Assignment 6 • code name: Idéfix

Dispersed: Thursday, September 30, 2014.
Due: By start of lecture, 1:00 pm, Thursday, October 23, 2014.
Some useful reminders:
Instructor: Peter Dodds
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Office hours: 2:30 pm to 3:45 pm on Tuesday, 12:30 pm to 2:00 pm on Wednesday
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2014-08UVM-300

All parts are worth 3 points unless marked otherwise. Please show all your workingses clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

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1. (3 + 3 + 3 + 3) This question is all about pure finite and infinite random networks

   We'll define a finite random network as follows. Take N labelled nodes and add links between each pair of nodes with probability p.

   (a) i. For a random node i, determine the probability distribution for its number of friends k, $P_k(p, N)$.
   ii. What kind of distribution is this?
   iii. What does this distribution tend toward in the limit of large N, if p is fixed?
   (No need to do calculations here; just invoke the right Rule of the Universe.)

   (b) Using $P_k(p, N)$, determine the average degree. Does your answer seem right intuitively?

   (c) Show that in the limit of $N \to \infty$ but with mean held constant, we obtain a Poisson degree distribution.

   Hint: to keep the mean constant, you will need to change p.

   (d) i. Compute the clustering coefficients C_1 and C_2 for standard random networks.
   ii. Explain how your answers make sense.
   iii. What happens in the limit of an infinite random network with finite mean?

2. (3 + 3)

   Determine the clustering coefficient for toy model small-world networks [1] as a function of the rewiring probability p. Find C_1, the average local clustering coefficient:

$$C_1(p) = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i = \frac{1}{N} \sum_{i=1}^{N} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}$$

   where N is the number of nodes, $a_{ij} = 1$ if nodes i and j are connected, and $\mathcal{N}_i$ indicates the neighborhood of i.

   As per the original model, assume a ring network with each node connected to a fixed, even number m local neighbors ($m/2$ on each side). Take the number of nodes to be $N \gg m$.

   Start by finding $C_1(0)$ and argue for a $(1 - p)^3$ correction factor to find an approximation of $C_1(p)$.

   Hint 1: you can think of finding C_1 as averaging over the possibilities for a single node.

   Hint 2: assume that the degree of individual nodes does not change with rewiring but rather stays fixed at m. In other words, take the average degree of individuals as the degree of a randomly selected individual.

   For what value of p is $C_1 \simeq 1/2$?

   (3 points for set up, 3 for solving.)

References

[1] D. J. Watts and S. J. Strogatz. Collective dynamics of 'small-world' networks. *Nature*, 393:440–442, 1998. pdf